SCIENCE, SOCIETY AND NEW TECHNOLOGIES SERIES

ENGINEERING, ENERGY AND ARCHITECTURE SET

Volume 3

Fluid Mechanics in Channel, Pipe and Aerodynamic Design Geometries 2

Christina G. Georgantopoulou
George A. Georgantopoulos

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Fluid Mechanics in Channel, Pipe and Aerodynamic Design Geometries 2

...To our family

Depy
Andreas
Nikos
Giannis
Lilian

...as without their support none of this would have ever been possible for us

Engineering, Energy and Architecture Set

coordinated by
Lazaros E. Mavromatidis

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Contents

Preface

This book presents an extended and detailed analysis of both the flow phenomena in closed and open channels and the flows around solid bodies. It comprises two volumes. This book is a specialized resource for those students, engineers and researchers who want to focus on the industrial applications of flows and study the fascinating world of internal and external flow phenomena.

We have both had extensive experience in teaching, studying and researching fluids since the completion of our respective PhD theses. We felt that it was time to write about the practical and analytical aspects of flow applications, all of which can be applied in industrial flows, to support researchers, engineering students and industrial engineers in the field of fluids in order to optimize their work in "flows".

For the first author, the "fluids direction" began in the early stages of her PhD thesis study in Computational Fluid Dynamics in 1998 at the National Technical University of Athens. The second author's knowledge of the fluids' path is very extensive, obtained from more than 45 years of studies and work involved in his PhD thesis and further research work at the University of Patras, as well as through his position as Professor of Aerodynamics at the Hellenic Air Force Academy, spanning more than 35 years.

We have both gained substantial experience in Fluid Mechanics research through numerous publications, presentations at international conferences, academic textbook authoring, teaching through international experiences and collaborations. However, we felt that more should be offered to the Fluid Mechanics community, and hence this book.

Although we both have experience in writing for academic textbooks, this is our first publication that caters to international students, researchers and engineers, considering the industrial phenomena that are met in international industries and we have tried to present most of the applications in flows inside or around bodies. This book is based on books written previously by us on Fluid Mechanics and on Aerodynamics, but for the first time our work focuses on the practical aspects of industrial internal and external flows.

Christina, the first author, offers this textbook to the Bahrain polytechnic engineering students and all the industrial delegates who have worked with her in “flows” for many years. She also wishes to express her appreciation for her colleagues, namely Payal Modi, for the thousands of hours of constructive discussions and collaborations in fluids aspects, to Lazaros E. Mavromatidis for his support during the publishing procedure, to her father George who has been her mentor for all these years and to Stephanie Sutton and Amerissa Kapela for their continuing support with the quality of the academic English language. Additionally, George, the second author, wishes to share his more than 40 years of experience in fluids with the fluids community around the world and support them in their “flows” work as best he can.

We both have a special sentimental feeling for this book in that we are extremely proud that we have been able to write, publish and offer it to you, hoping that it will really support you in your fluids journey. We have both worked on fluids with a passion not only for our students, but also to honor our colleagues around the world. We are equally happy to say that the Fluid Mechanics community has been served by the same family for more than 40 years. We hope that we will be physically and mentally healthy to continue to serve our students and support our colleagues in the fluids aspects in the future.

We hope that you will enjoy this book and be engaged with the fascinating world of flows.

Christina G. GEORGANTOPOULOU
George A. GEORGANTOPOULOS
February 2018

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Pipe Networks

1.1. Introduction

Pipe networks, which are interconnected and often have a netting structure, are used for transportation or even distribution of fluids from their storage or production areas to various other areas for certain purposes.

These networks are used in everyday life, such as plumbing networks of water transportation for productive applications such as fire extinguishing, networks of natural gas transportation, networks of fluid and gas transportation, and networks of water pipes, wet waste and compressed air.

Some of these networks are simple piping systems equipped with flow adjustment devices, whereas others are complicated, such as fluid distribution networks. Some of the most complicated networks are fire extinguishing networks (because of the fluid used for fire extinguishing), water distribution networks and plumbing networks.

Depending on the complication of mixed networks and the form they take for the feasibility of the distribution service, we distinguish them into the following three categories:

1) The tree-system-type pipe networks: these networks are characterized by the presence of a central pipe from which other pipes are branched (pipes of distribution) with a gradual decrease in their cross-section, taking the form of a tree (tree system), as shown in Figure 1.1.

2) The grid pattern: during development the pipes are formed into this pattern, resembling a chessboard, which covers the whole area of the distribution, with a decrease in their cross-sections with respect to the distance (grid system), as shown in Figure 1.2.

