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Mohammad Farshadnia

Advanced Theory of Fractional-Slot Concentrated- Wound Permanent Magnet Synchronous Machines

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Mohammad Farshadnia

Advanced Theory of Fractional-Slot Concentrated-Wound Permanent Magnet Synchronous Machines

Doctoral Thesis accepted by
the University of New South Wales, Sydney, Australia

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*To My Parents, Sister, and My Loving Wife
For Blessing My Life with Their
Unconditional Love and Support*

Supervisor's Foreword

Recent advances in multiphase fractional-slot concentrated-wound (FSCW) interior permanent magnet (IPM) machines have made them prime candidates for high-performance applications such as advanced electric drives in automobiles, aircraft, and marine vessels. The knowledge and understanding of the design and analysis of such revolutionary machines is limited, impeding further advances in research and development, both in the academia and the industry. This thesis addresses the knowledge gap by comprehensively analyzing, using first principles, the operation of such machines in a systematic manner.

In this book, which is the outcome of around 4 years of research, basic electromagnetic and machine theories are synthesized to develop the complex magnetic fields in generalized FSCW IPM machines. Two models are proposed and assessed for such machines: A detailed electromagnetic model for conceptual understanding and insight of the working principles; and a less complex electrical model for facilitating precise control of the machine. Furthermore, based on the rigorous analysis framework, for the first time, a general design methodology is proposed for multiphase machines using FSCW windings.

The original ideas presented in this book support the evolution of electric drive industries and can shape the further development of advanced electrical machines for the energy sector.

Sydney, NSW, Australia
January 2018

Prof. John E. Fletcher
UNSW Sydney

Abstract

This thesis focuses on the analytical modeling of fractional-slot concentrated-wound (FSCW) interior permanent magnet (IPM) machines and establishes a basis for their magnetic and electrical analysis. In the state-of-the-art methods for analyzing such machines, the nonhomogeneous magnetic saturation and the nonlinear B - H curve of the rotor iron are not considered. Moreover, the effect of the FSCW stator on the machine magnetic characteristics is overlooked.

Aiming at precise modelling of FSCW IPM machines' magnetic and electrical characteristics, a comprehensive mathematical treatment of the stator magnetomotive force (MMF), the IPM rotor nonhomogeneous magnetic saturation, and its airgap flux density are presented. The FSCW stator spatial MMF harmonics are analytically formulated, based on which, a novel heuristic algorithm is proposed for the design of optimal winding layouts for multiphase FSCW stators with different slot/pole combinations.

The nonhomogeneous magnetic saturation of the rotor iron due to its B - H curve and the residual flux of the embedded magnets is modeled and a saturation map is proposed for the rotor iron. Accordingly, a novel airgap function is proposed for FSCW IPM machines taking into account the effect of the FSCW stator and the nonhomogeneously saturated rotor. A precise mathematical model is then proposed for calculation of the airgap PM flux density.

The proposed mathematical models for the FSCW stator and the IPM rotor are combined to derive detailed mathematical expressions for its operational inductances, electromagnetic torque, torque ripple, and their respective subcomponents, as a function of the machine geometry and design parameters. Both normal operation of the machine and open-phase fault condition are considered in the aforementioned formulations. A "maximum torque per ampere" algorithm is then proposed for the machine under an open-phase fault condition in which customized currents are injected such that maximum average torque with a low torque ripple is

guaranteed. Lastly, the derived geometry-based models for the machine characteristics are used to propose an extended dq model for FSCW IPM machines which takes into account its non-sinusoidal parameters.

The proposed theories and analytical models are validated using finite element analysis and experimental tests on a prototype FSCW IPM machine.

Parts of this thesis have been published in the following articles:

Journal Publications

- [1] **M. Farshadnia**, M. A. Masood Cheema, A. Pouramin, R. Dutta and J. Edward Fletcher, “Design of Optimal Winding Configurations for Symmetrical Multiphase Concentrated-Wound Surface-Mount PMSMs to Achieve Maximum Torque Density Under Current Harmonic Injection,” in *IEEE Transactions on Industrial Electronics*, vol. 65, no. 2, pp. 1751–1761, Feb. 2018.
- [2] **M. Farshadnia**, M. A. M. Cheema, R. Dutta, J. E. Fletcher and M. F. Rahman, “Detailed Analytical Modeling of Fractional-Slot Concentrated-Wound Interior Permanent Magnet Machines for Prediction of Torque Ripple,” in *IEEE Transactions on Industry Applications*, vol. 53, no. 6, pp. 5272–5283, Nov.–Dec. 2017.
- [3] **M. Farshadnia**, M. A. M. Cheema, R. Dutta and J. E. Fletcher, “Analytical Modeling of Armature Reaction Air-Gap Flux Density Considering the Non-Homogeneously Saturated Rotor in a Fractional-Slot Concentrated-Wound IPM Machine,” in *IEEE Transactions on Magnetics*, vol. 53, no. 2, pp. 1–12, Feb. 2017.

