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Fundamentals of Electrical Circuit Analysis

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*To my parents, wife Asma Ara Bagum,
children Syeed Hasan, Yusra Salam, and
Sundus Salam for their constant support, and
all of my teachers' and well-wishers who
have helped me grow professionally over the
years.*

Md. Abdus Salam

*To my wife Munadil Rahman Tina, and our
children Mehrin Rahman and Mayaaz
Rahman for their selfless love, kindness and
endless support.*

Quazi Mehbubar Rahman

Preface

Four-year electrical and electronic engineering curriculums generally incorporate two courses on electrical circuit analysis to provide students with a strong foundation of electrical engineering practices. The contents of the *Fundamentals of Electrical Circuit Analysis* book have been arranged in a logical way to enable students to gradually make transition from simple topics to more complex ones. Therefore, this book is an essential resource for the undergraduate electrical engineering students especially in the first year of their studies. Electrical engineers need to be critical thinkers, and be competent with the design, construction and production of electrical equipment and systems for the technological advancement of society. In order to achieve such goals, proper fundamental knowledge in electrical circuit analysis is crucial. This knowledge is also important for other branches of electrical engineering such as power and energy, communication, mechatronics, robotics and control system engineering. In addition to electrical engineering, this book also covers areas such as mechanical, mechatronic and civil engineering. This book addresses all the necessary materials for accreditation given by Accreditation Board for Engineering and Technology (ABET), USA, Institution of Engineering and Technology (IET), UK and Canadian Engineering Accreditation Board (CEAB).

The main features of this book are as follows:

- Easy and simple presentation of each topic.
- Supporting mathematical equations for the presented circuit parameters.
- Step-by-step approach in worked out examples, preceded by the corresponding theoretical background.
- Practice problems after each topic.
- Inclusion of fundamentals of PSpice simulation.
- Some worked out examples in PSpice simulation.
- Significant number of exercise problems at the end of each chapter.
- Answers to all the exercises problems.

Organization of the Book

Chapter 1 focusses on the fundamentals of electrical parameters, measuring equipment and circuit parameters drawing and labelling in PSpice.

Chapter 2 deals with basic circuit laws, series–parallel circuit, delta–wye conversion, short and open circuit and source conversion technique.

Chapter 3 includes mesh analysis, nodal analysis, supermesh and supernode.

Chapter 4 discusses network theorems such as linearity property, superposition theorem, Thevenin's theorem, Norton's theorem and maximum power transfer theorem.

Chapter 5 looks after capacitor, inductor, charging and discharging activities, RL and RC circuit with step response.

Chapter 6 focusses on alternating current parameters, rms value, average values, form factor, peak factor, phasor algebra, AC circuits with resistance, inductance and capacitance and AC circuit with Kirchhoff's laws.

Chapter 7 deals with AC circuits with mesh analysis, nodal analysis, superposition, Thevenin's and Norton's theorem.

Chapter 8 discusses instantaneous power, average power, apparent power, reactive power, complex power, power factor and maximum power transfer theorem.

Chapter 9 deals with three-phase voltage generation, balanced and unbalanced wye and delta connections, wye and delta connections with balanced and unbalanced loads, three-phase power and power measurement.

Chapter 10 talks about frequency response with resistance, inductance and capacitance, transfer function, different types of filters and Bode plots.

The authors would like to thank all the faculty members who have contributed to the preparation of this book and the staff of Springer Publishers who have tirelessly worked to publish this book successfully.

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Chapter 1

Fundamental Electrical Terms



1.1 Introduction

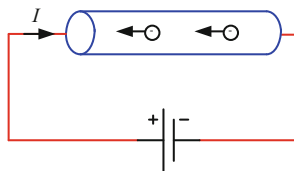
It is essential to understand the fundamental of electrical terms to comprehend any electrical phenomena. This chapter presents these necessary concepts and components as a first stepping stone in understanding any underlying electrical principles. In addition, it introduces the core measuring equipment in the electrical domain along with charge, current, voltage, power, energy, resistance, semiconductor and insulator.

1.2 Charge

Charge is an electrical property of matter, measured in coulombs (C). The charge in motion represents the current and the stationary charge represents static electricity. There are two types of charge namely the positive charge and the negative charge. Like charges repel each other and opposite charges attract each other. The charge is represented by the letter Q or q and its SI derived unit is coulomb (C). Coulomb is a large unit of the charge and, therefore, generally the smaller units of charge (pC , nC and μC) are used in practice. Note: charge on an electron, which is negative, has the magnitude of 1.602×10^{-19} C. In one Coulomb of charge, there are $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons [1–3].