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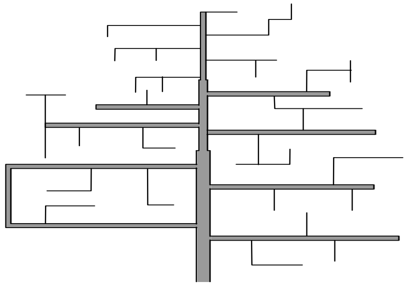


Figure 1.1. *Tree-system-type pipe network*

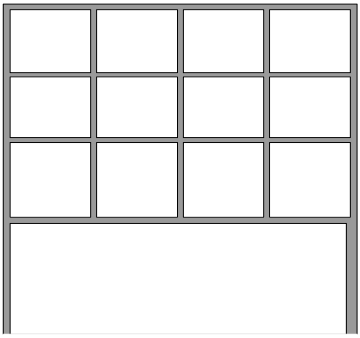


Figure 1.2. *Grid pattern networks*

3) The loop pattern: this pattern contains a central pipe, forming *loops* with smaller pipes at low flow rates (loop system), as shown in Figure 1.3.

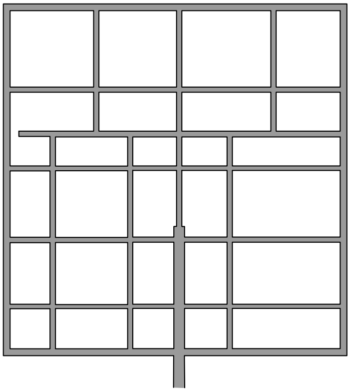


Figure 1.3. *Loop pattern network*

Of the three forms of networks, the loop and grid systems show high reliability due to their flexibility, expandability and ability to offer multiple paths to the fluid.

In general, in all the aforementioned network systems, we distinguish three groups of pipelines:

1) *Transportation lines*: these pipes transfer the fluid from the storage or production area to the distribution area.

2) *Central pipes*: these pipes transfer the fluid to the target area, e.g. transfer of water to a town or village or transfer of natural gas to an industrial installation.

3) *Supply lines*: these pipes of small diameters transfer the fluid from central pipes to the users.

Thus, the entire distribution system consists of pipes, valves and pumps. The fundamental aim of a network system is to supply sufficient amounts of fluid to target areas with desired pressures and flow rates. Therefore, the choice of materials, the diameter of pipes and the formation of pipelines in networks are mostly influenced by the necessity of ensuring sufficient pressures and flow rates, despite installation costs and operations.

1.2. Calculation of pipe networks

Simple pipe networks have procedures for connecting the pipes in a row or in parallel, as described in Chapter 5 of Volume 1 [GEO 18]. However, the same is not true for complicated pipe networks. The schematic representation of a typical plumbing network is shown in Figure 1.4.

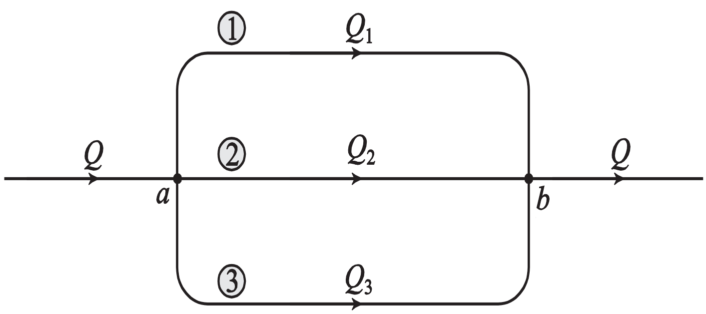


Figure 1.4. *Schematic representation of a pipe network*

The geometric convergence of three or more pipes is called a network *node*. In technical applications, nodes with more than four *branches* do not exist. Nodes with three branches (which are most common in practice) are classified into *branch nodes* (Figure 1.5a) and *convergence nodes* (Figure 1.5b). In branch nodes, the incoming branch with a flow rate Q is divided into two branches with flow rates Q_1 and Q_2, while in convergence nodes, two branches with flow rates Q_1 and Q_2 converge into one branch with a flow rate Q.

When the fluid passes through the node, there is some kind of energy loss, which can be attributed to a decrease in the cross-sectional areas of branches 1 and 2, so that the average velocity of the fluid in all the three branches of the node is approximately the same ($V \approx V_1 \approx V_2$), and the walls of the branches are rounded instead of being sharp, to avoid higher energy losses.