Conference Publications

- [4] **M. Farshadnia**, A. Pouramin, R. Dutta and J. E. Fletcher, “Airgap magnetic field estimation for IPM rotors considering their non-uniform local saturation,” 2017 20th International Conference on Electrical Machines and Systems (ICEMS), Sydney, NSW, 2017, pp. 1–6.
- [5] **M. Farshadnia**, M. A. M. Cheema, R. Dutta, J. E. Fletcher and M. F. Rahman, “Detailed analytical modelling of fractional-slot concentrated-wound interior permanent magnet machines for prediction of torque ripple,” 2016 IEEE Energy Conversion Congress and Exposition (ECCE), Milwaukee, WI, 2016, pp. 1–8.
- [6] **M. Farshadnia**, M. A. M. Cheema, R. Dutta, J. E. Fletcher, H. C. Lovatt and M. F. Rahman, “An extended dq model for concentrated-wound interior permanent magnet machines considering non-ideal machine parameters,” 8th IET International Conference on Power Electronics, Machines and Drives (PEMD 2016), Glasgow, 2016, pp. 1–6.
- [7] **M. Farshadnia**, M. A. M. Cheema, R. Dutta and J. E. Fletcher, “Analytical modeling of pulsating torque in concentrated-wound interior permanent magnet machines to achieve maximum average torque under an open-phase fault condition,” 2016 IEEE 8th International Power Electronics and Motion Control Conference (IPEMC-ECCE Asia), Hefei, 2016, pp. 2813–2819.
- [8] **M. Farshadnia**, M. A. M. Cheema, R. Dutta, J. E. Fletcher and M. F. Rahman, “Anomalies in experimental measurement of operational inducances of a concentrated-wound IPM machine under field-weakening region,” IECON 2015—41st Annual Conference of the IEEE Industrial Electronics Society, Yokohama, 2015, pp. 004880–004885.

- [9] **M. Farshadnia**, R. Dutta, J. E. Fletcher, K. Ahsanullah, M. F. Rahman and H. C. Lovatt, “Analysis of MMF and back-EMF waveforms for fractional-slot concentrated-wound permanent magnet machines,” 2014 International Conference on Electrical Machines (ICEM), Berlin, 2014, pp. 1976–1982.

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Contents

1	Introduction and Literature Review	1
1.1	Problem Statement and Research Motivations	1
1.2	Research Objectives and Contributions	4
1.3	Literature Review	6
1.3.1	Types of PMSMs	6
1.3.2	Machine Magnetic Characteristics	9
1.3.3	Detailed Electrical Models and Equivalent Circuits for the Machine	14
1.4	Thesis Outline	15
1.5	Conclusion	17
	References	18
2	Analytical Modelling of Stator Magnetic Characteristics in Fractional-Slot Concentrated-Wound Permanent Magnet Machines	23
2.1	Introduction	23
2.2	Properties of an Optimal FSCW Stator Winding Layout	24
2.2.1	Unbalanced Magnetic Pull	25
2.3	Derivation of MMF Expression in FSCW Stators	26
2.3.1	Basic Principles of MMF in FSCW Stators	27
2.3.2	Analysis and Formulation of the MMF Produced by FSCW Stators	30
2.3.3	MMF Expressions for the Most Common Three-Phase FSCW PMSMs	36
2.4	Fault-Tolerant Capability in FSCW Machines	46
2.5	Formulation of the Harmonic Winding Factors	47
2.5.1	MMF Expression as a Function of the Harmonic Winding Factor	53
2.6	Formulation of the Back-EMF	53