1.3 Current

The flow of electrons results in current. The direction of the current is opposite to the flow of electrons (negative charges) as shown in Fig. 1.1. The current is represented by the letter I and its SI unit is ampere (A) in honour of French Mathematician and Physicist Andre-Marie Ampere (1775–1836).

Fig. 1.1 Direction of current

In general, the current is defined as the total charge Q transferred in time t and it is expressed as,

$$I = \frac{Q}{t} \quad (1.1)$$

The rate of change of charge transferred at any particular time is known as instantaneous current, i , and it is expressed as,

$$i = \frac{dq}{dt} \quad (1.2)$$

$$dq = i dt \quad (1.3)$$

$$q = \int_{t_1}^{t_2} i dt \quad (1.4)$$

Equation (1.4) provides the expression for the total charge transferred through a conductor between time t_1 and t_2 .

Example 1.1 The expression of charge at a terminal is given by $q = 2t^3 + 5t^2 + t + 1$ C. Find the current at $t = 0.5$ s.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt}(2t^3 + 5t^2 + t + 1) = 6t^2 + 10t + 1 \text{ A} \quad (1.5)$$

At $t = 0.1$ s, the value of the current is,

$$i = 6(0.5)^2 + 10(0.5) + 1 = 7.5 \text{ A} \quad (1.6)$$

Practice Problem 1.1 The total charge entering a terminal is given by $q = 2e^{-2} + 3t^2 + t$ C. Find the current at $t = 0.1$ s.

Example 1.2 A current as shown in Fig. 1.2 passes through a wire. Determine the value of the charge.

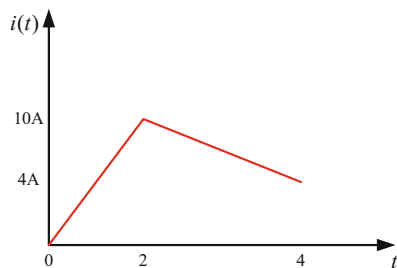


Fig. 1.2 Variation of current with time

Solution:

The expression of current from 0 to 2 s is,

$$i(t) = \frac{10}{2}t = 5t \text{ A} \quad (1.7)$$

The expression of current from 2 to 4 s can be written as,

$$\frac{i(t) - 4}{4 - 10} = \frac{t - 4}{4 - 0} \quad (1.8)$$

$$i(t) - 4 = -\frac{6}{4}(t - 4) \quad (1.9)$$

$$i(t) = 4 - \frac{6}{4}(t - 4) = 10 - 1.5t \quad (1.10)$$

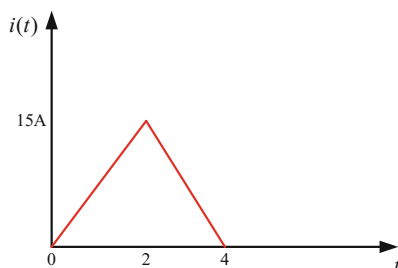
The value of the charge from 0 to 4 s can be calculated as,

$$q = \int_0^2 5t dt + \int_2^4 (10 - 1.5t) dt \quad (1.11)$$

$$q = 2.5(2^2 - 0) + 10(4 - 2) - 0.75(4^2 - 2^2) = 21 \text{ C} \quad (1.12)$$

Practice Problem 1.2 The value of an instantaneous current varying with time is shown in Fig. 1.3. Calculate the value of the charge.

Fig. 1.3 Triangular shape of current



1.4 Direct and Alternating Currents

There are two types of current: direct current, which is abbreviated as DC and alternating current, which is abbreviated as AC [4, 5]. When the magnitude of the current does not change with time, the current is known as direct current, while, in alternating current, the magnitude of the current changes with respect to the time. Both of these currents are shown in Fig. 1.4.

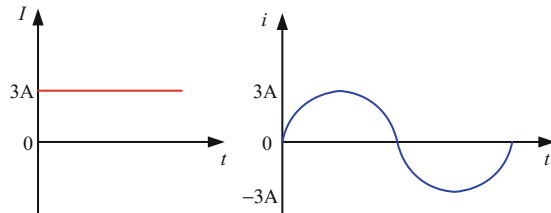
1.5 Conductor, Insulator and Semiconductor

The concept of conductor, insulator and semiconductor can be presented in terms of energy band models from the solid-state physics [6]. The band model consists of two energy bands, valance band and conduction band, and the energy gap between these two bands. The lower energy valance band contains the charges at their lowest energy levels while the highest energy conduction band, being generally empty, contains the energy-excited charges to make conduction. In case of a conductor, a large number of charges populate the conduction band even at room temperature, and as a result, a conductor provides least resistance to the flow of charges. In this case, there is no energy gap in between the valance band and the conduction band, and that means these two energy bands overlap with each other as shown in Fig. 1.5a. For a conductor, the charges in the conduction band are loosely bound with the parent atoms and hence, they can move easily under the influence of external energy source such as electric field.