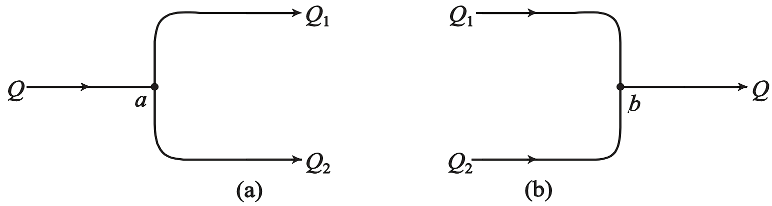


Figure 1.5. *Pipe network nodes*

Minor energy losses at a node can be calculated either by the method of losses coefficient or by the method of equivalent lengths. In Table 1.1, typical values of the losses coefficients for nodes T−90° with constant diameter are given. Furthermore, in Table 1.2, representative values of the equivalent lengths in diameters for *tees* are given. For a flow in parallel connection, the following two basic rules are applicable:

1) For an incompressible flow in a node, the algebraic sum of flow rates in its branches is zero. This means:

$$Q = \sum_{i=1}^{i=k} Q_i \qquad [1.1]$$

for the flow rates of the k branches of the node. During the addition of flow rates, streams entering the node are considered positive, while those leaving the node are considered negative. By applying this rule at node α, shown in Figure 1.6, we get:

$$Q = Q_1 + Q_2 + Q_3 \qquad [1.2]$$

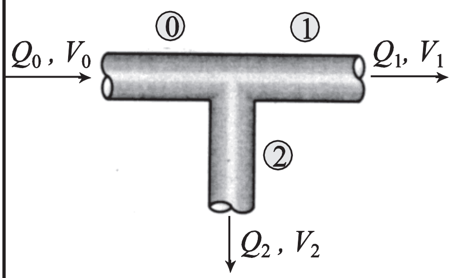
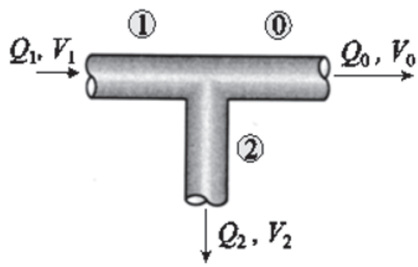
BRANCH NODE ($Q_0 = Q_1 = Q_2$)							**CONVERGENCE NODE** ($Q_1 = Q_2 = Q_0$)						
													
Q_2/Q	**0**	**0.2**	**0.4**	**0.6**	**0.8**	**1,0**	**Q_2/Q**	**0**	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
$K_{m,1}$	0.04	0.08	0.05	0.07	0.21	0.35	$k_{m,1}$	0.04	0.17	0.30	0.41	0.51	0.60
$K_{m,2}$	0.95	0.88	0.89	0.95	1.10	1.28	$k_{m,2}$	-1.20	-0.40	0.08	0.47	0.72	0.91
$h_{m,0\to 1} = k_{m,1}\frac{V_0^2}{2g}$ $h_{m,0\to 2} = k_{m,2}\frac{V_0^2}{2g}$							$h_{m,0\to 1} = k_{m,1}\frac{V_0^2}{2g}$ $h_{m,0\to 2} = k_{m,2}\frac{V_0^2}{2g}$						

Table 1.1. *Coefficient of losses for T-90 nodes*

KIND OF APPLIANCE	ℓ_e / d	KIND OF APPLIANCE	ℓ_e / d
Sudden dilatation $(d_1 / d_2 = 1/2)$	20	Tee, entrance from main line	20
Sudden systole $(d_2 / d_1 = 1/2)$	12	Tee, entrance from branch	60
Borda mouth	28	Valves (totally open):	
Mouth with sharp lips	18	back flow with swing	135
$45°$ curve	16	Angular	145
Standard $90°$ curve	30	Hydrant	18
$90°$ curve of large radius	20	Butterfly $(d \geq 6in)$	20
$90°$ angle	60	Sliding	13
$180°$ curve	65	Spherical	340

Table 1.2. *Equivalent length values*

Relationship [1.1] is a mathematical expression of the node theorem, also known as the first law of Kirchhoff in the theory of electrical networks

2) For a steady flow in a hydraulic network, for example, in the network shown in Figure 1.6, the total head loss h_ℓ between the nodes ***a*** and ***b*** is the same as the respective head losses, $h_{\ell,i}$, in each branch *i* of the network, which means:

$$h_\ell = h_{\ell,i} = h_{\ell,1} = h_{\ell,2} = h_{\ell,3} \qquad [1.3]$$

where $h_{\ell,1}, h_{\ell,2}$ and $h_{\ell,3}$ are the heads of the energy losses in branches 1, 2 and 3, respectively, for the corresponding flow rates Q_1, Q_2 and Q_3. Equation [1.3] constitutes the mathematical statement of the energy principle of the hydraulic network

