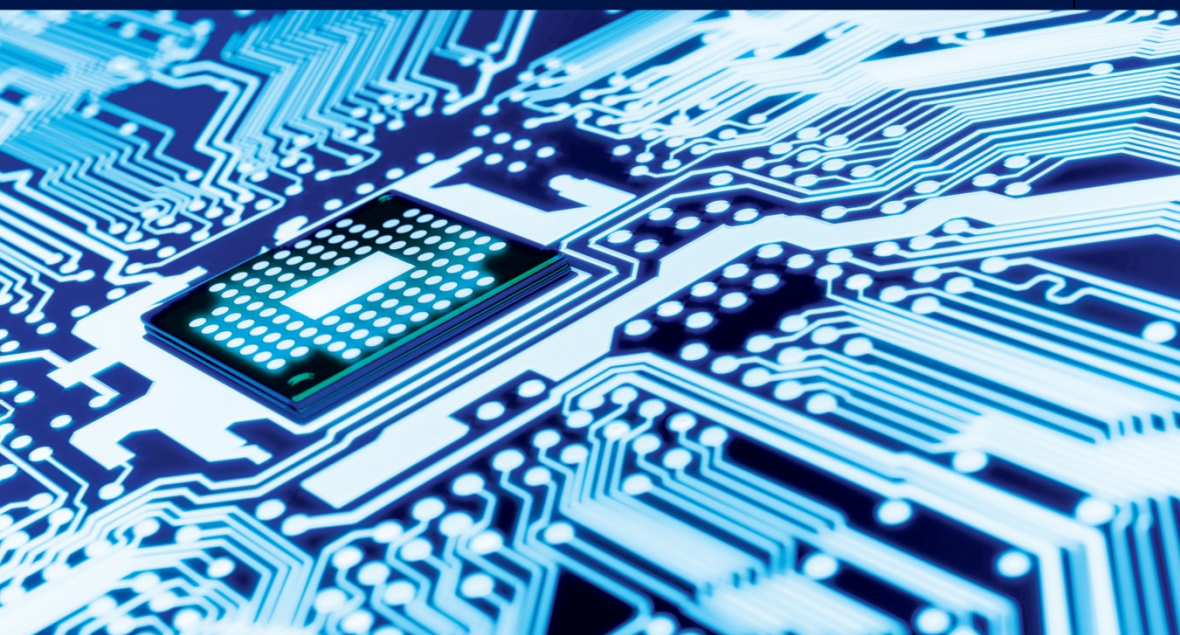


ELECTRONICS ENGINEERING SERIES



Fundamentals of Electronics 3

*Discrete-time Signals and Systems,
and Quantized Level Systems*

Pierre Muret

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First published 2018 in Great Britain and the United States by ISTE Ltd and John Wiley & Sons, Inc.

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USA

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Library of Congress Control Number: 2018930834

British Library Cataloguing-in-Publication Data
A CIP record for this book is available from the British Library
ISBN 978-1-78630-183-3

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Preface

Today, we can consider electronics to be a subject derived from the theoretical advances achieved during the 20th Century in areas comprising the modeling and conception of components, circuits, signals and systems, together with the tremendous development attained in integrated circuit technology. However, such development led to something of a knowledge diaspora that this work will attempt to contravene by collecting the general principles at the center of all electronic systems and components, together with the synthesis and analysis methods required to describe and understand these components and subcomponents. The work is divided into three volumes. Each volume follows one guiding principle from which various concepts flow. Accordingly, Volume 1 addresses the physics of semiconductor components and the consequences thereof, that is, the relations between component properties and electrical models. Volume 2 addresses continuous time systems, initially adopting a general approach in Chapter 1, followed by a review of the highly involved subject of quadripoles in Chapter 2. Volume 3 is devoted to discrete-time and/or quantized level systems. The former, also known as sampled systems, which can either be analog or digital, are studied in Chapter 1, while the latter, conversion systems, are addressed in Chapter 2. The chapter headings are indicated in the following general outline.