In the insulator, the energy gap in between the valance and conduction bands is very high as a result, no charge can gain enough energy to jump to the conduction band as shown in Fig. 1.5b. As a result, an insulator provides most resistance to the flow of charges.

A semiconductor falls in between the conductor and insulator and it offers a moderate resistance to flow the charge. A semiconductor has smaller energy gap in between the conduction band and the valance band, when compared to the insulators, as shown in Fig. 1.5c.

Fig. 1.4 Direct and alternating currents



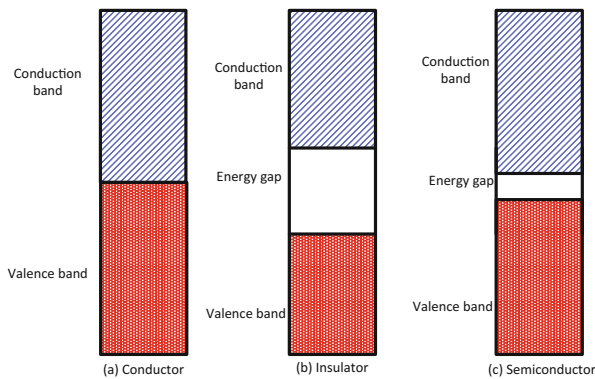


Fig. 1.5 Energy bands conductor, insulator and semiconductor

1.6 Resistance and Conductance

The property of a material, which opposes the flow of electric current through it, is known as resistance. The resistance is represented by the letter R and its SI unit is ohm (Ω). The symbol of a resistor is shown in Fig. 1.6. The resistance of any material is directly proportional to the length (l), and inversely proportional to the cross-sectional area (A) of the material as shown in Fig. 1.7, and can be written as,

$$R \propto l \quad (1.13)$$

$$R \propto \frac{1}{A} \quad (1.14)$$

Combining Eqs. (1.13) and (1.14) yields,

$$R \propto \frac{l}{A} \quad (1.15)$$

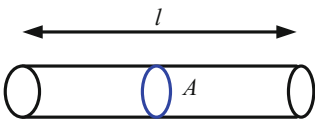
$$R = \rho \frac{l}{A} \quad (1.16)$$

where ρ is the proportionality constant which is known as the resistivity of a material and its unit is $\Omega\cdot\text{m}$. A good conductor has a very low resistivity, while an insulator has a very high resistivity.

Fig. 1.6 Symbol of the resistor



Fig. 1.7 A material with cross-sectional area A



In practice, the common commercial resistors are manufactured in a way so that their resistance values can be determined by the colour bands printed on their surfaces. Different coded values for different colours are shown in Table 1.1. The manufacturing tolerance for each resistor is also colour coded on its surface; Table 1.2 shows these coded values. From the colour bands, the value of a particular resistance can be determined as,

$$R = xy \times 10^z \, \Omega \tag{1.17}$$

where x is the first colour band, closest to the edge of the resistor, y is the second colour band next to x and z is the third colour band. The fourth colour band (if any) represents the tolerance in percent.

Tolerance is the percentage of error in the resistance value that says how much more or less one can expect a resistor’s actual measured resistance to be from its stated resistance.

As an example, the resistance value of a resistor shown in Fig. 1.8 can be determined from its colour code, with the help of Tables 1.1 and 1.2 as,

$$R = 52 \times 10^6 = 52 \, \text{M}\Omega \tag{1.18}$$

Fig. 1.8 Resistance with different colours



Table 1.1 Resistance colour codes with values

Colours name	Colours	Value
Black		0
Brown		1
Red		2
Orange		3
Yellow		4
Green		5
Blue		6
Violet		7
Grey		8
White		9

Table 1.2 Tolerance colour codes with values

Colours name	Colours	Value (%)
Orange		0.01
Yellow		0.001
Brown		1
Red		2
Gold		5
Silver		10
No colour		20

The tolerance is calculated as,

$$\text{Tolerance} = 52 \times 10^6 \times 10\% = 5.2 \text{ M}\Omega \quad (1.19)$$

The measured resistance may vary between $(52 + 5.2)$ and $(52 - 5.2) \text{ M}\Omega$.

Conductance, which is the reciprocal of the resistance, is a characteristic of materials that promotes the flow of electric charge. It is represented by the letter G and its SI unit is Siemens (S). Mathematically, it is expressed as,

$$G = \frac{1}{R} \quad (1.20)$$

Dependence of Resistance on Temperature The resistance of a good conductor such as, copper aluminium, etc., increases with an increase in temperature. Whereas the resistance of electrolyte (*An electrolyte is a substance that produces an electrically conducting solution when dissolved in a polar solvent, such as water*), alloy (*An alloy is a metal, made of the combination of two or more metallic elements*) and insulating material decreases with an increase in temperature. Let us assume that the resistance of a conductor at temperature T and T_0 degree centigrade are R and R_0 , respectively. In this case, the change in resistance is,

$$\Delta R = R - R_0 \quad (1.21)$$

While the change in temperature is,

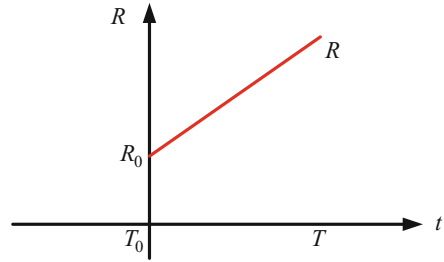
$$\Delta T = T - T_0 \quad (1.22)$$

The ratio of change in resistance to the resistance at T_0 degree centigrade is directly proportional to the change in temperature and it can be expressed as,

$$\frac{R - R_0}{R_0} \propto (T - T_0) \quad (1.23)$$

$$\frac{R - R_0}{R_0} = \alpha(T - T_0) \quad (1.24)$$

Fig. 1.9 Resistance with different temperatures



$$R - R_0 = \alpha R_0 (T - T_0) \quad (1.25)$$

$$R = [R_0 + \alpha R_0 (T - T_0)] \quad (1.26)$$

$$R = R_0 [1 + \alpha (T - T_0)] \quad (1.27)$$

where α is the proportionality constant which is known as the temperature coefficient of the resistance. Equation (1.27) provides the relationship between the resistance and the temperature, which has been presented in Fig. 1.9. In this figure, the temperature coefficient α represents the slope of the line.

Example 1.3 The length and diameter of a copper wire are found to be 100 and 0.002 m, respectively. Consider that the resistivity of the copper wire is $1.72 \times 10^{-8} \Omega\text{-m}$. Calculate the value of the resistance.

Solution:

The area of the wire is,

$$A = \pi d^2 = \pi(0.002)^2 = 1.26 \times 10^{-5} \text{ m}^2 \quad (1.28)$$

The value of the resistance can be calculated as,

$$R = \rho \frac{l}{A} = 1.72 \times 10^{-8} \times \frac{100}{1.26 \times 10^{-5}} = 0.14 \Omega \quad (1.29)$$

Practice Problem 1.3 The resistance of a 120 m length aluminium wire is found to be 10 Ω . The resistivity of the aluminium wire is $2.8 \times 10^{-8} \Omega\text{-m}$. Determine the diameter of the wire.

1.7 Voltage

The voltage is one kind of force required to move a charge in the conductor. Voltage is defined as the work done per unit charge when a charge to move it from one point to another point in a conductor. It is represented by the letter V and its unit is volts (V) in honour of Italian Scientist Alessandro Giuseppe Antonio Anastasio

Volta (February 1745–March 1827) who invented electric battery. Mathematically, the voltage is expressed as,

$$v = \frac{dw}{dq} \quad (1.30)$$

In general, the expression of voltage is,

$$V = \frac{W}{Q} \quad (1.31)$$

As common analogy, a water tank can be used to describe the charge, voltage and current. In this analogy, the amount of water in the tank represents the charge, the water pressure represents the voltage and the flow of water through the hose-pipe, which is connected at the bottom of the tank, represents the current as shown in Fig. 1.10.

The voltage difference is the difference in potential between the two points in a conductor, which is also known as potential difference.

Example 1.4 A charge is required to move between the two points in a conductor. In this case, the expression of work done is given by $w = 3q^2 + q + 1$ J. Find the voltage to move the charge $q = 0.5$ C.

Solution:

The expression of voltage is,

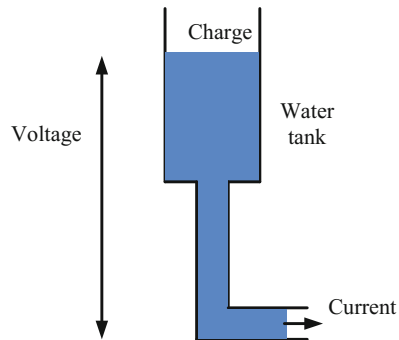
$$v = \frac{dw}{dq} = \frac{d}{dq}(3q^2 + q + 1) = 6q + 1 \text{ V} \quad (1.32)$$

The value of the voltage for $q = 0.5$ C is,

$$v = 6q + 1 = 6 \times 0.5 + 1 = 4 \text{ V} \quad (1.33)$$

Practice Problem 1.4 The work done is given by $w = 5q^2 + 2q$ J when a charge is moved between the two points in a conductor. Determine the value of the charge if the voltage is found to be 10 V.

Fig. 1.10 A tank with water



1.8 Voltage and Current Sources

Any electrical source (voltage or current) provides energy to the elements connected to this source. An ideal voltage source is a two-terminal circuit element that provides a specific magnitude of voltage across its terminals regardless of the current flowing through it. An ideal current source is a two-terminal circuit element that maintains constant current through its terminals regardless of the voltage across those terminals. Ideal voltage and current sources are shown in Fig. 1.11.

An active circuit element usually supplies voltage or current to the passive elements like resistance, inductance and capacitance. Active element is classified as either independent or dependent (controlled) source of energy. An independent voltage or current source is an active element that can generate specified voltage or current without depending on any other circuit elements. The symbols for independent voltage and current sources are shown in Fig. 1.12. A dependent source is an active element whose magnitude is fully dependent on other voltage or current value in the circuit. The symbols for dependent voltage and current sources are shown in Fig. 1.13. The dependent sources are classified into four types, and these are,

Fig. 1.11 Ideal voltage and current sources

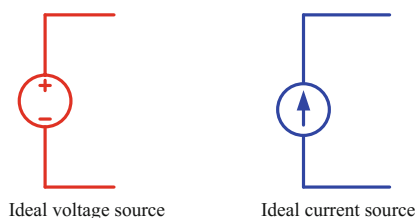


Fig. 1.12 Independent sources

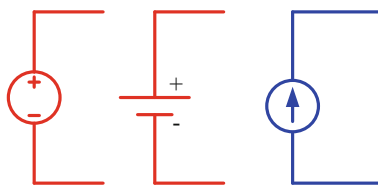
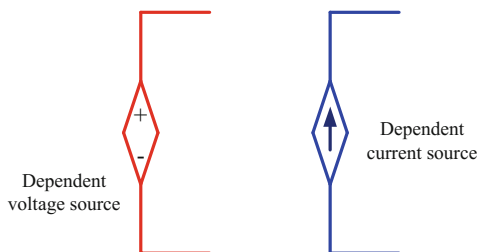


Fig. 1.13 Dependent sources



- (1) Voltage-controlled voltage source (VCVS)
- (2) Voltage-controlled current source (VCCS)
- (3) Current-controlled voltage source (CCVS)
- (4) Current-controlled current source (CCCS)

Different controlled (dependent) sources along with the corresponding application circuits are shown in Figs. 1.14, 1.15, 1.16 and 1.17. In Fig. 1.14, the magnitude of the voltage in the VCVS is α times the voltage across any element, and in the corresponding application circuit, this magnitude is equal to 0.2 (the value of α) times the voltage across the resistance R_1 .

In the VCCS application circuit in Fig. 1.15, the magnitude of the voltage source depends on the current through resistance R_1 . Similar explanation can be offered for CCVS and CCCS and their application circuits are shown in Figs. 1.16 and 1.17, respectively.

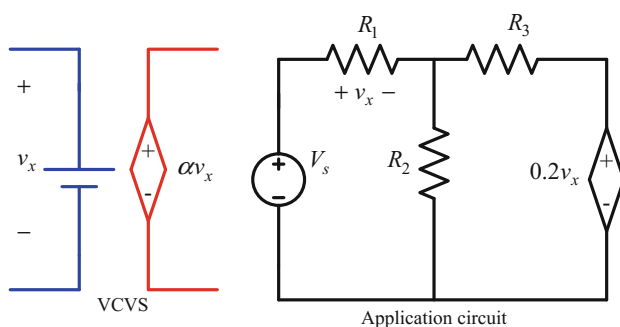


Fig. 1.14 Voltage-controlled voltage source and its application

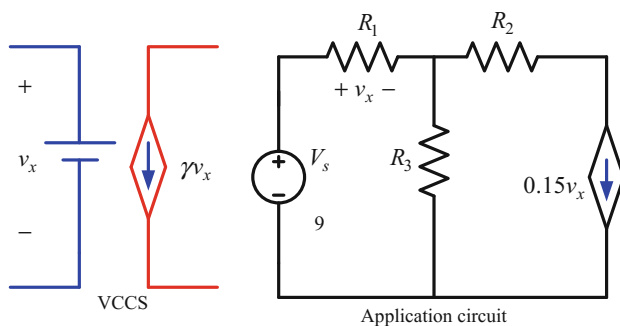


Fig. 1.15 Voltage-controlled current source and its application

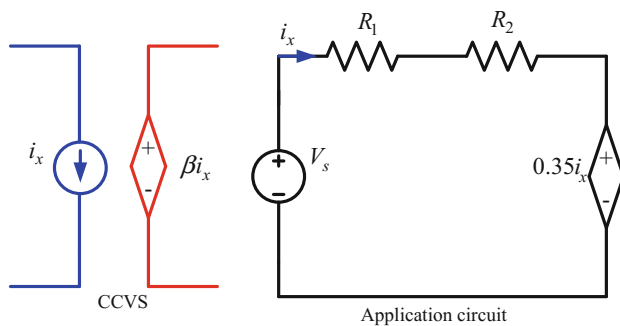


Fig. 1.16 Current-controlled voltage source and its application

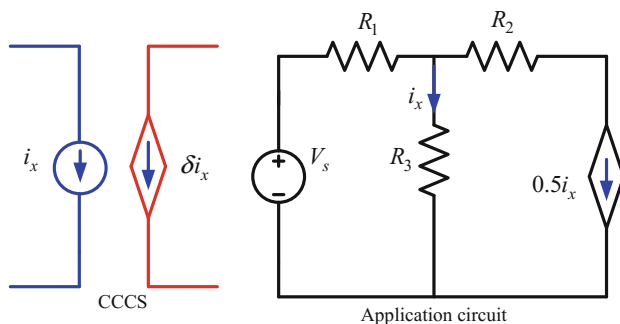


Fig. 1.17 Current-controlled current source and its application

1.9 Electric Power and Energy

The electric power is defined as the rate of receiving or delivering energy from one circuit to another circuit. The power is represented by the letter p and its unit is joules per sec (J/s) or watts (W) in honour to British Scientist James Watt (1736–1819). Mathematically, the expression of power can be written as,

$$p = \frac{dw}{dt} \quad (1.34)$$

where p is the power in watts (W), w is the energy in joules (J) and t is the times in seconds (s). In general, the total electric power can be written as,

$$P = \frac{W}{t} \quad (1.35)$$

Equation (1.34) can be re-arranged as,

$$p = \frac{dw}{dq} \frac{dq}{dt} \quad (1.36)$$

Substituting Eqs. (1.2) and (1.30) into Eq. (1.36) yields,

$$p = vi \quad (1.37)$$

Equations (1.1) and (1.31) can be re-arranged as,

$$t = \frac{Q}{I} \quad (1.38)$$

$$W = VQ \quad (1.39)$$

Substituting Eqs. (1.38) and (1.39) into Eq. (1.35) yields,

$$P = \frac{W}{t} = \frac{VQ}{\frac{Q}{I}} \quad (1.40)$$

$$P = VI \quad (1.41)$$

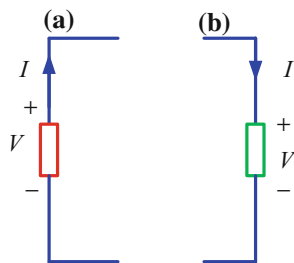
Both Eqs. (1.37) and (1.41) show that the power associated to any basic circuit element is the product of the voltage across that element and the current through that element. If the power, associated to a circuit element, has positive sign, then the power is absorbed by that element. Whereas, in a circuit element, the power with negative sign represents that the element delivers power to the other elements. The phenomena of power delivery and power absorption by the any element can be visualized with the aid of Fig. 1.18.

According to the circuit in Fig. 1.18, the expressions of power can be expressed as,

$$P_d = V(-I) = -VI \quad (1.42)$$

$$P_r = VI \quad (1.43)$$

Fig. 1.18 Power delivered and absorbed circuits



The negative sign in Eq. (1.42) indicates that the power is being delivered by the element, while the positive sign in Eq. (1.43) indicates the power is being absorbed by the element.

Energy is the capacity to do work. Since, the SI unit of work is joule (J), and for the same reason, it is represented by the letter w . From Eq. (1.34), the expression of energy can be written as,

$$dw = p dt \quad (1.44)$$

For a time window between t_0 and t , the total energy can be calculated as,

$$w = \int_{t_0}^t p dt \quad (1.45)$$

In practice, the energy supplied to the consumers is rated the term of kilowatt hour (kWh). One kWh is defined as the electrical power of 1 kW consumed in an hour, and it is expressed as,

$$E = P \times t \quad (1.46)$$

where E is the electrical energy in kWh, P is the power in kW and t is the time in hour.

Example 1.5 The expression of work done is given by $w = 5t^2 + 4t + 1$ J. Calculate the power at $t = 0.1$ s.

Solution:

The expression of power is,

$$p = \frac{dw}{dt} = \frac{d}{dt}(5t^2 + 4t + 1) = 10t + 4 \text{ W} \quad (1.47)$$

Power at $t = 0.1$ s is,

$$p = 10(0.1) + 4 = 5 \text{ W} \quad (1.48)$$

Example 1.6 A power with an expression of $p = 5 + 5t^2$ W is used from time 0.1 to 0.4 s to complete a specific task. Calculate the corresponding energy.

Solution:

$$\begin{aligned} w &= \int_{0.1}^{t=0.4} (5 + 5t^2) dt = 5[t]_{0.1}^{0.4} + \frac{5}{3}[t^3]_{0.1}^{0.4} \\ &= 5(0.4 - 0.1) + 1.67(0.4^3 - 0.1^3) = 1.61 \text{ J} \end{aligned} \quad (1.49)$$

Example 1.7 A 500 W electric toaster operates for 30 min, a 7 kW DVD player operates for 1 h and a 700 W electric iron operates for 45 min. Find the total energy used, and the associated cost if the energy price is 5 cents per kWh.

Solution:

The total energy used is,

$$\begin{aligned} E &= \frac{500}{1000} \times \frac{30}{60} + 7 \times 1 + \frac{700}{1000} \times \frac{45}{60} \\ &= 0.500 \times 0.5 + 7 + 0.700 \times 0.75 = 7.78 \text{ kWh} \end{aligned} \quad (1.50)$$

The cost of the energy used is,

$$C = 7.78 \times \frac{5}{100} = 0.39 \$ \quad (1.51)$$

Practice Problem 1.5 The power consumed at time t , for the work $w = 5t^2 + 4t + 1$ J, is found to be 20 W. Calculate time t .

Practice Problem 1.6 A power of $p = 10e^{-t} + 2$ W is used to complete a task. Calculate the energy for the time interval 0.01 to 0.2 s.

Practice Problem 1.7 Five 10 W energy-saving light bulbs operate for 8 h, a 60 W smart TV operates for 5 h and an 800 W hair dryer operates for 30 min. Calculate the total energy used, and the associated cost if the energy price is 20 cents per kWh.

1.10 Measuring Equipment

Measuring equipment are used to measure the electrical parameters accurately, both analogue and digital metres are available in practice. In an analogue metre, a pointer moves on a scale to read out the reading. Whereas the digital metre can display the reading directly in the form of numerical digits. Some of these widely used metres have been discussed below.

1.10.1 Ohmmeter

An ohmmeter, shown in Fig. 1.19, can measure the resistance of an electrical component. To measure the resistance of any component, it needs to be disconnected from the circuit. Ohmmeter is also used to identify the continuity of a circuit and the correct terminal ends in the application field where long cables are used. In the measurement process, the metre supplies current to the resistance, then measures the voltage drop across that resistance. Finally, the metre gives the value of the resistance.

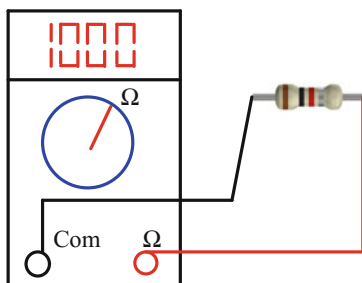


Fig. 1.19 Schematic of an ohmmeter

1.10.2 Ammeter

An ammeter is used to measure the current in the circuit. Ammeter is always connected in series with the circuit under test. This metre offers low internal resistance, which does not affect the actual measurement. The metre symbol and a circuit under test are shown in Fig. 1.20.

1.10.3 Voltmeter

A voltmeter is used to measure the voltage across any circuit element. A voltmeter is always connected in parallel to the circuit element under test. Voltmeters can be designed to measure AC or DC. The internal resistance of a voltmeter is usually kept very high to reduce the current flowing through it to a negligible small amount. Most of the voltmeters have several scales including 0–250, 0–500 V. The metre symbol and a circuit under test are shown in Fig. 1.21.

Fig. 1.20 Symbol and connection diagram of an ammeter

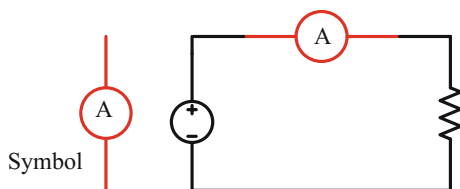
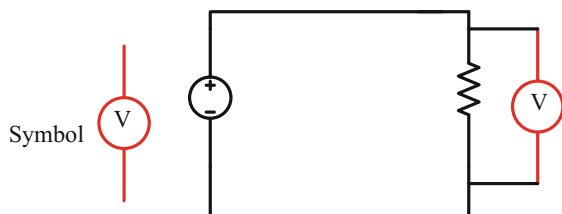


Fig. 1.21 Symbol and connection diagram of a voltmeter



1.10.4 Wattmeter

A wattmeter is used to measure either the power delivered by an electrical source or the power absorbed by a load. As shown in Fig. 1.22, a wattmeter has two coils, the current coil (CC) and the voltage coil (VC).