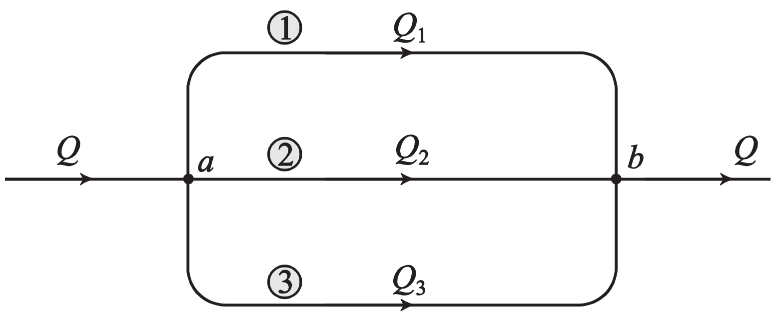


Figure 1.6. *Nodes and branches of a pipe network*

The procedure followed for obtaining the solutions of these problems depends on the type of information asked. Therefore, if the total head loss of the flow is known, then it is easy to calculate individual flow rates Q_i and finally their sum.

The reversed problem is solved with successive approximations because the distribution of the flow rate Q in the individual branches of the network is not known. In fact, in the first approximation, we consider zero energy loss at the nodes and, if they are considered important, we add them in the second approximation. In more complex hydraulic networks, *the balance sheet method*, *head losses in loops* and *flow rates in nodes* are applied. It is difficult to obtain the solution of such network problems; it can be obtained with a suitable computer program.

However, in addition to this form of network, there are more complicated distribution networks, particularly the *grid* and *loop patterns*. A loop with one inlet and two exits, as shown in Figure 1.7, is impossible to obtain by the methods that we developed in Chapter 5 of Volume 1 [GEO 18].

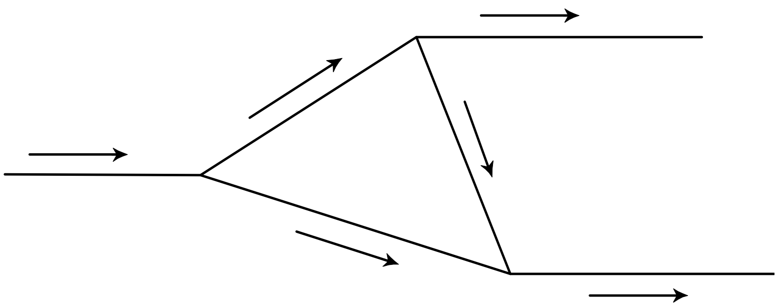


Figure 1.7. *Loop pattern network*

Before we develop a method for solving these types of networks, we should state the basic relation of these calculations. Therefore, considering that a network consists of pipes (*branches*) with *nodes* and forms *closed circuits* or *loops*, we will individually examine the relations of the nodes and the loops.

Viewing a network macroscopically and applying the *continuity equation* to the network, we have:

$$\Sigma \dot{m}_{ent} = \Sigma \dot{m}_{ex} \qquad [1.4]$$

and for incompressible fluids, the equality of flow rates is also applicable:

$$\Sigma Q_{ent} = \Sigma Q_{ex} \qquad [1.5]$$

A respective relationship is applicable for every node of the network. If we conventionally pre-label the flow rates (+ for entrance to the node and − for exit from the node), we have:

$$\Sigma Q_{node} = 0 \qquad [1.6]$$

For the node shown in Figure 1.8:

$$Q_1 - Q_2 + Q_3 - Q_4 - Q_5 = 0$$

or $Q_1 + Q_3 = Q_2 + Q_4 + Q_5$ [1.7]

In addition to the points of the branch or the nodes, a network is characterized by closed circuits or loops. A loop is a closed path formed by the sum of successive pipes, which will lead us back to the starting point if we follow them. Thus, in the pipe shown in Figure 1.9, starting from A and following a clockwise route, we will return to A again: A–B–C–D–A. The pipes that constitute the loop are called branches. For the formation of a loop, at least two branches are required.

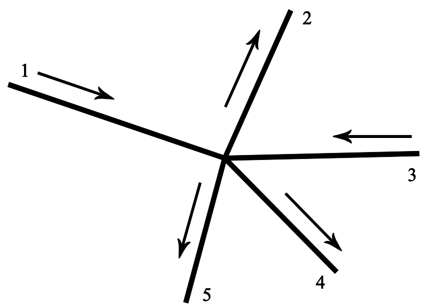


Figure 1.8. *Loop pattern of network*

The loop shown in Figure 1.9 constitutes four branches (AB, BC, CD and DA) and four nodes (A, B, C and D). If we consider the flow path in the pipes shown in Figure 1.9, by applying the Bernoulli equation between points A and C of the fluid paths ABC and ADC, it is given that:

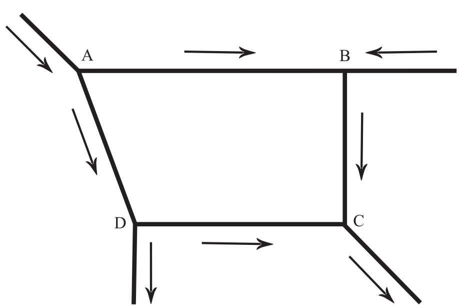


Figure 1.9. *Closed-loop pattern of a network*

$$h_{ABC} = h_{ADC} \Rightarrow h_{AB} + h_{BC} = h_{AC} + h_{DC} \Rightarrow$$

$$\Rightarrow h_{AB} + h_{BC} - h_{AD} - h_{DC} = 0 \qquad [1.8]$$

If we conventionally pre-label the losses, putting the sign + when the fluid flow is clockwise in the loop and the sign − when it is opposite, equation [1.8] becomes:

$$h_{AB} + h_{BC} + h_{AD} + h_{DC} = 0 \Rightarrow$$

$$\Rightarrow \Sigma h_{loop} = 0 \qquad [1.9]$$

Equation [1.9] is general and applicable for any closed system of pipes (loop). In combination with equation [1.6] of the branch points (nodes), it is the key for the solution of the network problems. We emphasize that both these relationships have resulted from conventional pre-labeling of the flow rates and the losses. In their specific application, we must take into account the rules of pre-labeling and specify the respective relationships. For example, for the nodes shown in Figure 1.8, the equation of the nodes will take the form [1.5], while for the node shown in Figure 1.9, the loop equation will take the form [1.8].

Finally, we remind ourselves that for each branch, we have the losses equation (Darcy–Weisbach):

$$h_i = \frac{8}{\pi^2 \cdot g \cdot d_i^2} \cdot \left(f_i \cdot \frac{\ell_i}{d_i} + \Sigma K_i \right) \cdot Q_i^2$$

which can take the form:

$$h_i = a_i \cdot Q_i^2 \qquad [1.10]$$

where $a_i = \frac{8}{\pi^2 \cdot g \cdot d_i^2} \cdot \left(f_i \cdot \frac{\ell_i}{d_i} + \Sigma K_i \right)$ [1.11]

The analysis and calculation of the distribution networks is a complicated and time-consuming procedure, which is based on equations [1.6], [1.9] and [1.10] (nodes, loops and branches, respectively).

1.3. Problem-solving methodology for pipe networks

Let us consider a pipe network in which the pipes have a common point, that is, a node or a bend, and the other side is connected to a tank, whose level is at a height *h*, as shown in Figure 1.10.

In this category of problems, the kinematic viscosity *v*, the lengths of the pipes ℓ_1, ℓ_2, ℓ_3, their diameters d_1, d_2, d_3, their heads h_1, h_2, h_3 and their roughnesses $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are usually given while their flow rates Q_1, Q_2, Q_3 are asked.

The following methodology is used to solve this category of problems:

1) We assume a value for the head h_A, where:

$$h_A = \frac{p_\Delta}{\gamma} + h_\Delta = \text{pressure head} \tag{1.12}$$

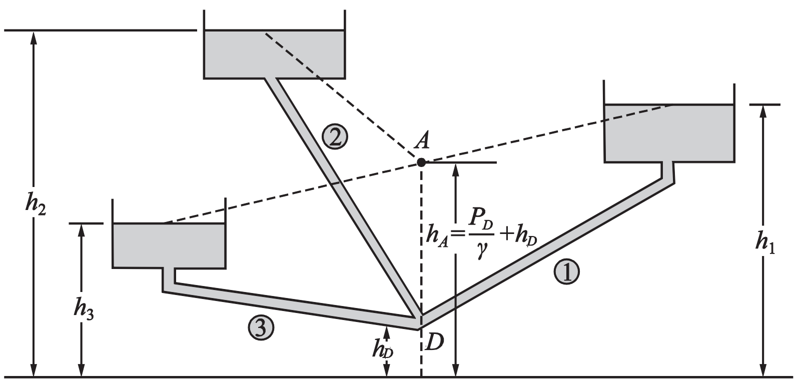


Figure 1.10. *Pipe network with a common point for the pipes*

2) From the Darcy–Weisbach relationship, the given data of the problem and the Moody diagram, we find the flow rates $Q_1, Q_2, Q_3, ..., Q_n$.

For example:

$$h_n = |h_n - h_A| = f_n \frac{\ell_n}{d_n} \cdot \frac{V_n^2}{2g} \tag{1.13}$$

where f_n is found by various tests, and V_n can be found by the relationship:

$$V_n = \sqrt{\frac{2gh_n d_n}{f_n \ell_n}} \quad [1.14]$$

Therefore, $$Q_n = \frac{\pi d_n^2}{4}\sqrt{\frac{2gh_n d_n}{f_n \ell_n}} \quad [1.15]$$

3) After calculating all the flow rates, we may use the continuity equation. This means that the values that we find for $Q_1, Q_2, Q_3, ..., Q_n$ have to satisfy the continuity equation.

