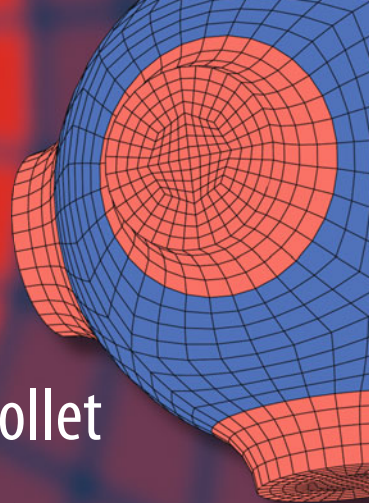


Advanced Structured Materials

Holm Altenbach · Joël Pouget
Martine Rousseau · Bernard Collet
Thomas Michelitsch *Editors*



Generalized Models and Non-classical Approaches in Complex Materials 1

Advanced Structured Materials

Volume 89

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*Dedicated to the memory of a great creative
spirit, G.A. Maugin*

Foreword

Gérard A. Maugin born at Angers (France) on December 2, 1944, married to Eleni Zachariadou in 1978, passed away in Villejuif (France) on September 22, 2016, 7 p. m.

He had retired from the University of Paris VI since 2010. His longstanding scientific activity in Continuum Mechanics and Continuum Physics is well known in the Community of Mechanics. In these fields he enjoys a well-established reputation. His interests covered almost all disciplines of Continuum Mechanics and his studies have been addressed fundamental problems of mechanics and electromagnetism and applications as well.



Gérard A. Maugin

One of his first papers (1965) is concerned with *The Race Tidal Power Plant*, a topical subject in the current engineering applications. A few years later he published a series of papers in the *Comptes Rendus de l'Académie des sciences, Paris* (1970-71), on the macroscopic description of magnetic media in the relativistic framework. His striking versatility in scientific research, which emerged since the very beginning of his career, cannot be unnoticed. In April 1971 he defended his PhD dissertation thesis on micromagnetism, supervisor Prof. Cemal A. Eringen from Princeton University. The Princeton University Press has published the thesis with the title *Micromagnetism and Polar Media*, 1-294, (1971). Four years later, in May 1975, Prof. Maugin achieved his “habilitation” (Doctorat d'Etat en Sciences Mathématiques) in Paris, supervisor Prof. Paul Germain de l'Académie des sciences.

By 1975, Gérard Maugin already had a ripe scientific *curriculum studiorum* in Mechanics and Physics: 45 papers published in the most well known scientific journals of mechanics and mathematics (*Ann. Inst. Henri Poincaré*, *J. of Physics*, *J.*

of Mathematical Physics, General Relativity and Gravitation, J. de Mécanique and others).

His favourite topics in this period are the behaviour of electromagnetic materials in the relativistic framework and in the Galilean approximation as well. Specifically, the behaviour of deformable dielectrics, ferro-magnetic and ferri-magnetic bodies are examined and explored in such frameworks.

His training in relativity and in electromagnetism presumably developed in his mind a specific sensitivity toward the mathematical description of Continua with coupled-fields, Continua with structures and/or Microstructures.

In 1980 Gérard Maugin published the paper *The method of virtual power in Continuum Mechanics: Application to Coupled Fields*, Acta Mechanica, 35, 1-70, (1980). The energetic approach therein proposed represents one of the most powerful methods for describing complex materials from the viewpoint of continua. The method also provides the proper tools, with which to attack problems of structured continua, both, from the theoretical viewpoint and from the standpoint of applications. The method of virtual power, such as expounded in the aforementioned paper, is formulated in its most general form and is applied to electromagnetic materials in their various aspects (thermo-elastic dielectrics with polarisation gradients, dielectrics with quadrupoles, ferromagnets, liquid crystals in external electromagnetic fields, et cetera). This contribution of Prof. Maugin stands as a referential point to many searchers in continuum mechanics.

Wave propagation was also one of his favourite topics of interest. To this topic he devoted his attention and his studies since the very beginning of his studies. Due to the interesting results achieved in applied problems of wave propagation, he was awarded by a scientific prize, the Prize of Mechanics Doisteau-Blutet of the French Academy of Sciences in 1982. His interest in wave propagation never ceased nor decreased in the subsequent years, even when his main efforts were focused on other fields. As a result of the expertise that he had acquired in this field, Gérard Maugin was invited to deliver a course on *Physical and Mathematical Models of Nonlinear Waves in Solids*, in Udine, at the International Centre for Mechanical Sciences (CISM), in 1993. Springer-Verlag will publish the lecture notes of this course in the series CISM Courses and Lectures. Afterward, he also published the book *Nonlinear Waves in Elastic Crystals*, Oxford University Press (1999).

This specific attention to the dynamical problems in continua is often transferred to his graduate students. Some of them investigated the possibility of “soliton-propagation” in structured materials, under his advice. Interesting and unexpected results are shown in evidence by their studies, with the help of numerical techniques.

Gérard Maugin not only provided his students with an excellent professional training in Continuum Mechanics and Physics he also transferred to his students and co-workers enthusiasm in research along with motivations and scientific curiosity. These qualities represent the primary source of his prolific scientific activity.

An impressive number of papers and many books and monographs (published by Springer-Verlag, McGraw-Hill, Elsevier, Oxford University Press UK, Cambridge University Press UK) emerge from his Curriculum and numerous awards and honours.

A detailed list can be found on the website:

<http://www.dalembert.upmc.fr/home/maugin/>.

G rard Maugin was a member of the editorial board of many scientific journals, among them

- International Journal of Engineering Science from 1976 to December 1995,
- Wave Motion since 1986,
- Journal of Thermal Stresses,
- Journal of Technical Physics of the Polish Academy of Science,
- International Journal of Applied Electromagnetics and Mechanics - one of its founders in 1990,
- Applied Mechanics Reviews - Associate Editor since 1985,
- Journal of Non-equilibrium Thermodynamics,
- Egyptian Journal of Mathematics,
- Archives of Applied Mechanics (formerly Ingenieur-Archiv),
- Yugoslav Journal of Mechanics,
- Archives of Mechanics of the Polish Academy of Science,
- ARI - Associate Editor since its creation 1997,
- Proceedings of the Estonian Academy of Science since 1997,
- Mechanics Research Communications - one of the five Editors since July 1999, and
- the Marocain Revue de M canique Appliqu e et Th orique.