Each chapter is paired with exercises and detailed corrections, with two objectives. First, these exercises help illustrate the general principles addressed in the course, proposing new application layouts and showing how theory can be implemented to assess their properties. Second, the exercises act as extensions of the course, illustrating circuits that may have been described briefly, but whose properties have not been studied in detail. The

first volume should be accessible to students with a scientific literacy corresponding to the first 2 years of university education, allowing them to acquire the level of understanding required for the third year of their electronics degree. The level of comprehension required for the following two volumes is that of students on a master's degree program or enrolled in engineering school.

In summary, electronics, as presented in this book, is an engineering science that concerns the modeling of components and systems from their physical properties to their established function, allowing for the transformation of electrical signals and information processing. Here, the various items are summarized along with their properties to help readers follow the broader direction of their organization and thereby avoid fragmentation and overlap. The representation of signals is treated in a balanced manner, which means that the spectral aspect is given its proper place; to do otherwise would have been outmoded and against the grain of modern electronics, since now a wide range of problems are initially addressed according to criteria concerning frequency response, bandwidth and signal spectrum modification. This should by no means overshadow the application of electrokinetic laws, which remains a necessary first step since electronics remains fundamentally concerned with electric circuits. Concepts related to radio-frequency circuits are not given special treatment here, but can be found in several chapters. Since the summary of logical circuits involves digital electronics and industrial computing, the part treated here is limited to logical functions that may be useful in binary numbers computing and elementary sequencing. The author hopes that this work contributes to a broad foundation for the analysis, modeling and synthesis of most active and passive circuits in electronics, giving readers a good start to begin the development and simulation of integrated circuits.

Outline

1) Volume 1: Electronic Components and Elementary Functions [MUR 17a].

- i) Diodes and Applications
- ii) Bipolar Transistors and Applications
- iii) Field Effect Transistor and Applications
- iv) Amplifiers, Comparators and Other Analog Circuits

2) Volume 2: Continuous-time Signals and Systems [MUR 17b].

i) Continuous-time Stationary Systems: General Properties, Feedback, Stability, Oscillators

ii) Continuous-time Linear and Stationary Systems: Two-port Networks, Filtering and Analog Filter Synthesis

3) Volume 3: Discrete-time Signals and Systems, and Quantized Level Systems.

i) Discrete-time Signals: Sampling, Filtering and Phase Control, Frequency control circuits

ii) Quantized Level Systems: Digital-to-analog and Analog-to-digital Conversions

Pierre MURET
February 2018

Introduction

This third volume covers signals and systems dealing with variables or quantities that are discrete or quantized. This leads to distinguishing two chapters: the first concerning the discrete-time case and the second that of discrete (or quantized) levels. The electronic circuits and applications implemented are of analog, digital or mixed nature, and some make use of both types of discretization. Similar to the previous volume, it is fundamental to explain the signals and their properties in detail as well as the basic circuits that transform these signals before considering the functions performed by more complex arrangements, which we will refer to as systems.

The first chapter begins with the study of discrete-time signals, obtained by sampling continuous-time signals, first by means of ideal sampling, then by actual sampling or by using interpolation. The use of the Fourier transform is essential and allows us to demonstrate, on the one hand, equivalences between a discrete variable in one domain and the periodic nature of the quantity depending on the dual variable in the other domain and, on the other hand, the fundamental theorem which determines the possibility to preserve (or not) all the information contained within a signal when shifting from continuous-time to discrete-time domains, called the sampling theorem or the Shannon theorem. Basic analog circuits are described. The other transforms, relevant in cases where discretization can be applied in both time and frequency domains, are also indicated since these are the ones that are used in practice. Next follows the study of the measurement of the time delay and of the phase shift between periodic signals in circuits comprising basic analog and logic functions, which are now widely used. Since this measurement is only achieved once per period,

the measured time and phase shifts become discrete quantities. However, in many cases, the approximation which consists of only considering the continuous-time domain, obtained by interpolation and assuming that stationarity is preserved, makes it possible to detail the operation of the analog phase-locked loop (PLL) and the correction strategies of this loop system. This approximation is also a means to establish a relation between phase and frequency, which proves very useful for the applications subsequently addressed. The PLL has undergone overly significant development since the 1970s, because it has allowed transformations of signals and their properties, which were very difficult or impossible to achieve without it, namely in areas such as instrumentation, computer sciences and communications (wireless broadcasting, wireline transmission, etc.) destined for conveying information. The main functions, grouped under the term “frequency synthesis,” are described. Digital PLLs are also covered in detail.