The current coil with very low resistance is connected in series with the load and responds to the load current, while the voltage coil is connected in parallel to the load circuit and responds to the load voltage. Depending on the test section (source or load), one terminal of each coil is shorted and connected either to the source or to the load as shown in Fig. 1.23. In an AC circuit, wattmeter is widely used to measure the single-phase and three-phase real power as shown in Fig. 1.24.

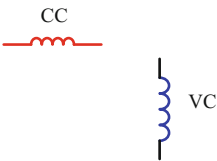


Fig. 1.22 Current coil and voltage coil of a wattmeter

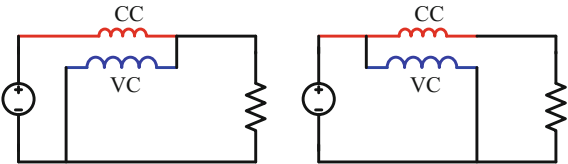


Fig. 1.23 Schematics of a wattmeter connection

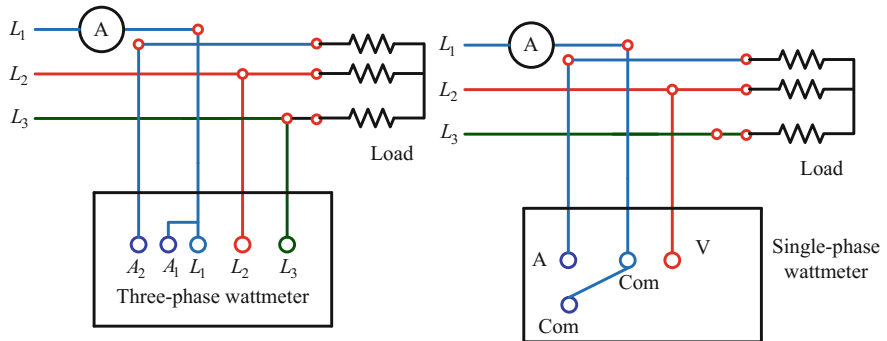


Fig. 1.24 Connection diagrams of three-phase and single-phase wattmeter

1.11 Electricity Bill

The generated power is usually transmitted and distributed by the power utility company to different types of consumers namely, residential, commercial and industrial for a set price. This price is set by the utility company, by taking into account the energy will charge based on production cost, transmission and distribution cost and the profit margin. The energy consumed by the customers is represented by kWh, based on which the consumers incur the associated costs. The rate at which electrical power is sold to the consumer is generally known as tariff. The main objective of the tariff is to recover the associated costs outlined above. There are different types of tariff namely, two-part tariff, three-part tariff, block rate tariff, maximum demand tariff and power factor tariff. The discussion on these different types of tariff is out of the scope of this text. Generally, the tariff is expressed as,

$$\begin{aligned}\text{Tariff} = & a + b \times \text{energy use for first block} \\ & + c \times \text{energy use for 2nd block} \\ & + d \times \text{energy use for next block}\end{aligned}\quad (1.52)$$

where

- a is the fixed charge in \$,
- b is the charge (in \$) per kWh for the first block,
- c is the charge (in \$) per kWh for the second block,
- d is the charge (in \$) per kWh for the next block

Example 1.8 The energy of a commercial building is supplied by a local power utility company, and this building consumes 1000 kWh of energy per month. The fixed charge of this demand is \$10. The cost for the first block of 100 kWh is \$ 0.10/kWh, the second block of 300 kWh is \$ 0.05/kWh and the next block is \$ 0.02/kWh. Determine the tariff of this demand.

Solution:

The tariff of this demand can be calculated as,

$$\text{Tariff} = 10 + 100 \times 0.1 + 300 \times 0.05 + 600 \times 0.02 = \$47 \quad (1.53)$$

Practice Problem 1.8 A local power utility company supplies power to a spinning mill that consumes 1500 kWh per month. The fixed charge of this demand is \$100. The cost for the first block of 500 kWh is \$ 0.15/kWh, and the second block of 1000 kWh is \$ 0.02/kWh. Calculate the tariff of this demand.

1.12 Efficiency of a System

The efficiency of a system is defined as the ratio of output power to the input power. The input power is equal to the output power plus the loss as shown in Fig. 1.25. Mathematically, the efficiency η is expressed as,