He held a membership in scientific societies (in most of the cases as member of the executive committee or of the advisory board)

- Society of Natural Philosophy (SNP), USA,
- Society of Engineering Science, (SES), USA, Life member,
- Society for Industrial and Applied Mathematics (SIAM), USA,
- American Physical Society (APS), USA,
- American Mathematical Society (AMS), USA,
- Acoustical Society of America (ASA), USA,
- Gesellschaft f r angewandte Mathematik und Mechanik (GAMM), Germany,
- International Society for the Interaction of Mathematics and Mechanics (ISIMM) - Member of the Executive Committee 1986-1990, 1997-2001,
- Soci t  Fran aise d'Acoustique (SFA), France,
- Association Fran aise de M canique (AFM), France, and
- EUROMECH Society (European Society of Mechanics),

and was appointed as consulting editor (for Springer, John Wiley & Sons, Kluwer, Oxford University Press) or as expert for research contracts and grants (in USA, Canada, UK, Belgium, France and other countries). In addition, he acted as a Series Editor of the CRC Series on Continuum Modelling and Discrete Systems (CRC, Boca Raton, Florida, USA) and Applied Electromagnetics and Mechanics (Elsevier, then I.O.S. Press, The Netherlands).

He was a visiting professor and visiting scientist at Princeton, Belgrade, Warsaw, Istanbul, at the Royal Institute of Technology in Stockholm, at the TU Berlin, Rome, Tel Aviv, the Lomonosov University, Kyoto, Darmstadt and Berkeley. In 2001 he

received the Max Planck Research Award, was the 1991/92 Fellow of the Berlin Institute for Advanced Study, and in 2001 received an honorary doctorate from the Technical University of Darmstadt. In 1982 he received the mechanics Prize of French Academy of Sciences and in 1977 the Medal of the CNRS in physics and engineering. He was a member of the Polish Academy of Sciences (1994), of the Estonian Academy of Sciences and was awarded an honorary professorship by the Moscow State University. In 2003, he received the A. Cemal Eringen Medal.

I would like rather to emphasise his natural attitude as searcher and as teacher. This attitude combined with his skill in finding the proper (and often the simplest) mathematical tools, through which to expound and to clarify the physical nature of the phenomenon under consideration.

The so-called Configurational Mechanics or Material Mechanics is the “novel” field, to which Gérard Maugin devoted his main interest during the last decades. He initiated the search and the studies of configurational forces in elasticity, being concerned with the elastic energy-momentum tensor, a notion introduced by Eshelby in a few seminal papers in the fifties. It is not difficult to show that the Eshelby tensor naturally applies to defective materials and in fracture mechanics. For instance, based on this tensor, one is able to recover all known invariant integrals around a defect, including the celebrated J-integral around the tip of a crack. In addition, fracture criteria can be (and indeed, are) properly extended to elastic dielectrics and to elastic magnetised materials.

The early studies of Gérard Maugin and others in this field are also concerned with inhomogeneous materials. Specifically, Maugin and others re-proposed the Eshelby tensor in finite elasticity, basing on Noll’s notion of homogeneity and uniformity. Such an extension of the Eshelby tensor shows in evidence important physical properties and relevant geometrical features, which are hidden in the linear framework. All these features eventually address the notion of configurational force. Gérard Maugin and others suddenly realised that the notion of configurational force confers to the Eshelby stress tensor a deeper physical meaning. They also realised that the notion of configurational (or material) force could not be confined to the in-homogeneities in the elasto-static framework. Hence, the important role of this force was enquired in dynamics. One of the relevant results is the natural relationship of the material force with the so-called material-momentum, or pseudo-momentum. Such a result also represents a turning point for the introduction of the so-called configurational mechanics, which now stands on firm bases. In addition, configurational mechanics is also shown to be the natural framework for thermodynamical transformations, such as solid-phase-transitions.

The notion of configurational force becomes even more powerful in complex materials and materials with structures. Based on this notion, Gérard Maugin (with a second author) contributed to disentangling the following quarrel in liquid crystals (*Int. J. Engng. Sci.*, 33, 1663-1678, 1995): as to whether the Ericksen stress tensor should be regarded as related to a configurational force or to the classical traction. The point is that the Ericksen tensor for liquid crystals has the form of an energy-stress tensor, just like the Eshelby stress. Hence, one could be tempted to incorrectly

identify the one with the other. It is worth noticing that the quarrel involved Ericksen and Eshelby themselves, along with Kröner and other prominent people.

Eventually, the interest arises in discriminating configurational forces from traction in the more general context of structured continua. This interest becomes a crucial need in the case of electromagnetic materials. In this regard, it is worth recalling that Eshelby was initially inspired by the Maxwell stress tensor of electromagnetism. The latter however, though possessing the form of an energy-stress, is undoubtedly related to the classical traction. In order to avoid misunderstandings, one envisages the existence of two meaningful energy-stress-tensors in continua and, more specifically, in electromagnetic materials. The introduction of the material energy-stress (namely, the Eshelby tensor) provides a novel standpoint, which allows one to enlighten unclear issues or rather obscure aspects of electromagnetic materials. One of these is the proper form of the electromagnetic momentum. Basing on a criterion established by Gérard Maugin and others, one is able to distinguish between momentum and *pseudo-momentum* or *crystal-momentum*, in the language of Solid State Physics. These themes are still nowadays open to further developments. New applications of these ideas are proposed from time to time in the community of continuum mechanics, in which a steadily increasing interest is recorded on this subject.