If the flow rates Q_1, Q_2 and Q_3 are higher than the flow rates of branches, then we obtain a higher value for h_A, and we repeat steps 1, 2 and 3. If the flow rate to the branch is lower than the flow rate from the branch, then we use a smaller value for h_A.

It is clear from the above procedure that for these problems, the following conditions are applicable:

1) For every pipe, the Darcy–Weisbach relationship must be satisfied.

2) $Q \rightarrow \text{branch} = Q \rightarrow \text{branch}$.

3) There has to be a flow from the higher tank to a lower one. This means that one of these relationships must be applicable:

$$Q_2 = Q_1 + Q_3 \quad [1.16]$$

or $$Q_3 = Q_1 + Q_2 \quad [1.17]$$

As $h_A < h_1$, we have a flow toward a tank, so it is given that:

$$Q_2 = Q_1 + Q_3 \quad [1.18]$$

4) Regardless of the losses in the flow in any pipe, equivalent lengths will be able to express them, which are added to the real length, as mentioned in section 1.2.

1.4. Overall approach for the network calculation

In the beginning of this chapter, we mentioned that the basic purpose of a distribution network is to ensure the necessary flow rate and pressure for the exit of the fluid from the network. Consequently, a water supply network must ensure a flow rate higher than 1 ℓ / s for every user and a pressure of 2 bar.

The flow rate is related to not only the selection of the pipes with suitable diameters but also the differences of the energy heads between the nodes of exit and inlet, as well as the presence or absence of pumps in the network. The collateralization of the required pressure is related to the energy heads: the pressure head at the entrance nodes, the heights and the height that the pumps attribute. More specifically, if we assume a distribution network that is supplied by a tank, as shown in Figure 1.11, the Bernoulli equation, if applied between the surface of tank *A* and the node *E*, will give:

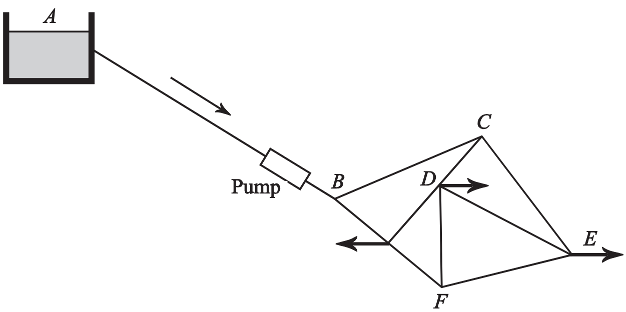


Figure 1.11. *Pipe network connected to a tank*

$$y_A - y_E \frac{V_A^2 - V_E^2}{2g} + \frac{p_A - p_E}{Y} = \Sigma h_{AE} - h_p \Rightarrow$$

$$\Rightarrow \frac{p_E}{Y} = y_A - y_E + \frac{V_A^2 - V_E^2}{2g} + \frac{p_A}{Y} - \Sigma h_{AE} + h_p \qquad [1.19]$$

As the variation of the kinetic energy is negligible because the flow velocities are smaller than 3 m/sec, as well as zero at the surface of the tank, the above relationship becomes:

$$\frac{p_E}{Y} = y_A - y_E + \frac{p_A}{Y} - \Sigma h_{AE} + h_p \qquad [1.20]$$

where $\sum h_{AE} = h_{AB} + h_{B\Gamma} + h_{\Gamma E}$, with the losses of every branch pre-labeled according to the assumption described in section 1.2.

1.5. The Hazen–Williams equation for network analysis

In the previous sections for the solution of the network problems, we used the Darcy–Weisbach losses equation as follows:

$$h_1 = f\frac{\ell}{d}\frac{V^2}{2g} = \frac{8f\cdot\ell Q^2}{\pi^2 d^5 g} = 8f\frac{\ell}{\pi^2 g}d^{-5}Q^2 \qquad [1.21]$$

Considering that the flow is turbulent, we calculate the friction coefficients f from this relationship and then the flow rates.

