The Finite Element Method

Fundamentals and Applications in Civil, Hydraulic, Mechanical and Aeronautical Engineering

BOFANG ZHU

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Preface

The finite element method (FEM) is so powerful that many very complicated engineering problems can be solved by it. This book is primarily written for engineers. It introduces the basic principles and applications of FEM. It may also be used as textbook in universities and colleges.

The first purpose of this book is to make an easy read for engineers, so the physical ideas are enhanced and the basic principles and computing methods are introduced in an easy but accurate way.

The second purpose of this book is to be of practical value to engineers, so the formulas that can be used to analyze problems in practical engineering are given in detail.

Thus, there are three distinguishing characteristics of this book: (1) it is easy to read; (2) the theory and computing formulas of finite element method are complete; (3) it is of practical use to readers, especially to engineers and professors and engineering students.

Before the publication of the first edition of this book in Chinese in 1979, the predicted readers were engineers, but after publication it was noticed that it was well received not only by engineers but also by professors and students in universities and colleges. It is now not only a widely accepted reference book for engineers but also widely used as textbook for professors and students in universities and colleges in China.

According to the Information Center of Chinese Academy of Science *The Finite Element Method, Theory, and Applications* (in Chinese) is one of the most well-received 10 books in China in water resources and hydropower domain.

Now the new book in English will be published, I hope it will be well received not only by engineers working in practical engineering project but also by professors and students.

About the Author

Bofang Zhu, an academician of the Chinese Academy of Engineering and a famous scientist of hydraulic structures and solid mechanics in China, was born on October 17, 1928, in Yujiang County, Jiangxi Province. In 1951, he graduated in civil engineering from Shanghai Jiao Tong University and then participated in the design of the first three concrete dams in China (Foziling dam, Meishan dam, and Xianghongdian dam). In 1957, he was transferred to the China Institute of Water Resources and Hydropower Research where he was engaged in the research work of high concrete dams. He was awarded China National Outstanding Young Scientist in 1984 and was elected the academician of the Chinese Academy of Engineering in 1995. He is now the consultant of the technical committee of the Ministry of Water Resources of China, a member of the consultant group of the three very high dams in the world: the Xiaowan dam, the Longtan dam, and the Baihetan dam. He was a member of the eighth and the ninth Chinese People's Political Consultative Conference, the board chairman of the Institute of Computer Application of China Civil Engineering Society, and a member of the standing committee of the China Civil Engineering Society and the standing committee of the China Hydropower Engineering Society.

He is the founder of the theory of thermal stresses of mass concrete, the shape optimization of arch dams, the simulating computation of concrete dam, and the theory of creep of mass concrete in the word.

He has established a perfect system of the theory of thermal stress and temperature control of mass concrete, including two basic theorems of creep of nonhomogeneous concrete structures; the law of variation and the methods of computation of the thermal stresses of arch dams, gravity dams, docks, sluices, tunnels, and various massive concrete structures; the method of computation of temperature in reservoirs and pipe cooling, thermal stress in beams on foundation, cold wave, heightening of gravity dam; and the methods and criteria for control of temperatures. He proposed the idea of "long-time thermal insulation as well as comprehensive temperature control" that ended the history of "no concrete dam without crack" and some concrete dams without crack that had been first constructed in China in recent years, including the Sanjianghe concrete arch dam and the third stage of the famous Three Gorges concrete gravity dam.

He proposed the mathematical model and methods of solution for shape optimization of arch dams, which was realized for the first time in the world and up to now had been applied to more than 100 practical dams, resulting in 10–30% saving of dam concrete, and the efficiency of design was raised a great deal.

He had a series of contributions to the theory and applications of the finite element method (FEM).

He proposed a lot of new methods for finite element analysis, including the compound element, different time increments in different regions, the equivalent equation of heat conduction for pipe cooling, and the implicit method for computing elastocreeping stresses by FEM.

He developed the method of simulating computation of high concrete dams by FEM. All factors, including the course of construction, the variation of ambient temperatures, the heat hydration of cement, the change of mechanical and thermal properties with age of concrete, and the pipe cooling, precooling, and surface insulation can be considered in the analysis of the stress state. If the tensile stress is larger than the allowable value, the methods of temperature control must be changed until the maximum tensile stress is not bigger than the allowable value. Thus cracks will not appear in the dam. Experience shows that this is an important contribution in dam technology.

He proposed the equivalent stress for FEM and its allowable values that had been adopted in the design specifications of arch dams in China; thus the condition for substituting the trial load method by FEM is provided.

The instrumental monitoring can give only the displacement of some particular points but cannot give the stress field and the coefficient of safety of concrete dams. In order to overcome this defect, he proposed a new method of numerical monitoring by FEM that can give the stress field and the coefficient of safety and raise the level of safety control of concrete dam and that had begun to be applied in practical projects in China.

A new idea for semimature age of concrete has been proposed by him. The crack resistance of concrete may be promoted by changing its semimature age.

A vast amount of scientific research works had been conducted under his direction for a series of important concrete dams in China, such as Three Gorges, Xiaowan, Longtan, Xiluodu, Sanmenxia, Liujiaxia, Xing'anjiang, and so on. Fourteen results of his scientific research were adopted in the design specifications of gravity dams, arch dams, docks, and hydraulic concrete structures.

He has published 10 books: *Theory and Applications of the Finite Element Method* (1st ed. in 1979, 2nd ed. in 1998, 3rd ed. in 2009), *Thermal Stresses and Temperature Control of Mass Concrete* (1st ed. in 1999, 2nd ed. in 2012), *Thermal Stresses and Temperature Control of Hydraulic Concrete Structures* (1976), *Theory and Applications of Structural Optimization* (1984), *Design and Research of Arch Dams* (2002), *Collected Works on Hydraulic Structures and Solid Mechanics* (1988), *Selected Papers of Academician Bofang Zhu* (1997), *New Developments in Theory and Technology of Concrete Dams* (2009), and *Thermal Stresses and Temperature Control of Mass Concrete (in English)* (2014). He has published more than 200 scientific papers.

He was awarded the title of China National Outstanding Young Scientist in 1984, the China National Prize of Natural Science in 1982 for his research work in thermal stresses in mass concrete, the China National Prize of Scientific Progress in 1988 for his research work in the optimum design of arch dams, and the China National Prize of Scientific Progress in 2001 for his research works in simulating computation and thermal stresses. He became ICOLD (International Commission on Large Dams) Honorary Member.

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Introduction to Finite Element Method and Matrix Analysis of Truss

This chapter first introduces the basic conception of finite element method. The basic principles of truss analysis are similar to finite element method but easier to be understood, so the matrix analysis of truss is introduced later as an introduction to the finite element method.

1.1 Introduction to Finite Element Method

A truss is shown in Figure 1.1(a) with all nodes pin jointed and each element is a member only bearing axial force. A frame is shown in Figure 1.1(b) with all nodes rigid jointed and each element is a member bearing bending moment, shearing force, and axial force.

A beam is shown in Figure 1.1(c). All of the above three types of structures may be analyzed by structural mechanics and the theory of strength of materials. The basic assumption of them is the plane section assumption; in other words, the plane perpendicular to the central axis of the member before deformation remains to be a plane after deformation. For a rectangular high beam with relatively high ratio of the height H to length L (H/L), as shown in Figure 1.1(d), the plane section assumption cannot be applied. The calculation must be made according to the theory of elasticity that is actually a complicated problem even though the shape is simple.

Figure 1.2 shows some engineering structures. Figure 1.2(a) shows a gravity dam on the rock foundation. The dam body is nearly a triangle. However, there are slopes on both upstream and downstream boundaries. The mechanical and thermal properties of the dam body and the base rock are different. Figure 1.2(b) shows a double-curvature arch dam, which is a parabolic shell with varied curvature and thickness, supported on the base rock. Figure 1.2(c) shows an underground cavern in rock foundation. Figure 1.2(d) shows a massive concrete block in the construction of the concrete dam. The block is great in volume with concrete placed layer by layer. Generally, a layer of concrete with thickness of 1.5–3.0 m is placed every 6–10 days. Due to different ages, the modulus of elasticity, creep, and heat of hydration are all different in each layer.

For the various types of actual engineering structures shown in Figure 1.2, it is obviously impossible to work out the theoretical solutions by means of theory of elasticity. Numerical method is probably the only solution for stress calculation. Previously, attempts have been made to analyze such complicated structures by finite difference method. For example, for plane problems, the structural sections are divided into

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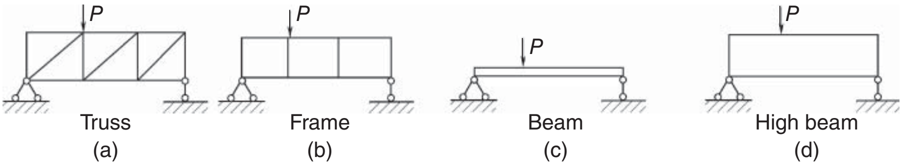

Figure 1.1 Truss, frame, beam, and high beam.

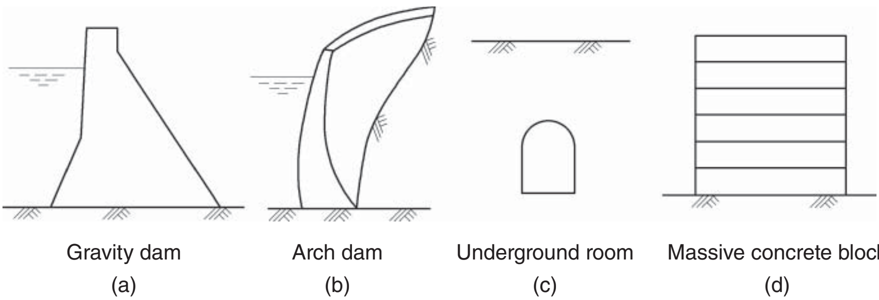

Figure 1.2 Practical engineering structures.

rectangular meshes, and the differential equations of equilibrium are transformed into finite difference equations. But the rectangular computing mesh is difficult to adapt to the boundary of the true structure, so it is rarely applied in the analysis of practical complicated structures.

The finite element method divides the original structure into finite elements, as shown in Figure 1.3.

The elements are a series of triangles of different size and shape; thus the computing mesh can adapt to the boundary of the true structure. Furthermore, different elements

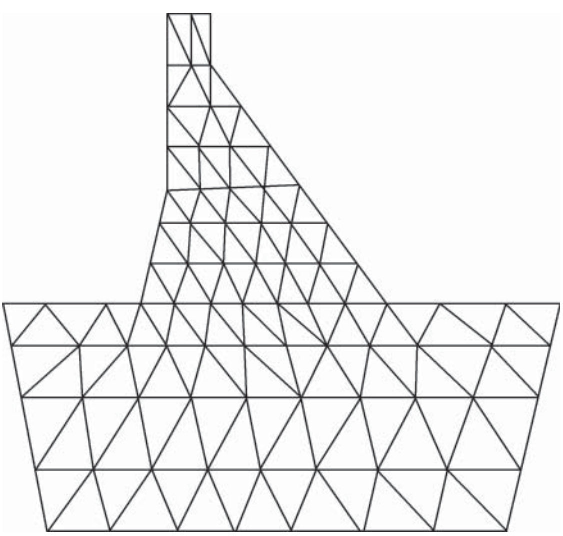

Figure 1.3 Cross section of gravity dam discreted into triangular elements.