It should be noted that Gérard Maugin delivered his knowledge and new research results immediately to the PhD and post graduated students. One of his loveliest places for lectures was the International Center of Mechanical Sciences (CISM, Udine, Italy), where he presented not only the aforementioned course on wave propagation. He was involved, for example, in the following activities:

- Non-Equilibrium Thermodynamics with Application to Solids (coordinated by W. Muschik in 1992): lectures on "Non-Equilibrium Thermodynamics of Electromagnetic Solids",
- Nonlinear Waves in Solids (coordinated by A. Jeffrey and J. Engelbrecht in 1993): lectures on "Physical and Mathematical Models of Nonlinear Waves in Solids",
- Configurational Mechanics of Materials (coordinated by R. Kienzler and G.A. Maugin in 2001): lectures on "Elements of Field Theory in Inhomogeneous and Defective Materials" and "Material Mechanics of Electromagnetic Solids" (together with C. Trimarco),
- Surface Waves in Geomechanics: Direct and Inverse Modeling for Soils and Rocks (coordinated by K. Wilmanski and C.G. Lai in 2004): 6 lectures on waves on interfaces, in thin layers on linear media, on surfaces with curvature, grating and roughness, nonlinear surface waves, propagation of surface soliton packages, interactions with nonmechanical fields,
- Generalised Continua and Dislocation Theory. Theoretical Concepts, Computational Methods and Experimental Verification (coordinated by C. Sansour in 2007): 4 lectures on defects, dislocations and the general theory of material inhomogeneities,
- Mechanics and Electrodynamics of Magneto- and Electro-Elastic Materials (coordinated by R.W. Ogden and D.J. Steigman in 2009): 7 lectures on the basics of electromagnetics in matter, with emphasis placed on the notions of electromagnetic forces, momentum and stresses, on the general thermomechanical framework,

and on applications to magnetoelasticity at different scales, the notions of internal stresses, internal variables, homogenization, ferromagnetic polycrystals and configurational force,

- Generalized Continua from the Theory to Engineering Applications (coordinated by H. Altenbach and V. Eremeyev in 2011): 6 lectures on electromagnetism and generalized continua, ponderomotive couple, electromagnetic microstructure, resonance couplings with classical deformation, effects on configurational forces (fracture and phase transformation).

G rard Maugin was also greatly attracted by researches in Epistemology and the History of Science. The naissance of fundamental concepts of Mechanics and Physics and their evolution through the centuries were fascinating topics for him. Toward these topics he had developed a unique sensitivity, since he was a young researcher. To them he devoted his efforts in the last years until the end of his life by writing a history of Continuum Mechanics in the following three volumes published by Springer, Solid Mechanics and Applications Series:

- Continuum Mechanics Through the Twentieth Centuries: A Concise Historical Perspectives (2013),
- Continuum Mechanics Through the Eighteenth and Nineteenth Centuries: Historical Perspectives from John Bernoulli (1727) to Ernst Hellinger (1914) (2014),
- Continuum Mechanics Through the Ages: From the Renaissance to the Twentieth Century (2016).

Last but not least, he offered clear and reliable explanations of over 100 keywords in Continuum Mechanics for better understanding the fundamental concepts

- Non-Classical Continuum Mechanics - A Dictionary (2017).

This book was published also by Springer in the Advanced Structured Materials Series (Series Editors: Andreas  chsner, Lucas F.M. da Silva, and Holm Altenbach) as volume 51.

His memory will endure among his many friends and in the Scientific Community of Mechanics.

Universit  di Pisa, Italia, January 2018

Carmine Trimarco

Preface

At the beginning of February 2017 the invitation letters for a special remembrance book were sent to approximately 70 friends and colleagues of the great French scientist in the field of Continuum Mechanics (or more general Continuum Physics) Gérard A. Maugin who died on September 22nd, 2016. As usual in such case that the response is 50% sending a kind reply that they will submit a paper and finally one gets 15-20 papers. In the case of Gérard the resonance was overwhelming - the editors got finally approximately 60 papers and the decision was made to publish two volumes. This is the first one including 40 papers from authors living in more than 20 countries.

The scientific interests of Gérard are well reflected by variety of subjects covered by the contributions to this book including the following branches of Continuum Mechanics

- relativistic continuum mechanics,
- micromagnetism,
- electrodynamics of continua,
- electro-magneto-mechanical interaction,
- mechanics of deformable solids with ferroic states (ferromagnetics, ferroelectrics, etc.),
- thermomechanics with internal state variables,
- linear and nonlinear surface waves on deformable structures,
- nonlinear waves in continua,
- Lighthill-Whitham wave mechanics,
- lattice dynamics,
- Eshelbian Mechanics of continua on the material manifold,
- geometry and thermomechanics of material defects,
- material equations and
- biomechanical applications (tissue and long bones growth).

In addition, he published several papers and books on the history of continuum mechanics. This was reason that the authors of this book have submitted so different papers with the focus on the research interests of Gérard.

We have to thank all contributors for their perfect job. Last but not least, we gratefully acknowledge Dr. Christoph Baumann (Springer Publisher) supporting the book project.

Magdeburg, Paris
January 2018

Holm Altenbach
Joël Pouget
Martine Rousseau
Bernard Collet
Thomas Michelitsch

Contents

1	Effective Coefficients and Local Fields of Periodic Fibrous Piezocomposites with 622 Hexagonal Constituents	1
	Ransés Alfonso-Rodríguez, Julián Bravo-Castillero, Renald Brenner, Raúl Guinovart-Díaz, Leslie D. Pérez-Fernández, Reinaldo Rodríguez-Ramos, and Federico J. Sabina	
1.1	Introduction	2
1.2	A Boundary Value Problem of the Linear Piezoelectricity Theory	3
1.3	Homogenization, Local Problems and Effective Coefficients	4
1.3.1	Explicit Form of the Homogenized Problem, Effective Coefficients and Local Problems	5
1.3.2	Local Fields	6
1.4	Application to a Binary Fibrous Piezocomposite with Perfect Contact Conditions at the Interfaces	7
1.5	Local Problems for Fibrous Composites with Constituents of 622 Hexagonal Class	9
1.5.1	Local Problems L^{23} and L^1	10
1.5.2	Effective Coefficients Related with the Local Problems L^{23} and L^1	14
1.5.3	Local Problems L^{13} and L^2 and Related Effective Coefficients	14
1.5.4	On the Computation of the Local Fields from the Solutions of the Local Problem L^{13}	17
1.6	Numerical Examples	18
1.6.1	Square Array Distribution	19
1.6.2	Rectangular Array Distribution	20
1.6.3	Spatial Distribution of Local Fields	21
1.7	Concluding Remarks	24
	References	24