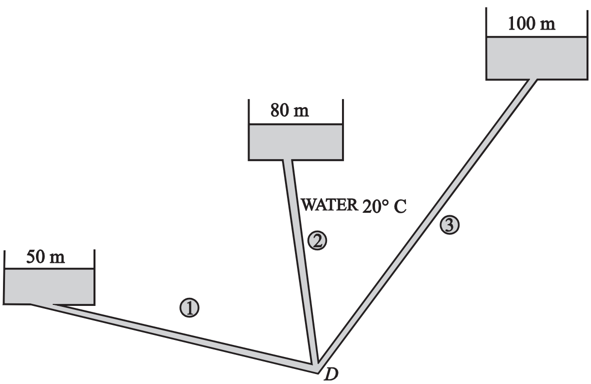


Figure 1.12. *Pipe network for water distribution*

Thus, when the flow is turbulent, the losses along a pipe with respect to the flow rate, the friction coefficient and the pipe's dimensions can be expressed by the following relationship:

$$h_1 = CQ^n \qquad [1.22]$$

where the coefficient C is a parameter that depends solely on the dimensions and the roughness of the pipe. Relationship [1.22] can be derived from [1.21] after we express the friction coefficient f with respect to the flow rate, which means that:

$$f = KQ^a \qquad [1.23]$$

Substituting [1.23] into [1.21] gives [1.22].

After a series of experiments, *Hazen* and *Williams*, who worked with pipes of various diameters and roughnesses, found that the value of the average water velocity in pipes is proportional to the hydraulic radius if it is raised to a power and to the square root of the inclination value of the pressure gauge line, which means that:

$$V \infty R_h^X \sqrt{S} \qquad [1.24]$$

where R_h is the hydraulic radius and S is the inclination of the pressure gauge line = h_1 / ℓ, with ℓ being the length of the pipe and h_1 being the losses during the length ℓ of the pipe.

Therefore, depending on the unit system (SI or British), the *Hazen–Williams* equation in its final form is given by the relationship:

1) In the metric system (SI):

$$V = 0{,}849 W_m R_h^{0{,}63} \cdot S^{0{,}54} \text{ in } m/\sec \qquad [1.25]$$

If R_h is in m, the coefficient W_m, depending on the type of the pipe, has the following values:

No.	**Kind and situation of pipes**	**Value of coefficient W_m**
1	Small, not of a good structure	up to 40
2	Old, in bad situation	60–80
3	Old, made of cast iron, nailed	95
4	Made of cast iron or mud brick	100
5	New, steel, nailed	110
6	Glazed, in good situation	110
7	Wooden, smooth	120
8	Very smooth	130
9	Made of concrete, of large dimensions	130
10	Asbestos tubes	140
11	Very smooth and rectilinear	140

Table 1.3. *Values of the coefficient W_m*

2) In the British system:

$$V = 1,318 W_E R_h^{0,63} S^{0,54} \text{ in ft/sec} \quad [1.26]$$

Table of values of the coefficient W_E		
No.	**Kind and situation of pipes**	**Value of coefficient W_E**
1	Made of cast iron, corroded	80
2	Made of cast iron, after some years of function	100
3	Glazed pipes of drainages	110
4	Average situation of cast iron	110
5	New, steel, nailed	110
6	New, smooth, made of cast iron	130
7	Very smooth and rectilinear	140

Table 1.4. *Values of the coefficient W_E*

If R_h is in ft, the coefficient W_E, depending on the type of the pipe, has the above values.

1.6. Hazen–Williams and Darcy–Weisbach identity

These two expressions that offer the possibility of solving complicated problems of pipe networks have some common points. Therefore:

1) If we assume relationship [1.25]:

$$V = 0,849 W_m R_h^{0,63} S^{0,54} \quad [1.27]$$

where $S = h_l / \ell$, and substituting S and rearranging, we have:

$$\left(\frac{h_1}{\ell}\right)^{0,54} = \frac{V}{0,849 W_m} R_h^{-0,63}$$

$$h_l = \ell \left[\frac{V}{0,849 W_m} R_h^{-0,63}\right]^{1/0,54} = \ell \left[\frac{V}{0,849 W_m} R_h^{-0,63}\right]^{1,852}$$

For a cylindrical pipe, $R_h = \dfrac{d}{4}$. Then, we have:

$$h_l = \frac{\ell \cdot V^{1,852}}{(W_m 0,849)^{1,852}}\left(\frac{d}{4}\right)^{-0,63\times 1,852} = \frac{\ell \cdot V - 0.148}{0,7385} \cdot \frac{V^2}{2g} \cdot \frac{2g}{W_m^{1,852}}\left(\frac{d}{4}\right)^{-1,167}$$

