

SCIENCE, SOCIETY AND NEW TECHNOLOGIES SERIES

ENGINEERING, ENERGY AND ARCHITECTURE SET



## Volume 2

# Fluid Mechanics in Channel, Pipe and Aerodynamic Design Geometries 1

**Christina G. Georgantopoulou**  
**George A. Georgantopoulos**

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Fluid Mechanics in Channel, Pipe and  
Aerodynamic Design Geometries 1

*...To our Family*

*Depy  
Andreas  
Nikos  
Giannis  
Lilian*

*...as without their support none of this would have ever been possible for us*

**Engineering, Energy and Architecture Set**

coordinated by  
Lazaros E. Mavromatidis

Volume 2

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## Preface

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This book presents an extended and detailed analysis of both the flow phenomena in closed and open channels and the flows around solid bodies. It comprises two volumes. This book is a specialized resource for those students, engineers and researchers who want to focus on the industrial applications of flows and study the fascinating world of internal and external flow phenomena.

We have both had extensive experience in teaching, studying and researching fluids since the completion of our respective PhD theses. We felt that it was time to write about the practical and analytical aspects of flow applications, all of which can be applied in industrial flows, to support researchers, engineering students and industrial engineers in the field of fluids in order to optimize their work in “flows”.

For the first author, the “fluids direction” began in the early stages of her PhD thesis study in Computational Fluid Dynamics in 1998 at the National Technical University of Athens. The second author’s knowledge of the fluids’ path is very extensive, obtained from more than 45 years of studies and work involved in his PhD thesis and further research work at the University of Patras, as well as through his position as Professor of Aerodynamics at the Hellenic Air Force Academy, spanning more than 35 years.

We have both gained substantial experience in Fluid Mechanics research through numerous publications, presentations at international conferences, academic textbook authoring, teaching through international experiences and collaborations. However, we felt that more should be offered to the Fluid Mechanics community, and hence this book.

Although we both have experience in writing for academic textbooks, this is our first publication that caters to international students, researchers and engineers, considering the industrial phenomena that are met in international industries and we

have tried to present most of the applications in flows inside or around bodies. This book is based on books written previously by us on Fluid Mechanics and on Aerodynamics, but for the first time our work focuses on the practical aspects of industrial internal and external flows.

Christina, the first author, offers this textbook to the Bahrain polytechnic engineering students and all the industrial delegates who have worked with her in “flows” for many years. She also wishes to express her appreciation for her colleagues, namely Payal Modi, for the thousands of hours of constructive discussions and collaborations in fluids aspects, to Lazaros E. Mavromatidis for his support during the publishing procedure, to her father George who has been her mentor for all these years and to Stephanie Sutton and Amerissa Kapela for their continuing support with the quality of the academic English language. Additionally, George, the second author, wishes to share his more than 40 years of experience in fluids with the fluids community around the world and support them in their “flows” work as best he can.

We both have a special sentimental feeling for this book in that we are extremely proud that we have been able to write, publish and offer it to you, hoping that it will really support you in your fluids journey. We have both worked on fluids with a passion not only for our students, but also to honor our colleagues around the world. We are equally happy to say that the Fluid Mechanics community has been served by the same family for more than 40 years. We hope that we will be physically and mentally healthy to continue to serve our students and support our colleagues in the fluids aspects in the future.

We hope that you will enjoy this book and be engaged with the fascinating world of flows.

Christina G. GEORGANTOPOULOU  
George A. GEORGANTOPOULOS  
February 2018

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# Introduction

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## I.1. Introduction

Fluid Mechanics consist of two main categories. The first one refers to the quantitative and qualitative analysis and study of fluids in motion, the velocity or acceleration as well as the forces exerted by nature. The second category analyzes the physical forces that are developed on solid–fluid interfaces, where the solids represent the containers. The first category can be called Theoretical Fluid Mechanics, while the second one is called Applied Fluid Mechanics.

This book primarily presents the aspects and problems of internal and external flows, including certain fundamental principles of fluids.

To develop an extended study of Applied Fluid Mechanics problems, mathematical modeling and analysis is considered necessary. On the other hand, the empirical or experimental investigation of fluid phenomena only provides us with certain measurements and information about individual cases, and it is often difficult to generalize our conclusions. Hence, the appropriate way to study fluid flows is to investigate the related phenomena with a combined analytical–computational and experimental approach in order to improve step by step the proposed fluid theories or solutions.

Industrial engineers have raised various issues related to the main assumption that all fluids are considered to be ideal. In order to overcome these issues, every technological problem is considered to be an individual one, resulting in a lack of theoretical background. Year after year, a huge gap has been created between theoretical and practical hydrodynamics researchers, which exists even today. This book bridges this gap between various industrial flows, and an attempt has been

made to present a common strategy. The flows inside pipes or channels as well as the flows around bodies are considered to be real life applications, setting the appropriate theoretical background simultaneously.

**I.2. Fluid Mechanics sections**

The Fluid Mechanics study comprises fluid motion and fluid balance. During the last decades, it has evolved in two major directions. Theoretical Fluid Mechanics includes the mathematical exploitation of fluid phenomena, and Technical Fluid Mechanics includes the applications of mechanical engineering, aeronautics, shipbuilding and meteorology. Technical Fluid Mechanics is considered an applied science, and hence it is often referred to as Applied Fluid Mechanics, which includes the possible solutions of fluid problems and the explanation of natural phenomena. Moreover, it aims to produce numerical predictions or experimental validation for direct practical applications.

Classic Fluid Mechanics can be derived from various areas according to the mechanical condition or fluid properties. The categories presented in Table I.1 are based on the motion of fluids as well as on compressibility, where the density varies according to the fluid condition.

Fluid mechanics	Fluids at rest	Fluids in motion
Hydrodynamics ( $\rho=ct$ )	Hydrostatics	Hydrodynamics
Aeromechanics ( $\rho\neq ct$ )	Aerostatics	Aerodynamics

**Table I.1. Fluid Mechanics categories**

**I.3. Systems of units**

**I.3.1. Definitions and general considerations**

Units are fundamental for physics, especially for all the applied sciences such as mechanical engineering. The number without units means absolutely nothing for Fluid Mechanics, as it represents a natural quantity such as pressure, velocity or force.

Historically, various systems of units have been developed according to the theoretical principle demands or to practical applications. In most countries (not including the USA), the metric system is the official system of measurement, which

is accepted by both scientists and engineers. The International System of Units (SI) was defined and established at the 11th General Conference on Weights and Measures, where more than 36 countries accepted it to be the most complete and appropriate one, including the USA. Since then, the USA has made huge progress in introducing SI units to engineering. For example, many NASA laboratories use SI units for their technical research results, and the AIAA (American Institute of Aeronautics and Astronautics) also supports the SI in its research papers.

Therefore, students who want to study engineering have to know both unit systems. The following table presents the corresponding basic units in both systems based on the theory that all the derived units at the metric system can be produced by the base ones.

Base quantity	SI	BS
Length	Meter (m)	Foot (ft)
Time	Second (s)	Second (sec)
Mass	Kilogram (kg)	Pounds of mass (lb <sub>m</sub> ) or slug
Temperature	Celsius (°C)	Fahrenheit (°F)
Absolute temperature	Kelvin (K)	Rankine (R)

**Table I.2. Base units in SI and BS (British system)**

As we have just mentioned, the derived units can be produced by the base units following the nature of interrelationships or the basic formulas with the need for adding any conversion factor, as in the following, using Newton's law:

$$F = m \times a \quad [I.1]$$

$$(1 \text{ newton}) = (1 \text{ kilogram})(1 \text{ meter/second}^2)$$

Thus, we further confirm the definition of Newton as the force that is required to accelerate a mass of 1 kg at a rate of 1 m/s<sup>2</sup>. Similarly, the ideal gas constant for air ( $R=287 \text{ J/(kg}\cdot\text{K)}$ ) can also be expressed in the following way:

$$R = 1716 \frac{\text{J}}{(\text{kg})(\text{K})} = 287 \cdot \frac{\text{N} \cdot \text{m}}{(\text{kg})(\text{K})} = 287 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \frac{\text{m}}{(\text{kg})(\text{K})} = 287 \frac{\text{m}^2}{(\text{s}^2) \cdot (\text{K})} \quad [I.2]$$

The BS is also a consistent system, and the same procedure can be followed for the derived quantities:

$$F = m \cdot a \quad [I.3]$$

$$(1 \text{ pound}) = (1 \text{ slug})(1 \text{ foot/second}^2)$$

$$R = 1716 \frac{\text{ft} \cdot \text{lb}}{(\text{slug})(R)} = 1716 \frac{\text{ft}^2}{(\text{s}^2)(R)} \quad [I.4]$$

However, more systems of units are not consistent; therefore, it is necessary to use a factor in order to produce the required conversion as shown below. These systems have been used in the past by engineers but have often not been convenient to be applied:

$$F = (1/g_c) \times m \times a \quad [I.5]$$

$$g_c = 32.2 \quad (\text{lb}_m) \cdot (\text{ft}) / (\text{s}^2) \cdot (\text{lb}_f)$$

$$F = (1/g_c) \quad m \times a \quad [I.6]$$

$$\begin{array}{cccc} \uparrow & \uparrow & \uparrow & \uparrow \\ \text{lbf} & (1/32.2) \text{lb}_m & \text{ft/s}^2 & \end{array}$$

The various temperature units are of high importance. We often denote absolute temperature by  $T$ , where the minimum temperature value can be zero. Kelvin (K) and Rankine (R) are the absolute temperature units, where  $0 \text{ R} = 0 \text{ K}$  indicates the temperature at which all the molecular motion theoretically stop. In addition, the relationships among the temperature units are:

$$0^\circ\text{F} = 460 \text{ R} \quad [I.7]$$

$$0^\circ\text{C} = 273 \text{ K} = 32^\circ\text{F} \quad [I.8]$$

It is worth mentioning that the temperature  $T$  in the ideal gas equation of state (equation [I.9]) is absolute:

$$p = \rho RT \quad [I.9]$$

where  $p$  is the pressure,  $\rho$  is the density of gas and the other symbols are defined as above.



### I.3.2. Definitions and fundamental units in fluids

Natural quantity	Units	Symbol
Force	Newton	$N = \text{kg m/s}^2$
Energy	Joule	$J = N \text{ m}$
Power	Watt	$W = J/s$

**Table I.3.** *Units of common quantities in physics and fluids*

$10^{-6}$	micro	$\mu$
$10^{-3}$	milli	m
$10^3$	kilo	k
$10^6$	mega	M

**Table I.4.** *Common metric prefix in SI*

Length $l$	
1 in	25.4 mm
1 ft	0.3048 m
1 yd	0.9144 m
1 mile	1.6093 km
Area $S$	
1 in <sup>2</sup>	645.16 mm <sup>2</sup>
1 ft <sup>2</sup>	0.0929 m <sup>2</sup>
1 yd <sup>2</sup>	0.8361 m <sup>2</sup>
1 mile <sup>2</sup>	2.590 km <sup>2</sup>
1 acre	4046.9 m <sup>2</sup>
Volume $v$	
1 in <sup>3</sup>	16387 mm <sup>3</sup>
1 ft <sup>3</sup>	0.02832 m <sup>3</sup>
1 UK gal	0.004546 m <sup>3</sup>
1 US gal	0.003785 m <sup>3</sup>

**Table I.5.** *Length, area and volume conversion factors*

<b>Mass <math>m</math></b>	
1 kg	$10^3$ g
1 oz	28.352 g
1 lb	453.592 g
1 cwt	50.802 kg
1 ton (UK)	1016.06 kg
<b>Density <math>\rho</math></b>	
1 lb/ft <sup>3</sup>	16.019 kg/m <sup>3</sup>
1 lb/UK gal	99.776 kg/m <sup>3</sup>
1 lb/US gal	119.83 kg/m <sup>3</sup>
<b>Force <math>F</math></b>	
1 dyne	$10^{-5}$ N
1 poundal	0.1383 N
1 lb-f	4.4482 N
1 kg-f	9.8067 N
1 ton-f	9.9640 kN
<b>Viscosity <math>\mu</math></b>	
1 poise (1 g/cm sec, 1 dyn sec cm <sup>2</sup> )	0.1 N sec/m <sup>2</sup>
1 lb/ft sec (1 poundal sec/ft <sup>2</sup> )	1.4882 N sec/m <sup>2</sup>
1 lb/ft hr (1 poundal hr/ft <sup>2</sup> )	0.4134 mN sec m <sup>2</sup>
<b>Pressure <math>p</math></b>	
1 bar (10 <sup>5</sup> dynes/cm <sup>2</sup> )	10 <sup>5</sup> N/m <sup>2</sup>
1 atm (1 kg-f/cm <sup>2</sup> )	98.0665 kN/m <sup>2</sup>
1 atm (standard)	101.325 kN/m <sup>2</sup>
1 psi (1 lb-f/in <sup>2</sup> )	6.8948 kN/m <sup>2</sup>
1 psf (1 lb-f/ft <sup>2</sup> )	47.880 N/m <sup>2</sup>

**Table I.6.** Conversion factor mass, density, force, viscosity and pressure

<b>Energy <math>E</math></b>	
1 erg	$10^{-7}$ J
1 ft poundal	0.04214 J
1 ft lb-f	1.3558 J
1 cal (international table)	4.1868 J

1 Btu	1055.06 J	
1 hph	2.6845 MJ	
1 kwh	3.6 MJ	
<b>Power <math>P</math></b>		
1 erg/sec	$10^{-7}$ W	
1 hp (British)	745.70 W	
1 hp (metric)	735.40 W	
1 ft lb-f/sec	1.3558 W	
1 Btu/hr	0.2931 W	
<b>Surface tension <math>\sigma</math></b>		
1 dyne/cm (1 erg/cm <sup>2</sup> )	$10^{-3}$ J/m <sup>2</sup>	
<b>Moment of inertia <math>M</math></b>		
1 lb.ft <sup>2</sup>	0.04214 kg m <sup>2</sup>	
<b>Momentum <math>J</math></b>		
1 lb-ft/sec	0.1383 kg.m/sec	
<b>Specific temperature <math>c</math></b>		
1 Btu/lb°F (1 cal/g.°C)	4.1868 kJ/kg.°C	
<b>Heat transfer coefficient <math>h</math></b>		
1 Btu/h.ft <sup>2</sup> .°F	5.6783 W/m <sup>2</sup> .K	
<b>Thermal conductivity <math>K</math></b>		
1 Btu/h.ft.°F	1.7307 W/m.K	
<b>Water (18°C and air properties (STP))</b>		
	<i>Water</i>	<i>Air</i>
Density (kg/m <sup>3</sup> )	$10^3$	1.3
Viscosity (N sec/m <sup>2</sup> )	$10^{-3}$	$1.7 \times 10^{-5}$
Specific heat (KJ/kg.K)	4	1
Thermal Conductivity (W/m.K)	0.6	0.024

Table I.7. Other conversion factors



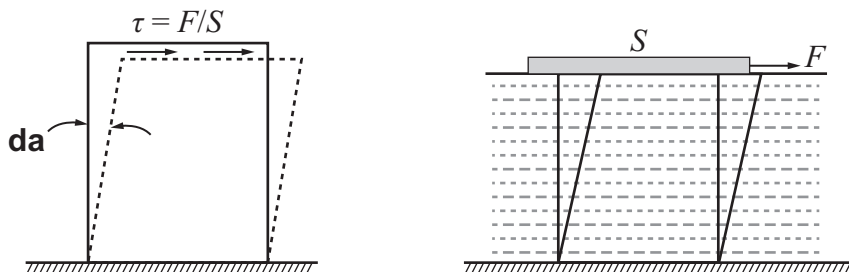
# Fundamental Principles in Fluids

## 1.1. Introduction

The most important characteristics of fluids are distinguished under shear conditions. As we have already mentioned, every fluid is incapable of maintaining its static balance under shear stress conditions.

As a matter of fact, many objects, even solids, show fluid properties if high-grade shear stresses are developed. In addition, some real fluids show a viscosity that is so high under certain conditions that they behave more like solids than fluids. However, the definition that has been given is adequate for fluids despite the occurrence of some extreme cases. In general, fluids are comprised of liquids and gases.

Figure 1.1 shows the shear stresses in both solids and fluids. A tangential force  $F$  is applied to the upper surface of each object, thereby developing shear stresses ( $F/S$ ). When the shear stresses are applied to a solid body, deformation is developed, which is depicted by its change in shape and the angle  $da$ .



**Figure 1.1.** *Shear stresses in solids and fluids*

However, if the same stresses are applied to fluids, deformation will not be permanent. Deformation is developed only during the stress application. If some objects sometimes behave like solids and other times like fluids, according to the size of the applied stresses, then they are called *plastics*, for example the paraffin. This type of material is deformed like solids if the shear stresses are less than a certain value  $\tau_0$ , and flows like fluids if the stresses are  $\tau < \tau_0$ . The official name of this type of material is *Bingham plastic*.

## 1.2. Incompressible and compressible fluids

According to the variation in physical properties, fluids can be classified as *incompressible* or *compressible*. Under certain conditions of static balance, some fluids show minimum density variation even at high pressure values. This type of fluid is called incompressible.

Consequently, incompressible fluids are fluids with constant density regardless of the applied pressure value. However, this definition is theoretical or can be considered as an assumption, as real fluids are not able to maintain their density totally constant under various pressure conditions. The density always shows a certain variation even if it is limited. If the variation is close to zero, then we can assume the fluids to be incompressible and neglect the aforementioned variation. For gases, this can be assumed only if the temperature is also constant.

Most liquids can be assumed to be incompressible. The study of incompressible fluids under static balance conditions is called hydrostatics. If the density of fluids cannot be considered as constant, then these fluids are called *compressible*. Although most liquids can be assumed to be incompressible, the parameter that determines the distinction between incompressible and compressible in air flows is the *Mach* number ( $M$ ). If  $M < 0.4$ , the air flow can be considered to be incompressible, while if  $M > 0.4$ , the flow is considered to be compressible.

## 1.3. Fluid properties

The main characteristics of fluids are called fluid properties. As these characteristics may vary according to the space or time, we will define these as characteristics of a fluid particle. Their definition will be presented on a point within a fluid or within a flow field.

### 1.3.1. Density ( $\rho$ )

Fluid density ( $\rho$ ) is defined as the mass  $dm$  of a fluid particle per volume  $dv$  unit. Consequently, if we want to define the density of the fluid in a point  $P$  of a flow field, we may write the following expression:

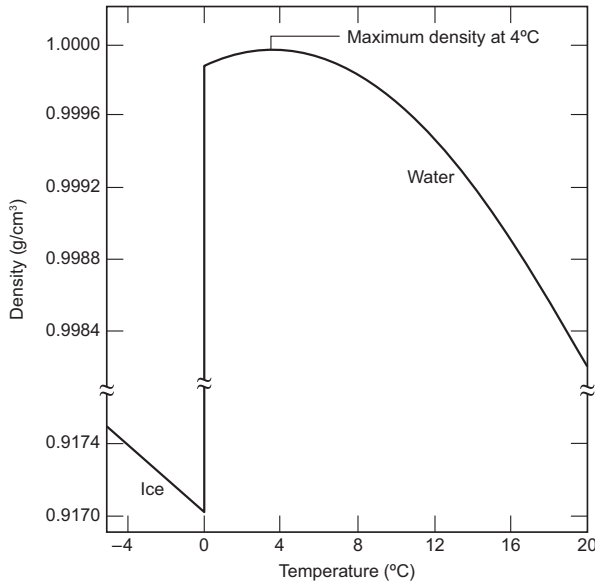
$$\rho(P) = \lim_{dv \rightarrow 0} \frac{dm}{dv}. \quad [1.1]$$

The SI unit of density is  $\text{kg/m}^3$ . Density depends on both the location of point  $P$  and the time:

$$\rho = \rho(x, y, z, t). \quad [1.2]$$

It can also be varied according to the pressure and temperature of the fluid.

If the density of a fluid remains constant regardless of the other property's variation, then the fluid is called incompressible. In the case of liquids, we may assume that their density remains constant even after high pressure or temperature variation. This assumption cannot be made for gases, and hence they are called *compressible* fluids. The density of most gases is proportional to the pressure and irreversibly proportional to the temperature.



**Figure 1.2.** Density of water variation according to the temperature

As the density is defined per volume unit, the *temperature* and *pressure* are the main conditions for its variation. In addition, the volume of the fluid and consequently the density are affected by the possible *dissolved substances*.

*Temperature* variation affects fluid density. As the temperature is increased, the volume of the body is increased (expansion) and consequently the density has lower values. There is an exception in the case of water at the temperature range of 0°C–4°C: the density is decreased when the temperature is increased in this range of values, and thus it reaches a minimum value (Figure 1.2).

T (°C)	ρ (kg/m <sup>3</sup> )	T (°C)	ρ (kg/m <sup>3</sup> )	T (°C)	ρ (kg/m <sup>3</sup> )	T (°C)	ρ (kg/m <sup>3</sup> )
0	999,839	7	999,901	30	995,647	65	980,502
1	999,898	8	999,848	35	994,032	70	977,771
2	999,940	9	999,781	40	992,215	75	974,850
3	999,964	10	999,699	45	990,213	80	971,799
4	999,972	15	999,099	50	988,037	85	968,621
5	999,964	20	999,204	55	985,696	90	965,321
6	999,940	25	997,045	60	983,200	100	958,365

**Table 1.1.** *Density of pure water according to various T values*

The expansion is more intense in gases than in liquids. For instance, the expansion coefficient of oil is four times lower than air's coefficient, and the water expansion coefficient is 17 times higher (Table 1.1). Consequently, if the temperature is increased from 20 to 30°C under constant pressure, the volume of the air will be increased by 3.3%, the volume of the oil by 0.9% and the volume of the water by approximately 0.2%. Table 1.1 presents the density of pure water for *T* ranging from 0 to 100°C. As can be easily seen, the maximum density is achieved at 4°C.

In the case of gases under conditions far below the dew point, the density as well as its variation in relation to the *T* and *p* can be calculated using the following state equation for gases (equation [1.4]):

$$p \cdot v = n \cdot R_w \cdot T = \frac{m}{m_r} \cdot R_w \cdot T \Rightarrow$$

$$\Rightarrow p = \rho \cdot R \cdot T \quad [1.3]$$



$$\rho = \frac{p}{R \cdot T} \quad [1.4]$$

where  $p$  is the pressure,  $T$  is the absolute temperature and  $R$  is the gas constant.

The gas constant  $R$  can be defined as the ratio of the universal gas constant over the molar mass of the gas:

$$R = \frac{R_w}{m_r} \quad [1.5]$$

where  $R_w = 8,314 \text{ J}/(\text{mole} \cdot \text{K}) = 8,314 \text{ J}/(\text{K mole} \cdot \text{K})$ .

In the case of air as a fluid that comprises two main components (21% oxygen and 79% nitrogen), the molar mass is:

$$m_r = 32 \cdot 0.21 + 28 \cdot 0.79 = 29 \text{ g/mole} = 29 \text{ kg/Kmole} \quad [1.6]$$

and then  $R = 287.1/\text{kgK}$ .

The *pressure* also significantly affects the density of fluids. If the temperature remains constant, a double value of pressure implies a double value of density. Concerning gas vapor, the relationship between pressure and density remains the same. For example, if the temperature of a vapor quantity is equal to  $150^\circ\text{C}$  ( $423 \text{ K}$ ) and the pressure to 1 bar (100 kPa), the density will be  $\rho = 0.590 \text{ kg/m}^3$  (if the state equation is applied, the results would be  $\rho = 0.512 \text{ kg/m}^3$ ). If the pressure is doubled, the density will be increased by 77%.

Unlike gases, the effect of pressure on the density of liquids is minimum. Hence, in most of the practical or technical applications in liquids, we assume that the density is constant despite possible pressure variation. As already mentioned, if the density remains constant when the pressure varies, the fluids are called incompressible, whereas these are called compressible if the density also varies. *In fact, compressible fluids are gases, as liquids are assumed to be incompressible.*

The *dissolved substance* in a mixture increases the fluids' density. When a solid substance is dissolved in a solvent, the mass is increased without any corresponding increase in the volume. This occurs due to the chemical relationship between the liquid and the dissolved substance, which can be explained by the dissolution theory and mechanism.

%	0°C	10°C	25°C	40°C	60°C	80°C	100°C
1	1,007.5	1,007.1	1,004.1	999.1	990.0	978.5	965.1
2	1,015.1	1,014.4	1,011.1	1,005.9	996.7	985.2	971.9
4	1,030.4	1,029.2	1,025.3	1,019.8	1,010.3	998.8	985.5
8	1,061.2	1,059.1	1,054.1	1,048.0	1,038.1	1,026.4	1,013.4
12	1,092.4	1,083.5	1,083.6	1,077.0	1,066.7	1,054.9	1,042.0
16	1,124.2	1,120.6	1,114.0	1,106.9	1,096.2	1,084.2	1,071.3
20	1,156.6	1,152.5	1,145.3	1,137.7	1,126.8	1,114.6	1,101.7
24	1,190.0	1,185.6	1,177.8	1,169.7	1,158.4	1,146.3	1,133.1
26	1,207.1	1,202.5	1,194.4	1,186.1	1,174.7	1,162.6	1,149.2

**Table 1.2.** *Sea water density*

This phenomenon receives great importance in the case of water. Natural water contains many dissolved solid substances such as salts. Therefore, the density of the natural water is slightly higher than the one presented in Table 1.1. For the temperature range of 0–20°C, the density of natural water is equal to 1,000 kg/m<sup>3</sup>. Sea water, which contains high concentrations of sodium chloride, has a higher density range (1,025–1,028 kg/m<sup>3</sup>). Table 1.2 presents the density of sea water analytically according to both the temperature and the concentration.

If we mix two liquids with densities  $\rho_1$  and  $\rho_2$ , the *density value of the mixture* will be between these two values, depending on the mixture ratio. The mass of the solution will be  $m = m_1 + m_2$ , while the volume  $v \leq v_1 + v_2$ . If we assume that  $v \cong v_1 + v_2$ , we are able to estimate the solution's density. However, the precise density estimation can be retrieved by the rating table or graphs.

The measurement of density can be achieved by specific instruments, which will be studied in the next chapter.

### 1.3.2. Specific weight ( $\gamma$ )

Specific weight ( $\gamma$ ) is defined as the weight per volume unit:

$$\gamma = \frac{dw}{dv} \quad [1.7]$$

where  $w$  is the weight of the body and  $v$  is the volume.

As can be easily seen according to the  $w$  and  $\rho$  definition, the specific weight can be calculated as follows:

$$\gamma = \rho \cdot g \quad [1.8]$$

where  $\rho$  is the density of the fluid and  $g$  is the acceleration of gravity.

The SI unit of specific weight is  $\text{N/m}^3$ .

Table 1.3 presents the density and specific weights for various fluids at  $20^\circ\text{C}$ ,  $p = 101.3 \text{ KPa}$ ,  $g = 9.807 \text{ m/s}^2$ .

Fluid	$\rho \text{ (kg/m}^3\text{)}$	$\gamma \text{ (kN/m}^3\text{)}$	Fluid	$\rho \text{ (kg/m}^3\text{)}$	$\gamma \text{ (kN/m}^3\text{)}$
Water (pure)	998.2	9.79	Pentane	630	5.71
Natural water	1,000	9.81	Hexane	659	6.46
Sea water	1,025	10.05	Heptane	684	6.71
Ether	708	6.94	Octane	703	6.89
Alcohol	789	7.74	Decane	730	7.16
Aceton	792	7.77	Hexadecane	774	7.59
Benzene	879	8.62	Petrol	700–750	6.86–7.36
Aniline	1,022	10.0	Kerosene	780–820	7.65–8.04
Chloroform	1,489	14.6	Diesel	810–860	7.94–8.43
Carbon tetrachloride	1,590	15.6	Crude oil	900–1,000	8.83–9.81
Sulfuric acid	1,834	28.0	Mineral oil	880–940	8.63–9.22
Mercury	13,550	133.0	Olive oil	910–920	8.92–9.02

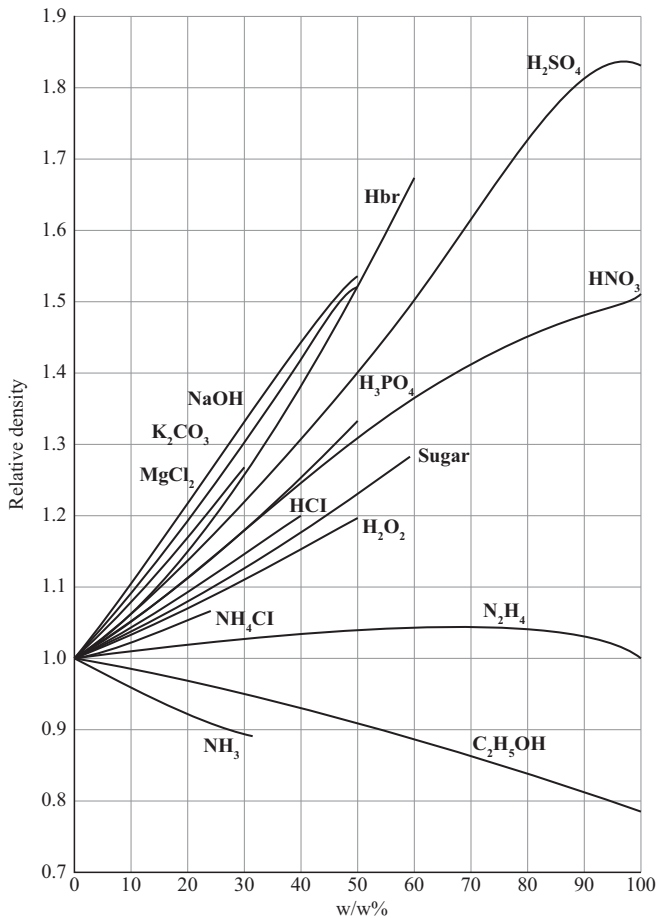
**Table 1.3. Density and specific weight of various fluids**

### 1.3.3. Relative density

Sometimes, especially in relation to industrial applications, the relative density ( $RD$ ) or specific gravity ( $SG$ ) is used instead of density.  $RD$  is a dimensionless quantity, which expresses the relationship between the densities of two fluids where the second one is chosen as a reference material:

$$RD = \frac{\rho}{\rho_f} \quad [1.9]$$

where  $\rho$  is the density of the fluid and  $\rho_f$  is the density of the reference material at a certain temperature. For liquids, the reference material is water and  $\rho_f$  is often equal to  $1,000 \text{ kg/m}^3$  at  $4^\circ\text{C}$ . In the case of gases, the corresponding reference density is the density of the air for  $T = 293 \text{ K}$  and  $p = 101.3 \text{ KPa}$ , which is  $\rho_f = 1.205 \text{ kg/m}^3$ . As the relative density is a dimensionless quantity, it is advantageous for most engineers, who thus prefer it to the density or specific weight. In Figure 1.3, various relative densities of liquid mixtures are depicted.



**Figure 1.3.** Relative densities of liquid mixtures

For non-homogeneous fluids, as some types of mixtures, the density definition differs. The concentration in this case is not steady, and mass motion is produced among various fluids' points regardless of the fluidity. This process which is called diffusion will be continued until a homogeneous mixture is formed. The concentration and density are connected according to their definition. Concentration  $A$  of a mixture component is defined as the mass per volume unit. The mass concentration or the partial density  $\rho_a$  is often used. The mixture density is equal to the sum of the mass concentration of the components:

$$\rho = \rho_\alpha + \rho_\beta + \dots + \rho_n = \Sigma \rho_n \quad [1.10]$$

where  $\rho_i$  are the densities of the  $n$  mixture components.

According to the data given, the mixture density can also be calculated as follows:

$$\rho = \frac{m}{v} = \frac{\rho_1 n_1 + \rho_2 n_2 + \dots + \rho_i n_i}{n_1 + n_2 + \dots + n_i} \quad [1.11]$$

where the symbols are defined as above.

Special density calculation is applied for the atmospheric air case, which is a mixture of oxygen, carbon dioxide, nitrogen oxide and noble gases containing water vapor simultaneously. The vapor precedence is not constant and depends on the pressure and temperature conditions. In these cases, the density of the air is given by:

$$\rho_0 = \rho_l - 0,337 \varphi \frac{p_a}{p} \quad [1.12]$$

where  $\rho$  is the density of the dry air,  $\varphi$  is the relative humidity,  $p$  is the pressure and  $p_a$  is the saturation pressure of water.

According to *API gravity* standards, the relative density of a fluid is defined as the ratio of the fluid density to the density of water in 60°F. Moreover, according to American Petroleum Institute standards, this ratio can be calculated based on the value of API gravity (API) as follows:

$$RD(60) = \frac{141.5}{131.5 + API} \quad [1.13]$$

The aforementioned formula can also be used for the API gravity calculation, which is known as the relative density value.

The relative density can be calculated at a different temperature using the following formula:

$$RD(T) = \frac{RD(60)}{1 + a(T - 60)} \quad [1.14]$$

where  $a$  is equal to:

$$a = \exp(0.0106 \cdot API - 8.05) \quad [1.15]$$

The *Baume scale* is also used by the industry for various fluids with different density values of the water (lower or higher). The corresponding relationship between the Baume scale and relative density is presented in Table 1.4.

Baume Degrees	RD for fluids with $< \rho_{water}$	RD for fluids with $> \rho_{water}$
0	—	1.000
10	1.000	1.074
20	0.933	1.160
30	0.875	1.261
40	0.823	1.381
50	0.778	1.526
60	0.737	1.706
70	0.700	1.933
80	0.669	2.231
90	0.636	—
100	0.609	—

**Table 1.4. Baume scale values**