2	High-Frequency Spectra of SH Guided Waves in Continuously Layered Plates	27
	Vladimir I. Alshits and Jerzy P. Nowacki	
2.1	Introduction	27
2.2	Statement of the Problem and Main Equations	28
2.3	The Propagator Matrix and Its Adiabatic Approximation	30
2.4	Boundary Problems and Their General Solutions	32
2.4.1	Spectral Regions Without Division Points	33
2.4.1.1	The Range $s < \min\{\hat{s}(y)\}$ (I_1 in Fig. 2.2)	33
2.4.1.2	The Range $s > \max\{\hat{s}(y)\}$ (I_2 in Fig. 2.2)	33
2.4.2	Spectral Regions with one Division Point	33
2.4.2.1	The Case $\hat{s}'(a) > 0$ (the Range II in Fig. 2.2)	34
2.4.2.2	The Case $\hat{s}'(a) < 0$	34
2.4.3	Spectral Regions with two Division Points	35
2.4.3.1	The Case $\hat{s}'(a) > 0, \hat{s}'(b) < 0$	35
2.4.3.2	The Case $\hat{s}'(a) < 0, \hat{s}'(b) > 0$ (the Range III in Fig. 2.2)	35
2.4.4	Extension for an Arbitrary Number of Division Points	36
2.4.4.1	An Odd Number of Division Points $N = 2n - 1$ ($n > 1$)	36
2.4.4.2	An Even Number of Division Points $N = 2n$ ($n > 1$)	38
2.5	The Low-Slowness Approximation and the Cut-Off Frequencies	39
2.6	Example of Inhomogeneity Admitting an Explicit Analysis	40
2.6.1	The Region $0 < s \leq \hat{s}_0$	41
2.6.1.1	The Cut-Off Frequencies of the Spectrum	41
2.6.1.2	Spectrum just Under the Level $s_l = \hat{s}_0$	42
2.6.2	The Region $\hat{s}_0 < s \leq \hat{s}_m$	43
2.6.2.1	Spectrum just Over the Level $s_l = \hat{s}_0$	44
2.6.2.2	Spectrum Under the Asymptote $s = \hat{s}_m$	44
2.7	Levels Related to Extreme Points on the Slowness Profile	45
2.7.1	An Absolute Minimum of the Function $\hat{s}(y)$	45
2.7.1.1	Spectral Features just Under the Level $s = \hat{s}_1$	45
2.7.1.2	Spectrum Features just Over the Level $s = \hat{s}_1$	47
2.7.2	The Level Related to an Inflection Point	49
2.7.3	Asymptote Related to Maximum at the Profile $\hat{s}(y)$	50
2.8	Conclusions	51
	References	52
3	Nonlinear Schrödinger and Gross - Pitaevskii Equations in the Bohmian or Quantum Fluid Dynamics (QFD) Representation	53
	Attila Askar	
3.1	Introduction	53
3.2	Polar Representation of the Wave Function	54
3.3	Conservation Laws	55

3.3.1	Mass Conservation Equation	55
3.3.2	Energy Conservation Equation	55
3.3.3	The Momentum Equation	56
3.3.4	Pressure Interpretation	56
3.3.5	The Lagrangian Representation	56
3.4	Adding a Dissipation Term as in Navier - Stokes Equation	57
3.5	Vorticity	58
3.6	Closing Remarks	59
	References	60
4	The Stability of the Plates with Circular Inclusions under Tension ..	61
	Svetlana M. Bauer, Stanislava V. Kashtanova, Nikita F. Morozov, and Boris N. Semenov	
4.1	Introduction	61
4.2	Problem Statement	63
4.3	Stability Loss	64
4.3.1	Case with Different Poisson's Ratio	64
4.3.2	A Plate with a Circular Inclusion under Biaxial Tension ..	67
	References	68
5	Unit Cell Models of Viscoelastic Fibrous Composites for Numerical Computation of Effective Properties	69
	Harald Berger, Mathias Würkner, José A. Otero, Raúl Guinovart-Díaz, Julián Bravo-Castillero, and Reinaldo Rodríguez-Ramos	
5.1	Introduction	70
5.2	Linear viscoelastic relations	71
5.3	Numerical Homogenization Model	72
5.4	Results	75
5.5	Conclusions	80
	References	80
6	Inner Resonance in Media Governed by Hyperbolic and Parabolic Dynamic Equations. Principle and Examples	83
	Claude Boutin, Jean-Louis Auriault, and Guy Bonnet	
6.1	Introduction	84
6.2	Dynamic Descriptions of Heterogeneous Linear Elastic Media Without and With Inner Resonance	87
6.2.1	Long Wavelength Descriptions	87
6.2.2	Short Wavelength Descriptions	90
6.3	Inner Resonance in Elastic Composites	91
6.3.1	Requirements for the Occurrence of Inner Resonance in Elastic Bi-Composites	91
6.3.2	Elastic Bi-Composites: High Contrast of Stiffness, Moderate Contrast of Density	93
6.3.2.1	Derivation of the Inner-Resonance Behavior by Homogenization	94

- 6.3.2.2 Comments 98
 - 6.3.3 Elastic Bi-Composites: Significant Contrast of Stiffness and of Density 101
 - 6.3.3.1 Co-Dynamics Regime at Anti-Resonance Frequencies 102
 - 6.3.3.2 Comments 106
 - 6.3.4 Synthesis on the Resonant and Anti-Resonant Co-Dynamic Regimes 107
 - 6.3.5 Reticulated Media: Inner Resonance by Geometrical Contrast 108
- 6.4 Inner Resonance in Poro-Acoustics 112
 - 6.4.1 Double Porosity Media: Inner Resonance by High Permeability Contrast 114
 - 6.4.1.1 Homogenized Behavior 116
 - 6.4.1.2 Comments and Generalization to Other Diffusion Phenomena 119
 - 6.4.2 Embedded Resonators in Porous Media: Inner Resonance by Geometrical Contrast 120
 - 6.4.2.1 Helmholtz Resonator 121
 - 6.4.2.2 Homogenized Behavior 122
 - 6.4.2.3 Comments 123
- 6.5 Inner Resonance in Poroelastic Media: Coupling Effect 125
 - 6.5.1 Double Porosity Poro-Elastic Media - Problem Statement 125
 - 6.5.2 Homogenized Behavior 127
 - 6.5.3 Comments 129
- 6.6 Conclusions 130
- Appendix: Elastic Bi-Composites: Moderate Stiffness Contrast and High Density Contrast 131
- References 132

- 7 The Balance of Material Momentum Applied to Water Waves 135**
Manfred Braun
- 7.1 Introduction 135
- 7.2 The Balance of Physical Momentum 137
- 7.3 The Balance of Material Momentum 139
- 7.4 The Energy Balance 144
- 7.5 Gerstner’s Wave 145
- 7.6 Change of Reference Configuration 150
- 7.7 Concluding Remarks 152
- Appendix: Derivatives of the Lagrangian 153
- References 154
- 8 Electromagnetic Fields in Meta-Media with Interfacial Surface Admittance 155**
David C. Christie and Robin W. Tucker
- 8.1 Introduction 155

8.2	Mathematical Preliminaries	157
8.3	General Maxwell Equations and their Fourier Transform	159
8.4	Maxwell Equations in a Source-Free Domain of an Ohmic, Homogeneous, Isotropic, Dispersive, Linear Medium	161
8.5	Electromagnetic Fields in a Source-Free Domain of an Ohmic, Homogeneous, Isotropic, Dispersive, Linear Medium	162
8.6	Plane Wave Solutions in Terms of the Complex Rotation Group . .	163
8.7	Interface Conditions for Media Containing Anisotropic, Homogeneous, Planar Interface Constitutive Relations	164
8.8	Consequences of the Interface Conditions	167
8.9	Solving the Interface Conditions	172
8.10	Conclusion	175
	References	177
9	Evolution Equations for Defects in Finite Elasto-Plasticity	179
	Sanda Cleja-Tîgăoiu	
9.1	Introduction	179
9.1.1	Defects in Linear eElasticity	180
9.1.2	Defects in Non-Linear Elasticity	180
9.1.3	Defects in Nonlocal Elasticity	181
9.1.4	Elasto-Plastic Models for Defects	181
9.1.5	Aim of this Paper	182
9.1.6	List of Notations	182
9.2	Elasto-Plastic Materials with Lattice Defects	184
9.2.1	Plastic Connection with Metric Property	186
9.2.2	Measure of Defects	187
9.3	Free Energy Imbalance Principle Formulated in $\mathcal{H}$	188
9.3.1	Free Energy Function	188
9.3.2	Free Energy Imbalance Principle	191
9.4	Constitutive Restrictions Imposed by the Imbalance Free Energy Principle	193
9.4.1	Elastic Type Constitutive Equations	193
9.4.2	Dissipation Inequality	194
9.5	Viscoplastic Type Evolution Equations for Plastic Distortion and Disclination Tensor	195
9.5.1	Quadratic Free Energy	197
9.5.2	Elasto-Plastic Model for Dislocations and Disclinations in the Case of Small Distortions	198
9.6	Conclusions	200
	References	201

10 Viscoelastic effective properties for composites with rectangular cross-section fibers using the asymptotic homogenization method . . . 203
Oscar L. Cruz-González, Reinaldo Rodríguez-Ramos, José A. Otero, Julián Bravo-Castillero, Raúl Guinovart-Díaz, Raúl Martínez-Rosado, Federico J. Sabina, Serge Dumont, Frederic Lebon, and Igor Sevostianov

10.1 Introduction 204

10.2 Statement of the Viscoelastic Heterogeneous Problem 205

10.3 Two-Scale Asymptotic Homogenization Method to Solve the Heterogeneous Problem 207

10.3.1 Contribution of the Level ξ^{-2} Problem 209

10.3.2 Contribution of the Level ξ^{-1} Problem 210

10.3.3 Contribution of the Level ξ^0 Problem 211

10.4 Two Phase Viscoelastic Composite 212

10.5 Numerical Results 214

10.5.1 Model I 214

10.5.2 Model II 216

10.5.3 Viscoelastic Effective Constants for Composites with Rectangular Cross-Section Fibers: Double Homogenization 218

10.6 Conclusions 220

References 221

11 A Single Crystal Beam Bent in Double Slip 223
Xiangyu Cui and Khanh Chau Le

11.1 Introduction 223

11.2 3-D Models of Crystal Beam Bent in Double Slip 225

11.3 Energy Minimization 229

11.4 Numerical Simulations 235

11.5 Non-Zero Dissipation 238

11.6 Numerical Simulation 242

11.7 Discussion and Outlook 244

References 245

12 Acoustic Metamaterials Based on Local Resonances: Homogenization, Optimization and Applications 247
Fabio di Cosmo, Marco Laudato, and Mario Spagnuolo

12.1 Introduction 247

12.2 Locally Resonant Microstructures 250

12.3 A Survey of Homogenization Techniques 253

12.3.1 Periodic Homogenization 254

12.3.2 Dynamic Homogenization and Willis-Type Constitutive Relations 255

12.3.3 Homogenization from Scattering Properties. 257

12.4 Topology Optimization 259

12.4.1 Topology Optimization for Local Resonant Sonic Materials 259

12.4.2	Topology Optimization for Hyperbolic Elastic Metamaterials	260
12.4.3	Topology Optimization for Hyperelastic Plates	262
12.5	Principal Applications: Phononic Crystals	263
12.6	Conclusions	268
	References	268
13	On Nonlinear Waves in Media with Complex Properties	275
	Jüri Engelbrecht, Andrus Salupere, Arkadi Berezhovski, Tanel Peets, and Kert Tamm	
13.1	Introduction	275
13.2	The Governing Equations	276
13.2.1	Boussinesq-Type Models	277
13.2.2	Evolution-Type (KdV-Type) Models	278
13.2.3	Coupled Fields	280
13.3	Physical Effects	281
13.4	Discussion	284
	References	285
14	The Dual Approach to Smooth Defects	287
	Marcelo Epstein	
14.1	Dedication	287
14.2	Summary of the Direct Approach	287
14.3	The Dual Perspective	289
14.4	A Brief Review of Differential Forms	290
14.4.1	Pictorial Representation of Covectors and 1-Forms	290
14.4.2	Exterior Algebra	291
14.4.3	The Exterior Derivative	292
14.4.4	Integration	293
14.5	An Application to Smectics	293
14.6	An Application to Nanotubes	296
14.7	A Volterra Dislocation	298
	References	300
15	A Note on Reduced Strain Gradient Elasticity	301
	Victor A. Eremeyev and Francesco dell'Isola	
15.1	Introduction	301
15.2	Reduced Strain Gradient Elasticity. Examples	303
15.2.1	Structural Mechanics	303
15.2.2	Continual Models for Pantographic Beam Lattices	304
15.2.3	Smectics and Columnar Liquid Crystals	306
15.2.4	Other Spatially Non-Symmetric Models	307
15.3	Conclusions	308
	References	308

16 Use and Abuse of the Method of Virtual Power in Generalized Continuum Mechanics and Thermodynamics	311
Samuel Forest	
16.1 Introduction	311
16.2 Micromorphic and Gradient Plasticity	313
16.2.1 The Micromorphic Approach to Gradient Plasticity	313
16.2.2 Direct Construction of Gradient Plasticity Theory	317
16.3 Gradient of Entropy or Temperature Models	318
16.3.1 A Principle of Virtual Power for Entropy	318
16.3.2 Gradient of Entropy or Gradient of Temperature?	321
16.4 The Method of Virtual Power Applied to Phase Field Modelling	322
16.5 On the Construction of the Cahn–Hilliard Diffusion Theory	325
16.5.1 Usual Presentation Based on the Variational Derivative	325
16.5.2 Method of Virtual Power with Additional Balance Equation	327
16.5.3 Second Gradient Diffusion Theory	328
16.5.3.1 Variational Formulation of Classical Diffusion	328
16.5.3.2 Variational Formulation of Second Gradient Diffusion	328
16.6 Conclusions	331
References	332
17 Forbidden Strains and Stresses in Mechanochemistry of Chemical Reaction Fronts	335
Alexander B. Freidin and Leah L. Sharipova	
17.1 Introduction	335
17.2 Chemical Affinity in the Case of Small Strains	338
17.3 Forbidden Zones	341
References	346
18 Generalized Debye Series Theory for Acoustic Scattering: Some Applications	349
Alain Gérard	
18.1 Introduction	349
18.2 Generalized Debye Series	351
18.2.1 Formulation of the Problem	351
18.2.2 "Local" Modal Reflection and Refraction Coefficients	354
18.2.2.1 Reflection and Refraction of a Wave Incident from Medium 1 (Fluid) on Medium 2 (Solid)	354
18.2.2.2 Reflection and Refraction of Wave Incident from Medium 2 on Medium 1	355
18.3 Transmitted Waves	356
18.4 Contribution to the Resonance Scattering Theory	357
18.4.1 Case of Solid Submerged Elastic Objects	357
18.4.2 Case of Solid Submerged Lossy Elastic Objects	358
18.4.3 Case of Submerged Elastic Shells	359

18.5	Non Resonant Background	361
18.6	Space-Time Dependence of a Bounded Beam Inside an Elastic Cylindrical Guide	364
18.6.1	Propagation Equations	365
18.6.2	Initial Conditions and Limiting Conditions	365
18.6.3	Solution of the Problem: Generalized Debye Series	366
18.6.4	Velocity Fields and Simulation	369
18.7	Conclusions	371
	References	372
19	Simplest Linear Homogeneous Reduced Gyrocontinuum as an Acoustic Metamaterial	375
	Elena F. Grekova	
19.1	Introduction	376
19.2	Basic Equations for the Linear Reduced Gyrocontinuum	377
19.3	Special Solution in Case $\omega = \omega_0$	379
19.4	Longitudinal Waves and Spectral Problem for the Shear-Rotational Wave	379
19.5	Shear-Rotational Wave. Reduced Spectral Problem	380
19.6	Shear-Rotational Wave Propagating Perpendicular to the Rotors' Axes ($\hat{\mathbf{k}} \cdot \mathbf{m} = 0$).	381
19.7	Shear-Rotational Wave Propagating Parallel to the Rotors' Axes ($\hat{\mathbf{k}} \times \mathbf{m} = \mathbf{0}$).	382
19.8	Shear-Rotational Wave Directed in General Way with Respect to the Rotors' Axes	383
19.9	Conclusions	385
	References	386
20	A Mathematical Model of Nucleic Acid Thermodynamics	387
	Sonia Guarguagli and Franco Pastrone	
20.1	Introduction	387
20.2	Denaturation of DNA	388
20.3	Mathematical Model	389
20.4	The Role of Parameter b	391
20.5	Discussion	391
20.6	Conclusion	393
	References	394
21	Bulk Nonlinear Elastic Strain Waves in a Bar with Nanosize Inclusions	395
	Igor A. Gula and Alexander M. Samsonov (†)	
21.1	Introduction	395
21.2	Refinement of the Model of a Continuous Microstructured Medium	397
21.3	Nonlinear Strain Waves in a Bar	401
21.3.1	The Model for Wave Propagation in an Isotropic Bar	401

- 21.3.2 The Refined Model Application for a Bar with Nanosize Inclusions 405
 - 21.4 Conclusions 411
 - 21.5 Supplement 412
 - 21.5.1 The Model for Longitudinal Nonlinearly Elastic Damping Waves Propagation in a Microstructured Medium 412
 - 21.5.2 Coefficients of the Coupled Equations in (21.24) 414
 - References 415
- 22 On the Deformation of Chiral Piezoelectric Plates 417**
Dorin Ieşan and Ramon Quintanilla
 - 22.1 Introduction 417
 - 22.2 Basic Equations 418
 - 22.3 Chiral Piezoelectric Plates 420
 - 22.4 General Theorems 423
 - 22.5 Equilibrium Theory 433
 - 22.6 Effects of a Concentrated Charge Density 434
 - 22.7 Conclusions 436
 - References 437
- 23 Non-Equilibrium Temperature and Reference Equilibrium Values of Hidden and Internal Variables 439**
David Jou and Liliana Restuccia
 - 23.1 Introduction 439
 - 23.2 Internal Variables and Hidden Variables 440
 - 23.3 Temperatures in Steady States 443
 - 23.3.1 Asymptotic Equilibrium Expressions for Caloric and Entropic Temperatures 443
 - 23.3.2 Dynamical Steady-State Expressions for Caloric and Entropic Temperatures 444
 - 23.4 A Model for System’s Aging 447
 - 23.5 Concluding Remarks 448
 - References 449
- 24 On the Foundation of a Generalized Nonlocal Extensible Shear Beam Model from Discrete Interactions 451**
Attila Kocsis and Noël Challamel
 - 24.1 Introduction 451
 - 24.2 The Mechanical Model 453
 - 24.3 Extensible Engesser Elastica 454
 - 24.3.1 Discrete Extensible Engesser Elastica 454
 - 24.3.1.1 Buckling Loads 456
 - 24.3.1.2 Analytical Solution for Short Linkages 458
 - 24.3.1.3 Numerical Solution 462
 - 24.3.2 Asymptotic Limit: the Local Extensible Engesser Elastica 462
 - 24.3.3 Continualized Nonlocal Extensible Engesser Elastica ... 466