2.7	Case-Study: The Effect of the Slot and Pole Combination on the Back-EMF	56
2.8	Conclusion	58
	References	59
3	Design of Optimal Winding Layouts for Multiphase Fractional-Slot Concentrated-Wound Permanent Magnet Machines	61
3.1	Introduction	61
3.2	Characteristics of an Optimal Winding Layout in Multiphase FSCW PMSM	62
3.3	Determination of the Optimal Winding Layout for Multiphase FSCW Stators	63
3.3.1	Definition of the Winding Performance Index	64
3.3.2	A Heuristic Algorithm for Design of Optimal Winding Layout	65
3.4	Case-Studies	68
3.4.1	Three-Phase FSCW PMSM with 14 Poles and 18 Slots	68
3.4.2	Five-Phase FSCW PMSM with 48 Poles and 50 Slots	68
3.4.3	Five-Phase FSCW PMSM with 18 Poles and 20 Slots	76
3.4.4	Five-Phase FSCW PMSM with 28 Poles and 30 Slots	79
3.5	Validation of the Results Using FEA and Experiment	82
3.5.1	Five-Phase FSCW PMSM with 48 Poles and 50 Slots	82
3.5.2	Five-Phase FSCW PMSM with 18 Poles and 20 Slots	86
3.5.3	Three-Phase FSCW PMSM with 14 Poles and 18 Slots	89
3.6	Conclusion	93
	References	93
4	Analytical Modelling of Rotor Magnetic Characteristics in an Interior Permanent Magnet Rotor	95
4.1	Introduction	95
4.2	Principles of Magnetic Circuits in an IPM Machine	95
4.2.1	Properties of the Magnetic Core Material	98
4.3	Geometrical Relationships in a V-shaped IPM Rotor	101
4.4	Geometric Analysis of the PM Flux Paths and Non-uniform Distribution of Saturation in a V-shaped IPM Rotor	104
4.5	Calculation of the PM Flux Density Distribution in the Air-Gap	111
4.5.1	State of the Art Rectangular Model for PM Flux Density	112
4.5.2	Proposed Trapezoidal Model for PM Flux Density	114
4.6	Case-Study: Validation of the Proposed Models	116
4.6.1	Calculation of the Non-uniform Saturation Model and the PM Flux Density	116

4.6.2	Estimation of the Back-EMF Based on the Calculated PM Flux Densities	123
4.7	Conclusion	124
	References	125
5	Calculation of Airgap Function and Inductance in Fractional-Slot Concentrated-Wound Interior Permanent Magnet Machines	127
5.1	Introduction	127
5.2	Inductances in FSCW PMSMs	127
5.2.1	Magnetizing Inductance and Airgap Leakage Inductance	128
5.2.2	Slot Leakage Inductance	131
5.2.3	Tooth-Tip Leakage Inductance	133
5.2.4	End-Winding Leakage Inductance	133
5.3	Modelling of the Equivalent Airgap Function	134
5.3.1	State of the Art Equivalent Airgap Function	134
5.3.2	Proposed Equivalent Airgap Function Taking into Account the Non-uniformly Saturated Stator	136
5.4	Case-Study: Validation of the Proposed Airgap Function	146
5.5	Conclusion	153
	References	154
6	Detailed Analytical Modelling of Inductances and Electromagnetic Torque in Fractional-Slot Concentrated-Wound Interior Permanent Magnet Machines Under Healthy and Open-Phase Fault Conditions	155
6.1	Introduction	155
6.2	Detailed Modelling of FSCW IPM Machine Operational Inductances	156
6.2.1	Analytical Model of the Self-inductance	156
6.2.2	Analytical Model of the Mutual Inductance	160
6.3	Detailed Modelling of FSCW IPM Machine Electromagnetic Torque Under Normal Operating Condition	162
6.3.1	Formulation of Alignment Torque	164
6.3.2	Formulation of Reluctance Torque Equation	168
6.3.3	Complete Torque Equation	172
6.3.4	Validation Through Finite Element Analysis	173
6.4	Detailed Modelling of FSCW IPM Machine Electromagnetic Torque Under an Open-Phase Fault Condition	175
6.4.1	Stator MMF Under an Open-Phase Fault Condition	177
6.4.2	Alignment Torque in a FSCW IPM Machine Operating Under an Open-Phase Fault Condition	179
6.4.3	Reluctance Torque in a FSCW IPM Machine Operating Under an Open-Phase Fault Condition	181