Now, if we set $g = 9,81m/sec^2$, we have:

$$h_l = \frac{4^{1,167}\times 2g}{0,7385W_m^{1,852}}\frac{\ell}{d}\cdot\frac{V^2}{2g}\cdot\frac{V^{-0,148}}{d^{0,167}} = \frac{134d^{-0,019}}{W_m^{1,852}}\left(\frac{\ell}{d}\right)\frac{V^2}{2g}\frac{1}{V^{0,148}d^{0,148}} =$$

$$h_l = \frac{\ell \cdot V^{1,852}}{(W_m 0,849)^{1,852}}\left(\frac{d}{4}\right)^{-0,63\times 1,852} = \frac{\ell \cdot V - 0.148}{0,7385} \cdot \frac{V^2}{2g} \cdot \frac{2g}{W_m^{1,852}}\left(\frac{d}{4}\right)^{-1,167}$$

$$= \frac{134d^{-0,019}}{\nu^{0,148}W_m^{1,852}}\left(\frac{\nu^{0,148}}{V^{0,148}d^{0,198}}\right)\left(\frac{\ell}{d}\right)\left(\frac{V^2}{2g}\right) =$$

$$\frac{134d^{-0,019}}{W_m^{1,852}\nu^{0,148}\,\mathrm{Re}^{0,148}}\left(\frac{\ell}{d}\right)\frac{V^2}{2g} = f\frac{L}{D}\frac{U^2}{2g}$$

However, because $f = \dfrac{134d^{-0,019}}{W_m^{1,852}\nu^{0,148}\,\mathrm{Re}^{0,148}}$, we have:

$$h_l = f\left(\frac{\ell}{d}\right)\frac{V^2}{2g} \qquad [1.28]$$

2) If we consider relationship [1.27] and substituting *S* and rearranging, we have for a cylindrical pipe, where $R_h = \dfrac{d}{4}$:

$$V = 1,318W_E\left(\frac{d}{4}\right)^{0,63}\left(\frac{h_1}{\ell}\right)^{0,54} = \frac{4Q}{\pi d^2}$$

So, solving for $\dfrac{h_1}{\ell}$, we find:

$$\frac{h_l}{\ell} = \left[\frac{\frac{4Q}{\pi d^2}}{1,318 W_E \left(\frac{d}{4} \right)^{0,63}} \right]^{\frac{1}{0,54}} \Rightarrow$$

$$\Rightarrow h_1 = \left[\frac{\frac{4^{1,63}}{1,318}}{W_E} \right]^{1,852} \cdot \frac{\ell}{d^{4,87}} Q^{1,852}$$

$$h_1 = \left[\frac{2,3136}{W_E} \right]^{1,852} \cdot \frac{\ell}{d^{4,87}} Q^{1,852} \quad [1.29]$$

where all the lengths are expressed in ft and the flow rate in ft/sec.

Comparing [1.29] and [1.22], we see that:

$$C = \left(\frac{2,3136}{W_E} \right)^{1,852} \frac{\ell}{d^{4,87}} \quad [1.30]$$

and n =1,852.

Combining relationship [1.29] with the nomograph of *Hazen–Williams*, we obtain solutions in various cases of pipe connection, which are much easier and faster than those obtained using the Moody diagram.

Moreover, the Hazen–Williams relationship is simpler than the Darcy–Weisbach relationship because the calculation of the coefficient C is easier. This can be solved by using tables or graphs, and therefore this method has been commonly used in the calculation of water networks. However, it also has some disadvantages of providing less accurate results than the Darcy–Weisbach equations, and it is applicable only if the fluid is water. Moreover, both relationships are empirical, but the Darcy–Weisbach equation is theoretically more close to an analytical method and compatible with the conclusions of the two-dimensional analysis.

1.7. Hardy–Cross method

Another method by which we can solve problems of pipe networks is the Hardy–Cross method, which is a relatively simple procedure. According to this methodology, we initially assume flow rate distribution under only one condition, which is to satisfy the mass conservation in each *node*. Continuing, we make corrections to these flow rates at each *loop* of the network, with corrections every time a new value of the flow rate comes up at every circle of calculations so that finally there is a better balance of flow rates in the network than the value we assumed before. If we get in the beginning a good value for the flow rate distribution, we can have convergence in the final value after two or three attempts.

For simple networks, as shown in Figure 1.12, the solution can be obtained easily using a calculator, while for more complicated ones, the assistance of a computer is necessary.

The procedure for finding a solution according to the Hardy–Cross method is as follows:

1) We initially assume the flow rate distribution $Q_{01}, Q_{02}, Q_{03}, \ldots$ with the only limitation at each node of the network being the mass conservation applicable, meaning that the amount of the mass of water that goes in is the same that comes out.

2) We calculate the losses at each pipe of the network based on the relationship:

$$h_{1_0} = C_1 Q_0^n \qquad [1.31]$$

Therefore, if there are seven pipes in the circuit, we will find seven losses using the Hazen–Williams nomograph.

$$\begin{aligned} h_{1_{01}} &= C_1 Q_{01}^n \\ h_{1_{02}} &= C_2 Q_{02}^n \\ &\;\vdots \\ h_{1_{07}} &= C_7 Q_{07}^n \end{aligned}$$

3) Choosing any direction and paying attention not to make a mistake in the sign, we find the following sum in each loop of the network: