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A Primer on the Kinematics of Discrete Elastic Rods

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To my sister Farha Jawed (MKJ)

To my parents Tjong Frendy Sugito and Tati Malina (AN)

To my wife Christina (OMO'R)

Preface

In the late 2000s, a novel formulation of Kirchhoff's celebrated rod theory was published by Bergou et al. [4]. In this formulation, an elastic rod is discretized into a series of segments (or edges) connecting vertices (or nodes). The edges are free to stretch and rotate relative to their adjacent neighbors. The relative rotations of the cross sections of the rod are modeled with the help of a pair of material vectors that are associated with each edge. The original formulation has been extended in a variety of directions including an extension to viscous threads and sound generation. The discrete elastic rod formulation is computationally cheap and, as a result, is used in computer graphics to render images of hairs and trees and is the technical underpinning behind the *Bristle Brush* feature in Adobe Illustrator and Adobe Photoshop.

Bergou et al.'s discrete elastic rod (DER) formulation uses ideas from the nascent field of discrete differential geometry and concepts such as holonomy from classic differential geometry. As a result, understanding the DER formulation (even for students who have exceptional backgrounds in continuum mechanics) can be challenging. Indeed, initially we were unable to rederive many of the key results in the papers by Bergou et al. [3, 4] and the related works by Audoly et al. [2] and Kaldor et al. [29]. The remarkable simulations in these four papers provided sufficient motivation for us to eventually prove the main results contained in [2–4, 29].

The present Brief is a result of our efforts to understand the DER formulation and we hope that it provides an accessible introduction to this remarkable formulation. We assume that the reader has a background in continuum mechanics at the level of Chadwick [8] or Gurtin [20]. The Brief starts with a pair of motivational examples. We then proceed to give a rapid summary of Kirchhoff's rod theory before discussing a discretized space curve and three frames that can be associated with it. Next, derivations of gradients and variations for various kinematical quantities that have appeared in the literature are discussed. One unusual feature of the DER formulation is the use of holonomy to help determine the twist of the rod. We devote an entire chapter to discussing results from differential geometry of spherical triangles and spherical quadrilaterals that are used to determine the twist of the rod.

The final chapter synthesizes the kinematical results and shows how they are used to formulate a set of ordinary differential equations for the position vectors of the nodes of the rod and the twisting of the edges. To help the reader, we present several examples of classic problems in the theory of rods that are solved using the discrete elastic rod formulation.

The C++ source code for the discrete elastic rod formulation discussed in this Brief can be found at

http://www.cs.columbia.edu/cg/elastic_coiling/

Source code for the input files used for the examples discussed in the Brief can be accessed at

http://dynamics.berkeley.edu/

We received a total of $500 from Springer-Nature for publishing this Brief. These funds have been donated to an organization that supports LGBTQ people who are held in immigrant detention in the United States: *Mariposas Sin Fronteras*.

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Acknowledgments

The discrete elastic rod formulation discussed in these pages was first brought to the attention of Oliver O'Reilly by Arun Srinivasa (Texas A&M University) during the Annual Meeting of the Society of Engineering Science at the University of Maryland in the fall of 2016. The formulation's capability of modeling knotted structures and potential application to simulating soft robot locomotion were the primary reasons that O'Reilly and Alyssa Novelia then began studying the papers by Bergou et al. [3, 4]. They had the good fortune at the time to be collaborating with Carmel Majidi's group at Carnegie Mellon University on soft robots. The third author, M. Khalid Jawed, was a postdoctoral researcher with Majidi's group whose Ph.D. thesis [25] at the MIT used Bergou et al.'s discrete elastic rod formulation to solve a variety of problems. Thus, by a series of fortunate coincidences and the support of colleagues, work on this Brief commenced. Our initial goal was to write a set of notes explaining all of the technical details in Bergou et al. [3, 4] but as the notes expanded substantially beyond our original expectations, we realized they would make a Brief that researchers on Bergou et al.'s discrete elastic rod formulation might hopefully find useful.

Part of the reason the notes expanded beyond our original horizon lay in our difficulty comprehending the holonomy results (7.1) and (7.2). These results play a central role in computing the torsional strain in the discrete elastic rod formulations presented in the papers [3, 4] and discussed in this Brief. We would not be able to explain the holonomy results were it not for the exceptionally helpful comments [65] and feedback provided by Etienne Vouga (University of Texas at Austin). Khalid Jawed is also grateful to Fang Da (Columbia University), Eitan Grinspun (Columbia University), Jungseock Joo (UCLA), Noor Khouri (MIT), and Pedro Reis (EPFL) who were involved in the adaptation, implementation, and experimental validation of the discrete elastic rod formulation with application to engineering problems.

As mentioned earlier, our primary motivation to study the discrete elastic rod formulation came from a desire to simulate the locomotion of soft robots. This research on soft robots is supported by grant number W911NF-16-1-0242 from the U.S. Army Research Organization administered by Dr. Samuel C. Stanton. Alyssa

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It has been a pleasure working with Michael Luby at Springer US on this Brief, and we are delighted that he chose to publish our work. We are also grateful that digital copies of this work will be freely available to students and faculty from the publisher's website.

It is impossible to remove all grammatical and typographical errors in a manuscript of the present size. We thank Evan Hemingway (University of California, Berkeley) for his careful proofreading and comments on an earlier draft of this Brief. The responsibility for all remaining errors, typographical and technical, rests on our shoulders and we would be most grateful if you could bring them to our attention.

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