6.4.4	Complete Torque Equation Under an Open-Phase Fault	183
6.4.5	Validation Through Finite Element Analysis	184
6.5	Comparison of the FSCW IPM Machine Performance Under Normal and Open-Phase Fault Conditions	190
6.6	Conclusion	191
	References	192
7	An Extended dq Model for Fractional-Slot Concentrated-Wound Interior Permanent Magnet Machines Considering Non-ideal Machine Parameters	195
7.1	Introduction	195
7.2	Standard dq Model of PMSMs	196
7.2.1	Machine Model in the abc -System	196
7.2.2	Transformation of the Machine Model to the $qd0$ -Reference Frame	198
7.2.3	Electromagnetic Torque in the $qd0$ -Reference Frame	201
7.3	Proposed Extended dq Model for FSCW PMSMs	202
7.3.1	Detailed Machine Model in the abc -System	203
7.3.2	Transformation of the Detailed Machine Model to the $qd0$ -Reference Frame	204
7.3.3	Formulation of the Electromagnetic Torque Using the Proposed Extended dq Model	209
7.3.4	Experimental Tests for Determination of the Parameters in the Proposed Extended dq Model	210
7.4	Experimental Results and Discussion	212
7.4.1	Measurement of the Extended dq Model Parameters	212
7.4.2	Estimation of the Average Torque and Torque Ripple for the Prototype FSCW IPM Machine	214
7.5	Conclusion	217
	References	218
8	Conclusions and Future Works	219
8.1	Conclusions	219
8.1.1	Stator Magnetic Characteristics	219
8.1.2	Rotor Magnetic Characteristics	220
8.1.3	Overall Performance Characteristics for the Machine	220
8.2	Suggestions for Future Work	221
	Appendix A: Analytical Calculation of Magnetic Reluctance and Magnetic Permeance in Objects of Different Geometrical Shapes	223
	Appendix B: Developed Code in Matlab for Computing the Proposed Equivalent Airgap Function	229

Appendix C: Experimental Measurement of the FSCW IPM

Machine Inductances 239

About the Author 251

Abbreviations

2D	Two-dimensional
3D	Three-dimensional
HCI	Harmonic current injection
DRM	Dynamic reluctance mesh
DW	Distributed-wound
EMF	Electromotive force
FEA	Finite element analysis
FFT	Fast Fourier transform
FSCW	Fractional-slot concentrated-wound
GCD	Greatest common divisor
IPM	Interior permanent magnet
LCM	Least common multiple
MCL	Magnetic circuit length
MEC	Magnetic equivalent circuit
MMF	Magnetomotive force
MTPA	Maximum torque per ampere
PM	Permanent magnet
PMSM	Permanent magnet synchronous machine
SPM	Surface permanent magnet
THD	Total harmonic distortion
VSI	Voltage-source inverter
WFT	Winding function theory
WPI	Winding performance index

Chapter 1

Introduction and Literature Review



1.1 Problem Statement and Research Motivations

Fractional-slot concentrated-wound (FSCW) permanent magnet synchronous machines (PMSMs) are an attractive candidate for many industrial and household applications. This is because of their simpler manufacturing process and attractive characteristics such as higher power-to-weight ratio, and wider constant power speed range, compared with their distributed-wound (DW) counterparts. Among the several types FSCW PMSMs, those that use an interior permanent magnet (IPM) rotor are becoming more popular. This is because of their better mechanical robustness and superior performance.

With an aim to improve the understanding of FSCW IPM machines and their utilization, the main motivations behind the research reported in this thesis, are:

1. *The lack of accurate analytical models in the literature for FSCW PMSM characteristics, such as the stator and rotor magnetic fields, flux linkages, back-EMFs, inductances, and the developed average torque and torque ripple under healthy and open-phase fault conditions.*

It is essential during the design and analysis of FSCW IPM machines to have a thorough understanding of the magnetic field interactions in the machine. In particular, precise analytical formulae are required for calculation of machine characteristics, such as stator and rotor magnetic fields, flux linkages, back-EMFs, inductances, and the developed average torque and torque ripple. This assists in determining the effect of different geometrical parameters on the machine performance characteristics.

Two sources of magnetic fields are present in a FSCW PMSM:

- The field generated by the FSCW stator, and
- the field developed by the IPM rotor.

Electromagnetic torque in a PMSM is developed by the interaction of the stator and rotor magnetic fields in the airgap. This makes accurate analytical models of the magnetic fields essential to correct prediction of the torque.

The magnetic field intensity of a FSCW stator in the airgap of the machine is generated by the stator magneto-motive force (MMF) across the airgap and has the following relationship:

$$H_s(t, \theta_s) = \frac{1}{g(t, \theta_s)} MMF(t, \theta_s) \quad (1.1)$$

where θ_s is the angle subtended along the stator inner circumference, and g is the equivalent airgap length and is a function of time, t , and θ_s . The MMF of a FSCW stator is rich in spatial harmonics with their amplitudes determined by the slot and pole combination of the machine. According to (1.1), this yields harmonics in the stator magnetic field.

The stator MMF is also used for calculation of the machine inductances. The self- and mutual inductances between phases x and y of a FSCW PMSM can be analytically calculated from their respective winding functions as follows:

$$L_{xy} = \mu_0 r l_{st} \int_0^{2\pi} \frac{w_x(\theta_s) w_y(\theta_s)}{g(t, \theta_s)} d\theta_s \quad (1.2)$$

where w_x and w_y are the winding functions of phases x and y . Equation (1.2) gives self-inductance of phase x by setting $x = y$. The winding function of phase x is found from the MMF by setting the current of their respective phases equal to unity:

$$w_x(\theta_s) = MMF(t, \theta_s)|_{i_x=1} \quad (1.3)$$

From (1.1) to (1.3) it is observed that knowledge of the MMF and the equivalent airgap function are essential to calculating the stator magnetic field and inductances. Thus, analytical modelling of the MMF and equivalent airgap function are the first steps to formulating the machine characteristics. Nevertheless, these topics have not received enough attention in the literature. There is only sparse research reporting analytical modelling of the MMF produced by FSCW stators with different slot and pole combinations. Moreover, the existing equivalent airgap function that is being used in the literature for IPM machines is adopted from the inset-permanent magnet (PM) machine theory and returns inaccurate results for IPM machines. Therefore, for a FSCW IPM machine, analytical modelling of the stator MMF and machine equivalent airgap function are essential to correct calculation of the stator magnetic field and inductances, and subsequently torque and torque ripple.

The magnetic field produced by an IPM rotor originates from the residual flux of the embedded magnets in the rotor. The magnetic field intensity in the airgap generated by the rotor PMs can be calculated from the PM flux density, B_{PM} , as a function of t and θ_s and as follows:

$$H_r(t, \theta_s) = \frac{1}{\mu_0} B_{PM}(t, \theta_s) \quad (1.4)$$

An accurate estimation of the PM flux density is essential for the calculation of the rotor field intensity. The residual flux of the embedded PMs in the rotor causes a non-uniform distribution of magnetic saturation in the rotor iron and should be taken into consideration in the calculation of the PM flux density. The current methods in the literature for analytical calculation of the PM flux density in IPM rotors are proven to yield inaccurate results, which is mainly due to neglecting the non-homogenously saturated rotor iron and the assumption of a rectangular PM flux density in the airgap. This makes correct analytical modeling of the magnetic saturation of the rotor iron and the PM flux density essential to accurate calculation of the rotor magnetic field. The non-homogeneous saturation of the rotor iron also affects the equivalent airgap function of the machine. However, this has not been taken into account in the existing literature.

Once accurate analytical models for the non-homogenously saturated rotor iron, rotor magnetic field, machine equivalent airgap function, and the stator MMF under healthy and open-phase fault conditions are obtained, in addition to the inductances and developed torque, analytical calculation of the back-EMF and PM flux linkages also becomes feasible.

2. *The lack of a comprehensive algorithm for designing multiphase FSCW stators that would yield maximum torque density considering all the torque production field harmonics.*

A thorough understanding of the stator and rotor magnetic fields through their analytical models enables systematic design of the stator winding configuration in a multi-phase FSCW PMSM. In an m -phase PMSM, injection of current harmonics with an odd order smaller than m increases the developed average torque. The existing methods in the literature for designing the winding configuration of multi-phase FSCW stators are based on the spatial field harmonics that are associated with the fundamental harmonic of the injected current, and no account is taken of the spatial field harmonics associated with the higher order current harmonics. In order to have a global design procedure for multiphase FSCW stators that would yield maximum torque density, all the field harmonics that can contribute to the average torque must be considered.

3. *The lack of an accurate dq model for FSCW PMSMs that considers all the machine non-ideal parameters.*

The standard dq model used for PMSMs is based on the assumption of a sinusoidal MMF which yields sinusoidal machine parameters. However, for a FSCW PMSM, the generated MMF by the stator is not sinusoidal and contains a wide range of spatial harmonics. This affects other parameters of the machine such as electromagnetic torque, back-EMFs, PM flux linkages, and inductances. Currently, the standard dq model of a PMSM is mainly used in the literature for

FSCW IPM machines. This requires formulating a general dq model for FSCW IPM machines that takes into account all the machine non-idealities.

1.2 Research Objectives and Contributions

As a first step in accurate calculation of the FSCW IPM machine characteristics, the accurate equivalent airgap function for the machine should be calculated. From Sect. 1.1, the main shortcomings in the literature regarding analytical calculation of the equivalent airgap function for FSCW IPM machines are ignoring the complicated flux path, magnetic characteristics of the machine magnetic material, and the non-homogeneous saturation of the rotor iron.

These shortcomings are addressed in this thesis and an equivalent airgap function is proposed based on the magnetic circuit length of the stator flux paths inside the IPM rotor iron. To this end, the non-homogeneous saturation of the rotor iron in a V-shaped FSCW IPM machine is modelled by deriving mathematical formulae for the saturation intensity and relative permeability at different regions of the magnetically saturated rotor. This model is governed by the B - H curve of the rotor magnetic material and the machine geometry. The proposed saturation model is used to obtain the magnetic circuit lengths of the stator flux paths in an IPM rotor, which is later used for computing the equivalent airgap function in an FSCW IPM machine.

In contrast with the existing methods where the equivalent airgap function is just dependent on the angle subtended along the rotor circumference, the research conducted in this thesis reveals that equivalent airgap length is a function of the rotor angular position as well as the angle subtended along the rotor circumference.

The proposed equivalent airgap function in this thesis is compatible for use in the standard formulae and techniques for analytical derivation of machine performance characteristics. Accordingly, PM flux linkages, armature reaction airgap flux density, and the machine inductance will be analytically formulated in this thesis.

Apart from the equivalent airgap function, correct analytical formulae for the stator MMF are also essential to field calculations in a FSCW IPM machine. With an aim to fill the knowledge gap in this regards, a thorough analysis of the MMF of FSCW stators with different slot and pole combinations are conducted, leading to proposition of lumped analytical formulae for the MMF. These analytical expressions are used to derive formulae for the harmonic winding factors of the machine which will be later used in formulating the PM flux linkages, back-EMFs, torque, and torque ripple.

The other essential component in the analytical calculations of the IPM machine characteristics is the PM flux density. As explained in Sect. 1.1, the current analytical models for the PM flux density are incapable of correctly predicting this quantity for IPM rotors. In order to address this shortcoming, a trapezoidal model is proposed in this thesis for the PM flux density that helps in a more accurate

calculation of the machine output characteristics. The parameters of the proposed PM flux density model are based on the proposed equivalent airgap function.

The other contribution of the research reported in this thesis is proposing detailed formulae for the electromagnetic torque and torque ripple for an FSCW IPM machine under both healthy and open-phase fault conditions. These models provide a tool for customizing the input currents to the machine such that the output torque under an open-phase fault condition gives maximum achievable average torque with less torque ripple compared with the state of the art methods.

The obtained detailed expressions for the FSCW IPM machine in this thesis are further used to propose an extended dq model that takes into account all the machine non-ideal parameters. An equivalent circuit is also proposed for the extended dq model which is later used in formulating a detailed expression for the developed torque of the machine in terms of the proposed extended dq model parameters.

The final contribution of this thesis is a heuristic algorithm for designing the winding configuration of multiphase FSCW stators with an objective of maximizing the torque density. As explained in Sect. 1.1, in the existing methods, only the fundamental torque producing harmonic of the magnetic field is considered. Contrary to the state of the art, in the proposed algorithm, all the torque producing harmonics in the magnetic field are taken into account in designing the winding configuration. A “*winding performance index*” is proposed for evaluating the torque density of different winding configurations. The proposed heuristic algorithm is based on the rigorous analysis performed on the MMF and harmonic winding factors in this thesis.

According to the above discussion, the main contributions of the research reported in this thesis are:

- Analytical modelling of the MMF produced by FSCW stators with different slot and pole combinations, leading to lumped formulae for the MMF and harmonics winding factors of the machine
- Formulation of a general heuristic algorithm for designing the optimal winding layout in multiphase FSCW PMSMs
- Analytical modelling of the non-homogeneous magnetic saturation in the rotor iron due to the residual flux of the embedded magnets
- Analytical modelling of the airgap function in a FSCW PMSM taking into account the effect of the non-homogeneously saturated rotor iron and the FSCW stator
- Analytical modelling of the airgap PM flux density
- Analytical modelling of the torque and torque ripple for FSCW PMSMs operating under healthy and open-phase fault conditions
- Proposing customized input currents for the FSCW IPM machine such that maximum average torque under an open-phase fault condition is generated
- Formulation of an extended dq model for FSCW PMSMs that takes into account all the machine non-idealities.

1.3 Literature Review

Several types of PMSMs have been investigated in the literature, for which, a brief review is first presented in this section. An overview on the choice of slot and pole combination in a FSCW PMSM which determines its electric and magnetic characteristics follows. The literature concerning analytical modelling of the electric and magnetic characteristics of FSCW PMSMs are then reviewed which fall into the following categories:

- Analytical modeling of the machine magnetic characteristics such as stator and rotor spatial field distributions, the equivalent airgap function, stator MMF, and rotor PM flux density. These magnetic characteristics can be used in formulating detailed expressions for the machine output characteristics such as inductances, flux linkages, back-EMFs, and electromagnetic torque.
- Detailed electrical models and equivalent circuits for the machine.

1.3.1 Types of PMSMs

Radial flux PMSMs are commonly categorized with respect to their rotor and stator types. The stator utilized in a PMSM is usually either from the FSCW or DW stator families as shown in Fig. 1.1, each of them with their own distinct features; while the rotor can be chosen from a vast range of structures including, surface PM rotors, inset PM rotors, and IPM rotors as shown in Fig. 1.2.

DW stator topologies are the major stator type used in PMSMs. This is due to their near sinusoidal MMF which yields a high main harmonic winding factor and low torque ripple. It was not until very recently that it was shown that the right choice of slot and pole combination for a FSCW stator could yield a high main harmonic winding factor which is essential to having a high average torque [1, 2]. As shown in Fig. 1.1, contrary to DW stators, the windings in a FSCW stator are non-overlapping which yields to a shorter end-winding resulting in a less copper usage, less copper losses, and a more compact design. Other advantages of FSCW stators over their DW counterparts are the easier manufacturing process, higher slot fill-factor, wider field-weakening region caused by their higher self-inductance, reduced cogging torque, and a better fault tolerant capability [2–6].

The important factors in designing a FSCW PMSM that affect the machine output characteristics are the number of slots and poles, and the number of winding layers in the slots. The most common FSCW stators are either single-layer or double-layer as shown in Fig. 1.1a and b, respectively. Each of these designs has its own benefits and characteristics. Compared with single-layer FSCW stators, double-layer stators feature lower spatial MMF harmonic content which lead to lower magnet eddy current losses, lower airgap leakage inductance, lower torque ripple at the cost of a reduction in the average torque, higher torque density leading

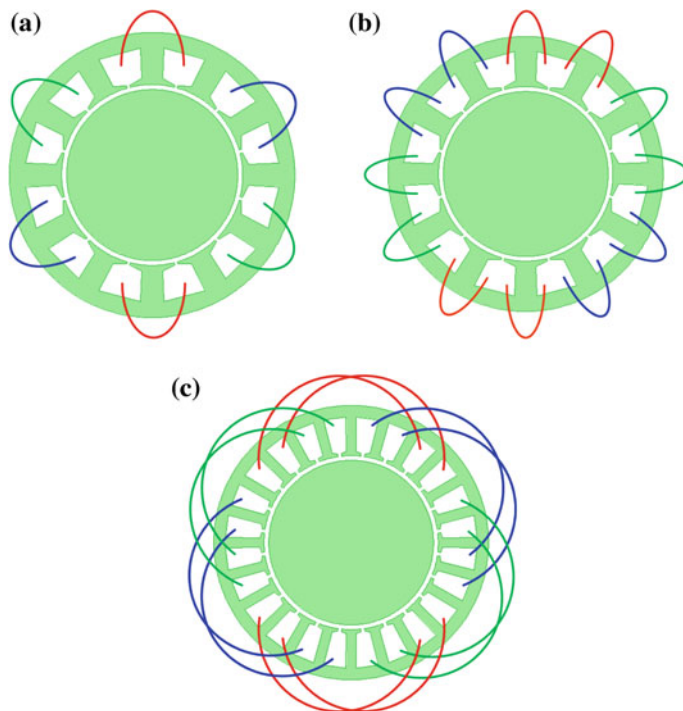


Fig. 1.1 Common stator types: **a** Single-layer FSCW stator. **b** Double-layer FSCW stator. **c** DW stator

to a more compact design, and higher efficiency [2, 7, 8]. The key advantage of single-layer FSCW stators over double-layer stators is their improved fault tolerance capability due to their phase windings being magnetically decoupled which yields a lower mutual inductance [3].

Not all combinations of slots and poles for an FSCW stator yield acceptable performance. Commonly, for a three-phase machine, the slot and pole combination should be chosen such that a sinusoidal back-EMF, a high main harmonic winding factor resulting in a high average torque, and reduced torque ripple and cogging torque, are achieved [9–11]. Various slot and pole combinations for a FSCW machines and design considerations for obtaining an optimal design are investigated in [3, 12–16].

It is the choice of the design engineer to choose the most suitable type of winding and slot and pole combination for designing application-oriented FSCW PMSMs.

Another design choice for PMSMs is the rotor topology for which the most common types was shown in Fig. 1.2. In Surface PM (SPM) rotors and some inset-PM rotors the magnets are glued to the rotor surface, directly accessing the airgap, as shown in Fig. 1.2a and b, respectively. However, in IPM machines, the

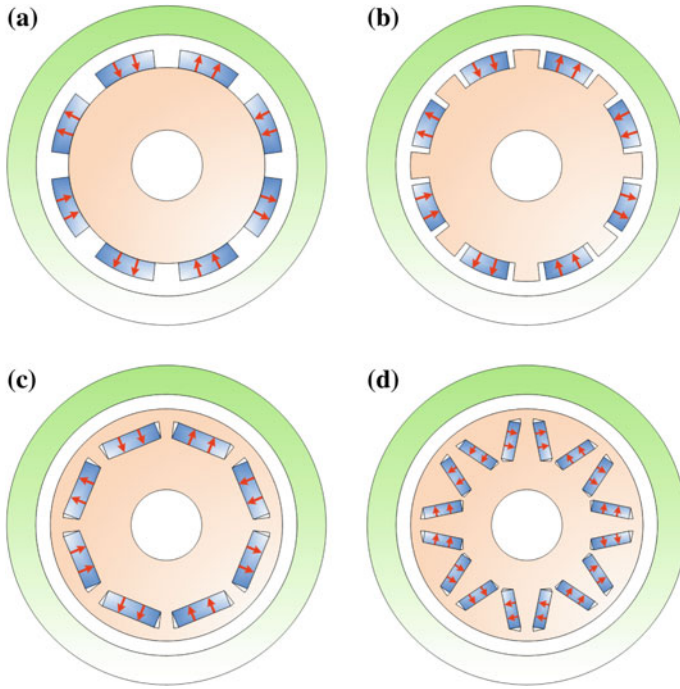


Fig. 1.2 Common PM rotor types: **a** Surface PM rotor. **b** Inset PM rotor. **c** Flat-shaped IPM rotor. **d** V-shaped IPM rotor

magnets are buried inside the rotor iron, which makes IPM machines superior over SPM and inset PM machines in several aspects, including [5, 17–20]:

- Higher power rating.
- Smaller eddy current losses in the PMs and smaller possibility of their demagnetization since they are not directly exposed to the airgap.
- Lower iron losses under open-circuit conditions.
- Higher overall efficiency.
- Smaller airgap, making the machine more suitable for flux-weakening operation. In SPM machines, since the permeability of the magnets is close to that of air, the effective airgap becomes relatively large.
- Increased mechanical robustness of the rotor because the magnets are buried inside the rotor iron. This makes the rotor capable of withstanding higher speeds.
- Presence of a reluctance torque component compared with SPM machines due to the saliency of IPM machines. This leads to production of more torque and can be used to extend and enhance the flux-weakening operation.
- Easier sensorless control of the machine because of the saliency effect.

1.3.2 Machine Magnetic Characteristics

Electromagnetic torque generation in a PMSM originate from the interaction between the rotor and stator magnetic fields in the airgap. Several methods have been explored in the literature for obtaining the magnetic characteristics of electric machines, among which, numerical techniques such as dynamic reluctance mesh (DRM) modelling [21–23] and finite element analysis (FEA) [24–27] are the most accurate ones that consider magnetic saturation in the rotor and stator iron. This has led to their extensive use in the past two decades for analysis and enhanced design of electric machines. Nevertheless, these techniques suffer from the common drawback of being time consuming. Besides, they cannot provide an insight into the effect of different geometry parameters on the machine performance characteristics.

In order to overcome the shortcomings of numerical techniques, analytical methods have been used in the literature for computing the magnetic fields in electric machines. Among analytical techniques, direct analytical methods that are based on the solution of Laplacian or quasi-Poissonian field equations are most accurate [28–31]. These equations are solved by applying boundary conditions on the interface between the stator slots, magnets, and the airgap. The main assumption in these techniques is a negligible reluctance for the stator and rotor yoke which means neglecting the saturation.

Direct analytical methods are commonly used for inset-PM and SPM rotors [28–30, 32]. However, for an IPM rotor, the embedded magnets in the rotor saturate the iron bridges, complicating the boundary conditions which in turn make analytical solution of Laplacian or quasi-Poissonian equations unfeasible. This has led to introduction of analytical methods that are based on magnetic equivalent circuit (MEC) model and winding function theory (WFT) [33–36] of the IPM machine [37–39]. In these techniques, flux sources are used to model the saturated rotor iron bridges. These flux sources produce a constant flux density of B_{sat} which is the saturated flux density of the rotor iron and obtained from its B - H curve. In MEC- and WFT-based methods, similar to direct analytical methods, the rotor and stator yoke reluctances are often neglected, thus saturation due to the stator currents is not considered. Such assumptions could lead to negligible errors in the airgap magnetic field calculation [23, 37–41].

1.3.2.1 Equivalent Airgap Function

Magnetic reluctance and the MMF across the reluctance are the crucial elements in MEC- and WFT-based techniques. In these methods, a constant MMF equal to that of the stator is assumed across a position variant equivalent reluctance. This equivalent reluctance is realized by an equivalent airgap length that is a function of the angle subtended along the rotor circumference. Magnetic potential distribution in SPM and inset-PM machines is uniform on the rotor surface. Thus, because of