24.3.3.1	Numerical Solution: Discrete Versus Nonlocal Extensible Engesser Elastica	467
24.4	Extensible Haringx Elastica	467
24.4.1	Discrete Extensible Haringx Elastica	469
24.4.1.1	Buckling Loads	470
24.4.1.2	Analytical Solution for Short Linkages	472
24.4.1.3	Numerical Solution	475
24.4.2	Asymptotic Limit: the Local Extensible Haringx Elastica	475
24.4.3	Continualized Nonlocal Extensible Haringx Elastica . . .	479
24.4.3.1	Numerical Solution: Discrete Versus Nonlocal Extensible Haringx Elastica	480
24.5	Conclusions	480
	Appendix A	480
	Appendix B	483
	References	484
25	A Consistent Dynamic Finite-Strain Plate Theory for Incompressible Hyperelastic Materials	487
	Yuanyou Li and Hui-Hui Dai	
25.1	Introduction	487
25.2	The 3D Governing Equations	489
25.3	The 2D Dynamic Plate Theory	491
25.3.1	Dynamic 2D Vector Plate Equation	492
25.3.2	Edge Boundary Conditions	496
25.3.2.1	Case 1. Prescribed Position in the 3D Formulation	496
25.3.2.2	Case 2. Prescribed traction in the 3D formulation	497
25.3.3	Examination of the Consistency	497
25.4	The Associated Weak Formulations	499
25.4.0.1	Case 1. Edge position and traction in the 3D formulation are known	501
25.4.0.2	Case 2. Edge position and traction in the 3D formulation are unknown	501
25.5	Conclusions	502
	References	503
26	A One-Dimensional Problem of Nonlinear Thermo-Electroelasticity with Thermal Relaxation	505
	Wael Mahmoud, Moustafa S. Abou-Dina, Amr R. El Dhaba, Ahmed F. Ghaleb, and Enaam K. Rawy	
26.1	Introduction	505
26.2	The Nonlinear Equations	507
26.3	The Associated System of Linear Equations	508
26.4	Numerical Scheme	511
26.5	Conclusions	513

References	515
27 Analysis of Mechanical Response of Random Skeletal Structure	517
María-Belén Martínez-Pavetti and Shoji Imatani	
27.1 Introduction	517
27.2 Material Characterization	519
27.2.1 Extended Voronoi Tessellation	519
27.2.2 Basic Equations	521
27.2.3 Finite Element Discretization	522
27.3 Analyses and Discussions	524
27.3.1 Preparation of Skeletal Model	524
27.3.2 Static Tension	525
27.3.3 Dynamic Loading	527
27.4 Concluding Remarks	529
References	530
28 On the Influence of the Coupled Invariant in Thermo-Electro-Elasticity	533
Markus Mehnert, Tiphaine Mathieu-Pennober, and Paul Steinmann	
28.1 Introduction	534
28.1.1 Kinematics	535
28.1.2 Balance Laws in Electrostatics	536
28.1.2.1 Spatial Configuration	536
28.1.2.2 Material Configuration	538
28.1.3 Heat Equation	539
28.1.4 Energy Function	541
28.2 Non-Homogeneous Boundary Value Problems	543
28.2.1 Deformation of a Cube with a Uniaxially Applied Electric Field	544
28.2.2 Extension and Torsion of a Cylindrical Tube	546
28.3 Conclusions	551
References	552
29 On Recurrence and Transience of Fractional Random Walks in Lattices	555
Thomas Michelitsch, Bernard Collet, Alejandro Perez Riascos, Andrzej Nowakowski, and Franck Nicolleau	
29.1 Introduction	556
29.2 Time Discrete Markovian Random Walks on Undirected Networks	558
29.3 Probability Generating Functions - Green's Functions	561
29.4 The Fractional Random Walk	565
29.5 Universality of Fractional Random Walks	567
29.5.1 Universal Behavior in the Limit $\alpha \rightarrow 0$	567
29.5.2 Recurrence Theorem for the Fractional Random Walk on Infinite Simple Cubic Lattices	568
29.5.3 Universal Asymptotic Scaling: Emergence of Lévy Flights	571

29.6	Transient Regime $0 < \alpha < 1$ for the Infinite One-Dimensional Chain	573
29.7	Conclusions	577
	References	579
30	Micropolar Theory with Production of Rotational Inertia: A Rational Mechanics Approach	581
	Wolfgang H. Müller and Elena N. Vilchevskaya	
30.1	Review of the Current State-of-the-Art	582
30.2	Productions of Microinertia and the Coupling Tensor for Transversally Isotropic Media	586
30.3	Discussion of Special Cases for the Production Term for the Moment of Inertia, χ_J	588
30.3.1	Examples for the Isotropic Case	589
30.3.2	Structural Change I: Purely Deviatoric Production	590
30.3.3	Structural Change II: Purely Axial Production	593
30.4	Dynamics of Micropolar Media with Time-Varying Micro-Inertia	599
30.4.1	General Remarks	599
30.4.2	Axial Elongation and Shrinkage	600
30.5	Conclusions and Outlook	601
	Appendices	602
	Representation of the Production of Moment of Inertia	603
	Restrictions on the Production of Moment of Inertia by the Second Law	604
	References	605
31	Contact Temperature as an Internal Variable of Discrete Systems in Non-Equilibrium	607
	Wolfgang Muschik	
31.1	Introduction	607
31.2	Contact Temperature	608
31.2.1	Definition	608
31.2.2	Contact Temperature and Internal Energy	609
31.3	State Space and Entropy Rate	610
31.4	Equilibrium and Reversible "Processes"	611
31.5	Brief Overview of Internal Variables	613
31.6	Contact Temperature as an Internal Variable	615
	Appendices	616
	Heat Exchange and Contact Temperature	616
	Contact Temperature and Efficiency	618
	References	620
32	Angular Velocities, Twirls, Spins and Rotation Tensors in the Continuum Mechanics Revisited	621
	Konstantin Naumenko and Holm Altenbach	
32.1	Introduction	621

32.2	Rotation Tensor and Angular Velocity Vector	622
32.3	Rotation Tensors and Spins in the Classical Continuum Mechanics	623
32.3.1	Rotations of Principal Directions and Twirls	624
32.3.2	Logarithmic Spin	627
32.4	Conclusions	630
Appendix: Some Operations with Second Rank Tensors		630
	Dot Products of a Second Rank Tensor and a Vector	630
	Cross Products of a Second Rank Tensor and a Vector	630
	Vector Invariant	631
	References	632
33	Towards Continuum Mechanics with Spontaneous Violations of the Second Law of Thermodynamics	633
	Martin Ostoja-Starzewski and Bharath V. Raghavan	
33.1	Dissipation Function in Thermomechanics within Second Law	633
33.2	Dissipation Function in Statistical Physics beyond Second Law	635
33.3	Stochastic Dissipation Function	637
33.3.1	Basics	637
33.3.2	Atomic Fluid in Couette Flow	638
33.4	Closure	639
	References	640
34	Nonlocal Approach to Square Lattice Dynamics	641
	Alexey V. Porubov, Alena E. Osokina, and Thomas M. Michelitsch	
34.1	Introduction	641
34.2	Linear Local Model	644
34.3	Nonlocal Linear Model	646
34.4	Dispersion Relations Analysis	648
34.5	Continuum Equations	649
34.5.1	Local Model	650
34.5.2	Nonlocal Model	650
34.5.3	Nonlinear Interaction	652
34.6	Conclusion	653
	References	653
35	A New Class of Models to Describe the Response of Electrorheological and Other Field Dependent Fluids	655
	Vít Průša and Kumbakonam R. Rajagopal	
35.1	Introduction	655
35.2	Preliminaries	657
35.3	Constitutive Relation	659
35.4	Simple Shear Flow	662
35.4.1	Extra Stress Tensor $\mathbb{S}$ is a Linear Function of the Symmetric Part of the Velocity Gradient $\mathbb{D}$	666
35.4.2	Symmetric Part of the Velocity Gradient $\mathbb{D}$ is a Linear Function of the Extra Stress Tensor $\mathbb{S}$	667

35.4.3	Extra stress tensor $\mathbb{S}$ is a function of the symmetric part of the velocity gradient	667
35.4.4	Fully Implicit Constitutive Relation – Constitutive Relation with Bilinear Tensorial Terms	668
35.5	Conclusion	670
	References	671
36	Second Gradient Continuum: Role of Electromagnetism Interacting with the Gravitation on the Presence of Torsion and Curvature	675
	Lalaonirina R. Rakotomanana	
36.1	Introduction	675
36.2	Electromagnetism in Minkowski Spacetime	676
36.2.1	Maxwell’s 3D Equations in Vacuum	676
36.2.2	Covariant Formulation of Maxwell’s Equations	678
36.3	Electromagnetism in Curved Continuum	679
36.3.1	Variational Method and Covariant Maxwell’s Equations	679
36.3.2	Field Equations and Conservation Laws	681
36.4	Electromagnetism in Twisted and Curved Continuum	684
36.4.1	Faraday Tensor in Twisted Continuum	684
36.4.2	Field Equations, Wave Equations	685
36.4.3	Electromagnetism and Continuum Defects	688
36.5	Concluding Remarks	691
	References	693
37	Optimal Calculation of Solid-Body Deformations with Prescribed Degrees of Freedom over Smooth Boundaries	695
	Vitoriano Ruas	
37.1	Introduction	695
37.2	Method Description	697
37.3	Method Experimentation	699
37.3.1	Deflections of an Elastic Membrane	699
37.3.2	Torsion of an Elastic Annulus	701
37.4	Final Comments	703
	References	703
38	Toward a Nonlinear Asymptotic Model for Thin Magnetoelastic Plates	705
	Sushma Santapuri and David J. Steigmann	
38.1	Introduction	705
38.2	Summary of the Three-Dimensional Theory for Conservative Problems	706
38.3	Reformulation	708
38.4	Legendre-Hadamard Conditions	709
38.5	Equations Holding on the Midplane and Small-Thickness Estimates	711
38.6	Potential Energy of a Thin Plate	712
38.7	Reduction of the Plate Energy	714

References	716
39 Modelling of an Ionic Electroactive Polymer by the Thermodynamics of Linear Irreversible Processes	717
Mireille Tixier and Joël Pouget	
39.1 Introduction	717
39.2 Description and Modelling of the Material	720
39.2.1 Average Process	721
39.2.2 Interface Modelling	721
39.2.3 Partial Derivatives and Material Derivative	722
39.2.4 Balance Laws	723
39.3 Conservation Laws	724
39.3.1 Conservation of the Mass	724
39.3.2 Electric Equations	724
39.3.3 Linear Momentum Conservation Law	725
39.3.4 Energy Balance Laws	726
39.3.4.1 Potential Energy Balance Equation	726
39.3.4.2 Kinetic Energy Balance Equation	726
39.3.4.3 Total Energy Balance Equation	727
39.3.4.4 Internal Energy Balance Equation	727
39.3.4.5 Interpretation of the Equations	727
39.4 Entropy Production	728
39.4.1 Entropy Balance Law	728
39.4.2 Fundamental Thermodynamic Relations	728
39.4.3 Entropy Production	729
39.4.4 Generalized Forces and Fluxes	730
39.5 Constitutive Equations	731
39.5.1 Rheological Equation	731
39.5.2 Nafion [®] Physicochemical Properties	732
39.5.3 Nernst-Planck Equation	733
39.5.4 Generalized Darcy's Law	734
39.6 Validation of the Model: Application to a Cantilevered Strip	735
39.6.1 Static Equations	735
39.6.2 Beam Model on Large Displacements	737
39.6.3 Simulations Results	739
39.7 Conclusion	740
39.8 Notations	741
References	742
40 Weakly Nonlocal Non-Equilibrium Thermodynamics: the Cahn-Hilliard Equation	745
Péter Ván	
40.1 Introduction	745
40.2 Variational derivation of Ginzburg-Landau and Cahn-Hilliard equations	748