The last part of this chapter is dedicated to samples systems analysed with the z - transform (ZT), just as with the Laplace transform in the case of continuous-time systems. The properties of the ZT are carefully presented in order to provide all the tools that will be used in the end of the first chapter and in the next chapter, including the new meaning for the plane of the complex z variable. The study of switched-capacitor circuits is then discussed in a didactic manner, because it is based on the principles of electrostatics which are simple but nonetheless not necessarily familiar to the readers when applied to capacitor networks. These circuits have experienced major developments because of the possibility to integrate them naturally within CMOS technology and they constitute the basic building blocks for analog sampled filters, and modern digital-to-analog conversion (DAC) or analog-to-digital conversion (ADC). The first chapter logically proceeds through the study of two types of sampled filters (with infinite impulse response [IIR] and with finite impulse response [FIR]) and their properties, as well as approximations useful to recover second order transfer functions in the frequency domain. The notion of transmittance in the plane of the z variable is developed for all the basic functions useful for building these filters. The synthesis methods of these filters are briefly described in order to introduce the readers to the use of numerical functions available in MATLAB® or SciLab software. On the one hand, it should be noted that FIR filters allow one to access properties inaccessible to IIR and analog filters and, on the other hand, that all processing and analyses based on the ZT can be applied without the need for specifying if the technology being used for the implementation is either analog or digital. In the first case, these

are switched-capacitor circuits that were previously studied and that are used, while the basic principle of the numerical functions necessary for the second case is described to conclude this chapter. Finally, we proceed with showing the power of state variable analysis in the discrete-time domain and in the plane of the z variable for sampled systems. In effect, it provides direct access to the mathematical modeling of these systems characterized by their fundamental parameters, namely transmittance poles in the plane of the z variable. Provided that the computation of successive samples is performed by computerized means, it enables, in addition, the avoidance of all the approximations previously employed. This model paves the way for the exact computation of the sampled time response in the case of nonlinear systems and/or undergoing frequency variations strong enough so that certain parameters of their transfer characteristics depend thereupon, which is the case in PLLs.

In the second chapter, we consider the principles and implementations of systems dealing with quantized signals, as is the case for ADCs and DACs. The digital quantity is a number encoded onto n bits in the binary system. The quantization of a signal induces some degradation, the first of which being quantization noise, which is presented and analyzed. The other imperfections, which can be likened to errors disturbing the original signal after its conversion, are then connected to the electrical characteristics of these converters. DAC is detailed through the various principles that can be implemented, on the one hand, with resistor ladder networks, historically the first ones to have been used, and, on the other hand, with switched-capacitor circuits, well-adapted to CMOS technology. The reverse conversion is then presented along with its different possible principles, which all have in common the development of an approximation of the analog quantity in digital form, then reconverted and compared to the original quantity, increasingly more precise during the successive stages of the conversion. Looped systems are therefore the main subject. In general, the complexity of systems increases if it is desirable to reduce the conversion time, and the quality of the analog comparator (or analog comparators) determines a very significant part of the accuracy of the conversion.

Finally, “sigma-delta” or “delta-sigma” conversions are addressed, which are the most recent in the field. In its basic principle, the “delta-sigma” conversion is easily understood if deduced from that using a ramp voltage and a count. However, when it is desirable to increase the performance of this type of converter, one is confronted, on the one hand, with a significant sophistication of the modeling, especially in optimizing the signal-to-noise

ratio and, on the other hand, with the stability problems of the loop because it is necessary to increase the order of the filter(s) beyond two. In order to solve these problems, a large number of concepts presented in the first chapter are utilized. The core of this type of converter is formed by the modulator, a closed system that processes signals with a significantly much lower number of bits than the initial or final number, desired or imposed, but at a much faster rate than that of the input or the output. The operations carried out by the modulator yield a loss of resolution that will be recovered later by the decimator filter, and a displacement of the noise spectrum toward higher frequencies where it will be more easily filtered. The first-order modulator is examined in the first place and then followed by a generalization to higher-order modulators, which makes it possible to establish the transfer functions for the signal and for the quantization noise. Several types of stable modulator are examined. The role and the way to build the decimator filter are discussed and, based on this analysis, a scaling of the different frequencies to be used can be proposed. Finally, the principle and the implementation of the digital-to-analog “delta-sigma” converter are presented. Although in theory it is deduced from the ADC by swapping digital and analog functions, it is preferable to describe it by proceeding to digital resolution and rate conversions before the final DAC and the associated filtering, which conforms to the practical implementation.

This entire volume thus presents discrete-time and quantized-level signals and systems, the transformations of these signals into continuous-time or continuous-level signals as well as reverse transformations, analog, digital or mixed circuits, effective to achieve these operations and the models capable of calculating, predicting and scaling the responses of these systems. Corrected exercises are provided in order to address specific cases not fully detailed in the course, in order to illustrate it, to complete it and to show the methods adapted to solve the presented problems.

Discrete-time Signals and Systems

1.1. Discrete-time signals

In the discretized time domain, where only specific moments are taken into consideration and identified, signals are represented by series of samples.

1.1.1. “Dirac comb” and series of samples

The “Dirac comb” distribution is the basic tool in the discrete-time domain.

1.1.1.1. Dirac comb in the phase space and in the time domain

Signals can be expressed by means of linear combinations of complex exponentials $\exp(jn\theta)$ in the time and frequency domains (see Chapter 1 of Volume 2 [MUR 17b]), n being an integer and θ an angle proportional to the angular frequency-time product. In the case of a linear combination of $2N + 1$ terms having the same amplitude, n varying from $-N$ to $+N$,

$I_N(\theta) = \sum_{n=-N}^N \exp(jn\theta)$, a function $I_N(\theta)$ is obtained which gives very sharp

maxima (or lines) every time $\theta = 2k\pi$ (Figure 1.1) with k integer, because it is the only case in which the images of $\exp(jn\theta)$ in the complex plane are colinear and add up while their sum tends to cancel out when $\theta \neq 2k\pi$:

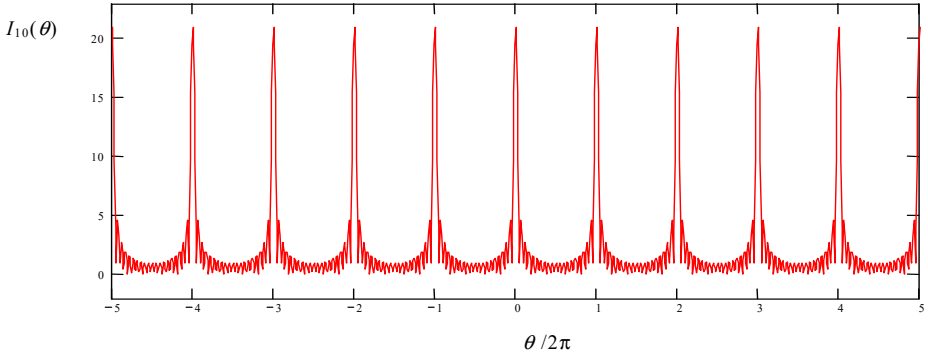


Figure 1.1. Function $I_N(\theta)$ with $N = 10$

The “Dirac comb” is a series of periodic Dirac impulses and can be defined from $I_N(\theta)$ by taking the limit for $N \rightarrow \infty$.

Provided that $\lim_{T_0 \rightarrow \infty} \left[2f_0 \frac{\sin(2\pi f_0 t)}{2\pi f_0 t} \right] \rightarrow \delta(t)$ and

$\lim_{T_0 \rightarrow \infty} \left[2T_0 \frac{\sin(2\pi T_0 f)}{2\pi T_0 t} \right] \rightarrow \delta(f)$ (see Chapter 1 and the Appendix in

Volume 2 [MUR 17b]), or more generally $\lim_{n \rightarrow \infty} \left[n \frac{\sin(n\theta)}{n\theta} \right] \rightarrow \delta(\theta)$, we define the “Dirac comb” distribution by means of a sum of Dirac impulses (Figure 1.2):

$$\sum_{k=-\infty}^{\infty} \delta\left(\frac{\theta}{2\pi} - k\right) = \lim_{N \rightarrow \infty} \sum_{n=-N}^N \exp(jn\theta) = \lim_{N \rightarrow \infty} \frac{\sin\left(\frac{2N+1}{2}\theta\right)}{\sin\left(\frac{\theta}{2}\right)}$$

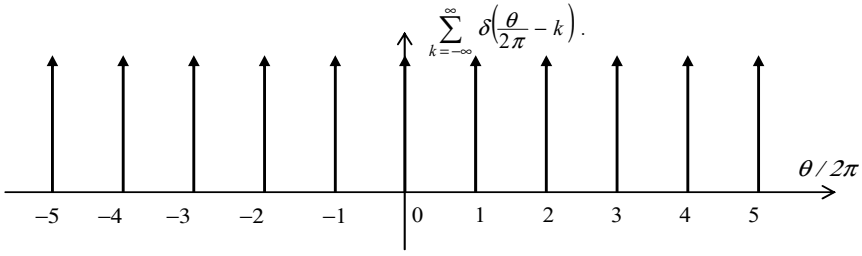


Figure 1.2. Dirac comb in the phase space

If $\theta = 2\pi f_0 t$, where $\frac{1}{T_0} = f_0$ is a fixed frequency and t is the time variable, we obtain, by changing variable in the distribution $\sum \delta_k$, a Dirac comb in the time domain $T_0 \sum \delta_{kT_0} = T_0 \sum_{k=-\infty}^{\infty} \delta(t - kT_0) = \lim_{N \rightarrow \infty} \sum_{n=-N}^N \exp(j2\pi n f_0 t)$ with a single line whenever $f_0 t = k$ (k being an integer), that is $t = k T_0$ (Figure 1.3).

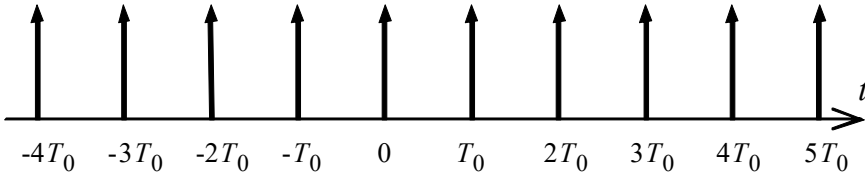


Figure 1.3. Time "Dirac comb" distribution of time period T_0

$T_0 \sum \delta_{kT_0}$ is dimensionless and $\sum \delta_{kT_0}$ has the dimension of a frequency, in order to preserve the dimension of the function to which the distribution is applied.

1.1.1.2. Frequency Dirac comb

Alternatively, if $\theta = 2\pi f T_0$, where $\frac{1}{f_0} = T_0$ is a fixed period and f is the frequency variable, we have a Dirac comb in the frequency domain

$f_0 \Sigma \delta_{nf_0} = f_0 \sum_{n=-\infty}^{\infty} \delta(f - nf_0) = \lim_{N \rightarrow \infty} \sum_{k=-N}^N \exp(j2\pi k f T_0)$ with a single line every time $f T_0 = n$ (n being an integer) or $f = n f_0$ (Figure 1.4).

$\Sigma \delta_{nf_0}$ has the dimension of time, and the presence of the factor $\frac{1}{T_0} = f_0$ can be verified by calculating the coefficient of the Fourier series of the frequency Dirac comb, which is periodic.

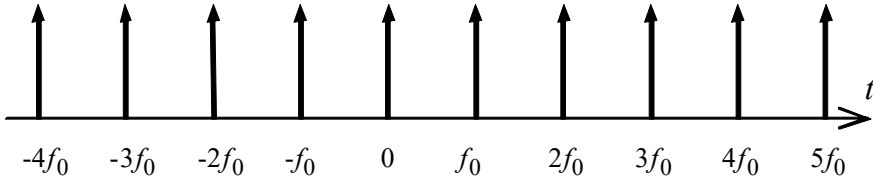


Figure 1.4. Frequency “Dirac comb” distribution of frequency period f_0

1.1.1.3. Fourier series

These series of “Dirac comb” impulses $\Sigma \delta_n$ do not directly represent temporal signals or their spectrum but rather virtual signals, since δ_n is not a regular function. These are operators associated with the “Dirac comb” distribution acting on a time or frequency function g , which can generally be denoted as a functional associated to $\Sigma \delta_n$: $\langle T_{\Sigma \delta_n}, g \rangle$ (see Appendix in Volume 2 [MUR 17b]). We can also consider them as bases of the vector spaces of series or discrete functions upon which the continuous-time signal or the continuous-frequency spectrum is projected by computing either the distribution applied to the signal or to the spectrum, or the distribution applied to the product of the signal or spectrum by $\exp(\pm j2\pi f t)$ when looking for the Fourier transform (FT) or its inverse.

The use of the “frequency Dirac comb” distribution corresponds to the decomposition of a periodic signal into Fourier series, already demonstrated in Chapter 1 of Volume 2 [MUR 17b] and derived from a description of the periodic time signals built on exponential function bases.

As a matter of fact, by taking the FT of a periodic signal $y_{T_0}(t) = y_{T_0}(t + kT_0)$ in the time domain, then reducing the interval to the period T_0 and finally by performing a summation over all periods, we find:

$$\begin{aligned} \text{TF}[y_{T_0}(t)] &= \int_{-\infty}^{\infty} y_{T_0}(t) \exp(-j2\pi f t) dt = \sum_{k=-\infty}^{\infty} \int_{-T_0/2}^{T_0/2} y_{T_0}(t) \exp[-j2\pi f(t + kT_0)] dt \\ &= \left[\int_{-T_0/2}^{T_0/2} y_{T_0}(t) \exp(-j2\pi f t) dt \right] \sum_{k=-\infty}^{\infty} \exp(-j2\pi f k T_0) \end{aligned}$$

expression of the product of the Dirac comb $f_0 \sum_{n=-\infty}^{\infty} \delta(f - n f_0) = \lim_{N \rightarrow \infty} \sum_{k=-N}^N \exp(j2\pi k f T_0)$ by the integral between

brackets, to be evaluated for the only line frequencies $f = n f_0 = \frac{n}{T_0}$. It is therefore possible to place the result of the integral calculated at $f = n f_0$ inside the summation by replacing f by $n f_0$ and write:

$$\text{TF}[y_{T_0}(t)] = \sum_{n=-\infty}^{\infty} c_n \delta(f - n f_0) \text{ with the complex Fourier coefficient}$$

$$c_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} y_{T_0}(t) \exp[-j2\pi n f_0 t] dt.$$

The inverse FT again yields the sum of the complex Fourier series, calculated by the distribution associated with the frequency Dirac comb applied to $\exp(j2\pi f t)$:

$$y_{T_0}(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} c_n \delta(f - n f_0) \exp(j2\pi f t) df = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_0 t).$$

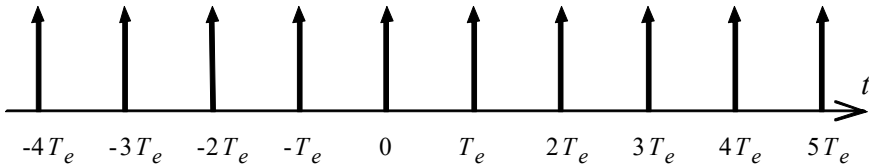


Figure 1.5. Time “Dirac comb” distribution of period T_e