Studies in Systems, Decision and Control 148

Anthony Tri Tran C. · Quang Ha

A Quadratic Constraint Approach to Model Predictive Control of Interconnected Systems

Springer

Studies in Systems, Decision and Control

Volume 148

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ISSN 2198-4182 ISSN 2198-4190 (electronic)
Studies in Systems, Decision and Control
ISBN 978-981-10-8407-2 ISBN 978-981-10-8409-6 (eBook)
https://doi.org/10.1007/978-981-10-8409-6

Library of Congress Control Number: 2018932517

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer Nature Singapore Pte Ltd.
The registered company address is: 152 Beach Road, #21-01/04 Gateway East, Singapore 189721, Singapore

Preface

The attraction of achieving higher efficiency and reliability for industrial plants and networked systems has created new research opportunities in the control and optimisation field. Among different design methods, the model predictive control (MPC) strategies, first developed for the petroleum refining industry, have proved to be effective in many applications. Originally found a widespread use in the stand-alone sites, the non-centralised adaption such as distributed and decentralised MPCs have been progressing towards more heterogeneous architectures that are able to cope with system complexities and variations in application domains.

This book presents a stabilising method for the control of interconnected systems having mixed connection configurations with distributed and decentralised model predictive control schemes. The novel notions of asymptotically positive realness constraint (APRC) and quadratic dissipativity constraint (QDC) are introduced as a fundamentally constituent part of this book. In both constraints, the function of inputs and outputs in the form of a supply rate, or a 'supply power', is quadratic. From the communication and information perspective, the quadratic constraint packs two pieces of information, the control and state vectors, into one variable, before carrying to different locations, and then unpacks them for use with the local control algorithm. The employment of quadratic constraints in two distinct approaches, segregation from and integration into the control algorithms, for the constrained stabilisation of interconnected systems is another contribution of this book.

Solutions for linear systems are given in distributed and decentralised strategies whereby the communication between subsystems is either fully connected, partially connected or totally disconnected. The interconnected systems and their distributed computerised-control platforms are considered within the realm of a cyber-physical system consisting of the physical connections between subsystems and the communication links between local computing processors. Within the auspice of the integrated construction method, the distributed and decentralised MPC strategies deal with the communication links from the cyber-connection side—the subsystems are wholly or partially connected in a distributed MPC scheme while being totally disconnected in a decentralised MPC.

By having the control inputs entirely or partially decoupled between subsystems, and no additional constraints imposed on the interactive variables, rather than the coupling constraint itself, the proposed approaches outreach various types of networked systems and applications. The effects of coupling delays and device networks are also resolved in part of the development. For parallelised connections that emulate parallel redundant structures and have unknown splitting ratios, a fully decentralised control strategy is developed as an alternative to the hybrid system approach. For the semi-automatic control systems, involved with both closed-loop and human-in-the-loop regulatory controls, the stability-guaranteed method of decentralised stabilising agents, that are interoperable with different control algorithms, is germinated and implemented for each single subsystem.

For nonlinear input-affine systems, the extended output vector including the vector field and the state vector are introduced such that the dissipativity criterion can be rendered in linear matrix inequalities. The compound vector can be viewed as manifest variables in the beviourial framework for dynamical systems. From the perspective of the dissipative system theory, both the storage function and supply rate with the extended output vectors are parameterised to avoid any conservativeness that may incur to the stabilisation of nonlinear systems.

In this book, MPC is formulated with state-space models having a standardised cost function. The stability constraint here is a constraint imposed on the current-time control vector, independently to the MPC objective function. For interconnected systems, the terminal constraint computations are formidable when dealing with subsystems having dissimilar dynamics whose settling times are heterogeneous. The quadratic constraint approach resolves this difficulty by having a constraint on the current-time control vector. The state constraint and recursive feasibility are, nevertheless, not included in this book.

An extension to new applications with the Internet of Things (IoT) is also presented with some dependable control schemes in which multiple controllers and sensors are cross-connected via the IoT communication network to ensure the duty-standby architecture for achieving quantitatively higher reliability of cyber-physical systems.

A broad range of applications in the process and manufacturing industries, networked robotics, networked control systems and network-centric systems such as power systems, telecommunication networks and chemical processes will benefit from the approaches in this book. Illustrative examples of networked interconnected systems are provided with numerical simulations in MATLAB environment. Specifically, a power system having four control areas, a dependable controller for cyber-physical systems and some other numerical examples are implemented with the distributed and decentralised MPC strategies employing the quadratic constraint approach to demonstrate the theoretical appraisals.

The developments are presented in seven chapters. This book starts with an introduction to the quadratic constraint in the time domain with a different perspective, as stated in Chap. 1. Here, the differences between this closed-loop perspective on the dissipation-based constraint and the other open-loop dissipative system approaches in the well-known interconnection stability conditions with passivity and small-gain theorems will be highlighted. A brief review on the MPC applications and the stabilising methods for the previously developed distributed MPC strategies is also given in the first chapter. Chapter 2 is dedicated to the quadratic constraints and their applications to the decentralised MPC of interconnected systems as the enforced attractivity constraints. In the next chapter, the attractivity conditions for the complex interconnected systems that have parallelised connections with unknown splitting ratios are presented, Chap. 3. An alternative constructive method of stabilising agents with the QDC is then delineated following in Chap. 4. Chapter 5 outlines a deterministic approach to the data lost processes with the presented dissipation-based quadratic constraints. A virtual perturbed cooperative-state feedback (PSF) strategy will be presented in the second part of this chapter. The available communication network in a cyber-physical system is capitalised on for improving the control performance with the PSF strategy. The developments for interconnected systems having a coupling delay element with the accumulative quadratic constraints are subsequently provided in Chap. 6. Chapter 7 is dedicated to the QDC application to the dependable control systems.

The general dissipativity constraint (GDC) method for the control design and synthesis of multi-variable systems in the discrete-time domain is presented in Appendix A. APRC and QDC with quadratic supply functions are the two special cases of the GDC. The dissipation-based constraints with a general supply function and the stability with a relaxed non-monotonic Lyapunov function and the input-to-power-and-state stabilisability (IpSS) are presented in this appendix. The GDC method for stabilising the interconnected systems with distributed, decentralised and dependable control architectures is well suited to the modern cyber-physical systems incorporating scalable and flexible communication networks. With emerging technologies in the Internet of Things (IoT) and cloud computing, the new architecture and algorithms will provide the tractability for implementations in a connected and 'smart' environment, yet help achieve the required reliability and continuity of the operational systems. The well-known MPC algorithms that employ plant models in the future state prediction for computing the control moves with convex optimisations have been found agile for deploying with cyber-physical systems.

During the course of preparation of this monograph, there were a series of invaluable discussions with Profs. Jan Maciejowski, Hung T. Nguyen and Tuan D. Hoang, to whom the authors are much indebted. In particular, the first author would like to gratefully acknowledge support obtained from the Singapore National Research Foundation (NRF) under its Campus for Research Excellence And Technological Enterprise (CREATE) programme and the Cambridge Centre for Advanced Research and Education in Singapore (Cambridge CARES), C4T project. Support received from various internal grant schemes at the Faculty of Engineering and Information Technology and the University of Technology Sydney, Australia, is also acknowledged.

Sydney, Australia
November 2017

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Quang Ha

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Chapter 1
Introduction

1.1 General

Automatic and semi-automatic control of large-scale interconnected systems, also known as network systems [10, 101], remains a challenging problem due to the increasing interactions between multi-variable subsystems connected in parallel, parallelised (Chap. 3), serial and recirculation. In a broad sense, a network system can be modelled as a large set of interconnected nodes, in which a node is a fundamental unit with specific contents [81]. The network system notion is extended over many application domains such as, but not limited to, chemical and petrochemical processes, power systems, telecommunication systems, transportation systems, supply chain systems, networked robotics and biological systems.

Figure 1.1 depicts a conceptual block diagram of such network systems with parallelised, serial and recirculated connections. In this book, we consider an interconnected system with a distributed computer system and a sensor/device network, as shown in Fig. 1.2a, b, that form a cyber-physical system (Chap. 5). The term cyber-physical system came into effect recently to describe the integration of control, computation, networking, physical processes and human–machine–machine interactions. In other words, cyber-physical systems (CPS) are "smart systems that include co-engineered interacting networks of physical and computational components", as defined by the National Institute of Standards and Technology (NIST). Despite the generic nature of the content covered in this book, some case studies and numerical examples provided herein are mainly for chemical process systems and power systems.

Feedback control of large-scale systems is a classical topic in the control theory, see, e.g. [1–3, 86, 100, 131, 140]. The coordination methods for the control of interconnected systems and various decomposition techniques have been used for many years, see, e.g. [142]. It is not, however, trivial to apply those approaches directly to the modern control problems with mathematical programming and flexible features of on-the-fly adjustments in a real-time computing environment. As a matter of course, the distributed and decentralised model predictive control of large-scale

A. Tri Tran C. and Q. Ha, *A Quadratic Constraint Approach to Model Predictive Control of Interconnected Systems*, Studies in Systems, Decision and Control 148, https://doi.org/10.1007/978-981-10-8409-6_1

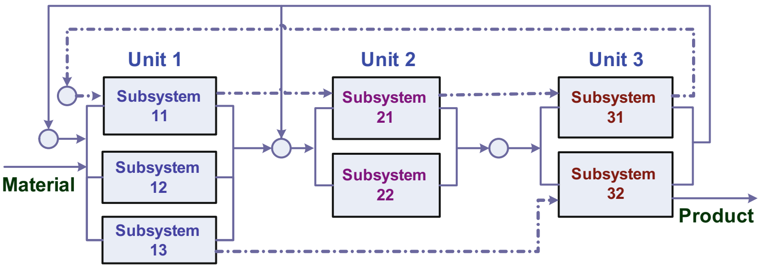
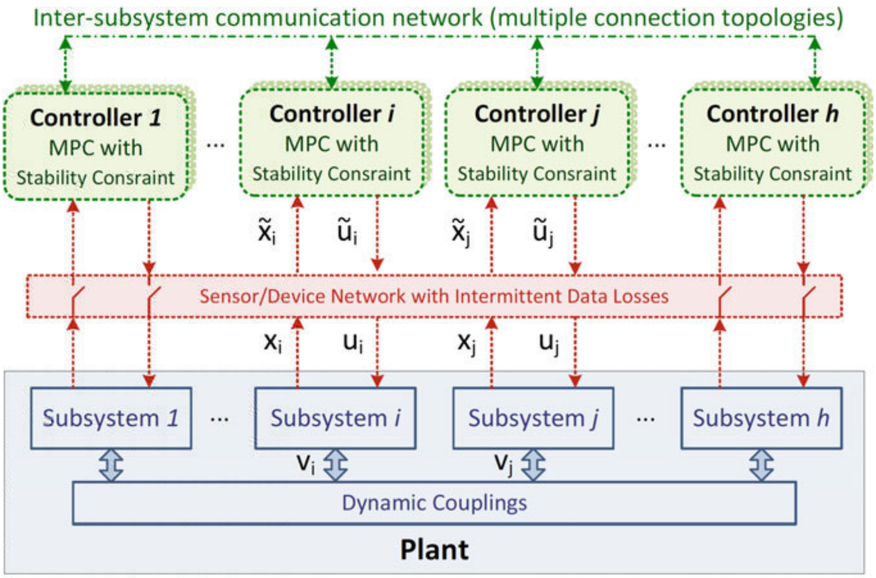
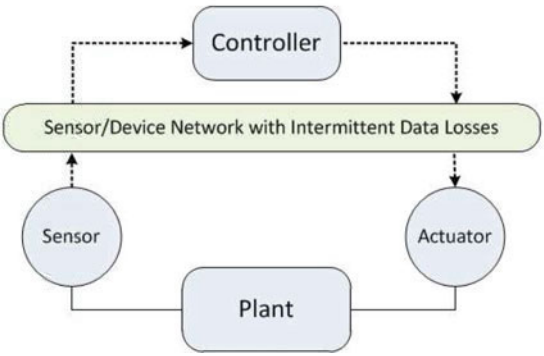


Fig. 1.1 Mixed connection configuration of an interconnected system



(a) System architecture



(b) Networked control system - a feedback loop

Fig. 1.2 An interconnected system under the cyber-physical system auspice interconnected systems [89, 125] have only been developed during the last two decades. The field has been progressing towards a heterogeneous architecture and outreaching different application domains.

The MPC algorithms cannot assure the closed-loop system stability in a cyber-physical environment without having an extra measure. To this end, the stability-guaranteed problem of MPC with state and control constraints has long been a research topic that draws keen interests from both academia and industry, see, e.g. [4, 5, 13, 19, 20, 23, 74, 75, 80, 88, 90, 93, 115, 118, 124, 128]. A thorough review on the stability and optimality of different MPC formulations and stability-guaranteed methods can be found in [93, 117]. The constrained closed-loop system with MPC is not guaranteed stable if the predictive horizon N is not sufficiently long [88], and further, the MPC optimisation may not be recursively feasible if the initial state vector does not belong to an appropriate admissible set [93]. The recursive feasibility may imply system stability regardless of the length of the predictive horizon N if an additional constraint is imposed onto the MPC optimisation. Among the developed stability constraints, the terminal state constraint associated with the maximal output admissible set [42, 73] is well known in the MPC literature. However, the terminal constraint approach may provide a conservative feasible region for distributed MPCs, especially for heterogeneous, dynamically coupled systems, see, e.g. [24, 121]. The quadratic constraint approach offers a non-conservative alternative for the stability assurance of decentralised and distributed MPCs.

The quadratic constraint in this book has been developed on the ground of the dissipative system theory [172], which is attractive to the control design problem for interconnected systems owing to its input and output usage and related properties. Although the theory has been applied to the stability analysis and control design of linear and nonlinear systems since the 1970s, see, e.g. [17], the practical implementations have not been found widespread used yet. Inspired by the (Q, S, R)-dissipativity in [59, 133, 134, 159, 172], the quadratic supply-rate function has been extensively employed in this work. Our work is focused on the open-loop approach (or the closed-loop perspective), as we believe it is suitable for the decentralised model predictive control schemes. The interconnection stability conditions with the two open-loop dissipative systems are not applicable in this open-loop approach, as a result, but the conditions on the closed-loop system are derived instead. Under this open-loop realm, the quadratic supply rate can be developed into a stability constraint for the MPC, which is a constraint on the current-time control vector. The coefficient matrices of the stability constraint can be updated online, or simply pre-computed off-line, for guaranteeing the closed-loop system stability. The notions of asymptotically positive realness constraint (APRC) and quadratic dissipativity constraint (QDC) are introduced here in this context.

The simplest form of APRC that employs a real-valued quadratic supply function with respect to the state and input pair, $\xi(x_i(k), u_i(k))$, is expressed as a constraint in $u_i(k)$, when $x_i(k)$ and $\xi(x_i(k-1), u_i(k-1))$ are known. For example, the key conditions with the APRC consist of the dissipation inequality with a storage function, as in the dissipative systems theory [172], of the form

$$V(x_{k+1}) - \tau V(x_k) \leqslant -(x_k^T \; u_k^T) N (x_k^T \; u_k^T)^T, \quad V(x_k) \geqslant 0,$$

where $(x_k^T\ u_k^T)^T$ is the stacking vector comprising x_k and u_k, and a dissipation-based inequality of the form

$$0 \geqslant (x_k^T\ u_k^T)N(x_k^T\ u_k^T)^T \geqslant \gamma\,(x_{k-1}^T\ u_{k-1}^T)N(x_{k-1}^T\ u_{k-1}^T)^T \ \ \forall k \in (0, k_s],$$

$$\text{and}\ \ (x_k^T\ u_k^T)N(x_k^T\ u_k^T)^T \geqslant 0\ \ \forall k > k_s > 0,$$

where $N := \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix}$, $\gamma \in (0, 1)$ and $\tau \in (0, 1)$.

The key conditions with the QDC also consist of the dissipation inequality with a storage function and a dissipation-based inequality, as follows:

$$V(x_{k+1}) - \tau V(x_k) \leqslant -(x_k^T\ u_k^T)N(x_k^T\ u_k^T)^T, \ \ V(x_k) \geqslant 0,$$

$$0 \geqslant (x_k^T\ u_k^T)N(x_k^T\ u_k^T)^T \geqslant \gamma\,(x_{k-1}^T\ u_{k-1}^T)N(x_{k-1}^T\ u_{k-1}^T)^T \ \ \forall k > 0.$$

The differences between the APRC and the QDC are in the second inequalities whose fulfilment may not incur at every time steps in $\mathbb{Z}$ or only within a finite-time interval. We will prove that the attractivity can be obtained from the above APRC and QDC inequalities for $V(x_k) = x_k^T P x_k$, where P is full row rank. Furthermore, a generalised form of the dissipation-based inequality with a $\mathcal{KL}-$bounded supply function (see Sect. 1.3.2) will also be introduced, the general dissipativity constraint (GDC). In the GDC method, we often deal with the non-negative $\triangle V(x_k)$, i.e. $V(x_k) - V(x_{k-1}) \geqslant 0$ (and decreasing, not necessarily monotonically), but not with $\triangle V(x_k) \leqslant 0$, as in Lyapunov's method.

The APRC and QDC are subsequently recast and incorporated into the online optimisation problem of a local MPC as an enforced attractivity constraint, or employed in the algorithm of stabilising agent independently to the associated control algorithms. These quadratic constraint approaches are non-conservative for the decentralised control problem by virtue of its non-monotonic storage function that plays a similar role as a relaxed non-monotonic Lyapunov function whose non-conservativeness has been pointed out in [96].

Methodologically, the online-centric stabilisation of interconnected systems with decentralised model predictive controllers is the main theme of this book. Convex programming numerical methods with their elegant solutions [15, 103] are used throughout the numerical examples. By the same token, the linear matrix inequality (LMI) is extensively employed in the derivations for dissipative criteria and stability conditions for interconnected systems.

For semi-automatic control with quadratic constraints in Chap. 4, the control system is associated with a stabilising agent that is interoperable with different control algorithms. This segregated stabilising method can be applied to the stability assurance problem for the local and remote operations of a cyber-physical system. The stabilising agent relies on the quadratic constraint trajectory to initiate corrective actions to stabilise the operational system. Both intermittent data losses and coupling delays are inclusive and resolved deterministically by extending the quadratic constraint and dissipative conditions to recover the convergence property.

We formulate the redundant processes as continuously parallelised, or parallel splitting, systems in Chap. 3 as an alternative to the hybrid system with Boolean variables. As a result, the global system is effectively stabilised in different operational scenarios, including concurrent and duty-standby modes. The parallelism characteristic comes from the splitting ratios of parallelised plants, which are unknown and usually time dependent. The concept and mechanism of self-recovery control and dependable control systems using the Internet of Things (IoT) are introduced for the building of a reliable operational technology (OT) (computerised control) system, within a connected environment of cyber-physical systems in Chap. 7.

The implementation aspect of the quadratic constraint method, but not the mathematical theory, is emphasised in this book. The remaining of this chapter is reserved for introducing the quadratic constraints in the time domain with a practical perspective. A brief review of passivity and small-gain theorems is provided at the end of the chapter to simply show that they are not applicable to the quadratic constraint approaches in this book.

1.2 Quadratic Constraint—a Time-Domain Perspective

A practical and time-domain perspective of the quadratic constraint in a control problem is presented in this section.

We first discuss the meaning of the passivity term, $-y^T u$, or $-y \times u$ for systems with single variables, in a feedback control system shown in Fig. 1.3, and various trajectories of $-y^T u > 0$, or $-y^T u \geqslant 0$, cases, as shown in Fig. 1.4, within the time domain.

From the theoretical perspective, the passivity term represents the phase property of an output and input pair, see, e.g. [139]. It is well known that the phase and gain properties are important for the control system design in the frequency domain with Bode and Nyquist plots in the control literature [3, 38]. However, the motion of the passivity term,

$$-y(t)^T u(t),$$

or of the quadratic supply rate of the form

$$-\boldsymbol{\alpha}\, y(t)^T y(t) - \boldsymbol{\beta}\, y(t)^T u(t) + \gamma\, u(t)^T u(t),$$

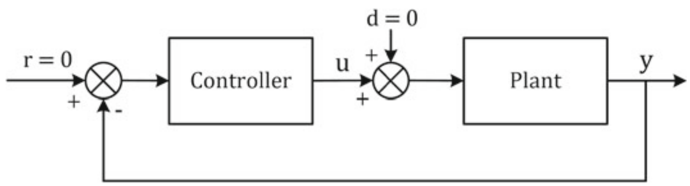


Fig. 1.3 A feedback control system

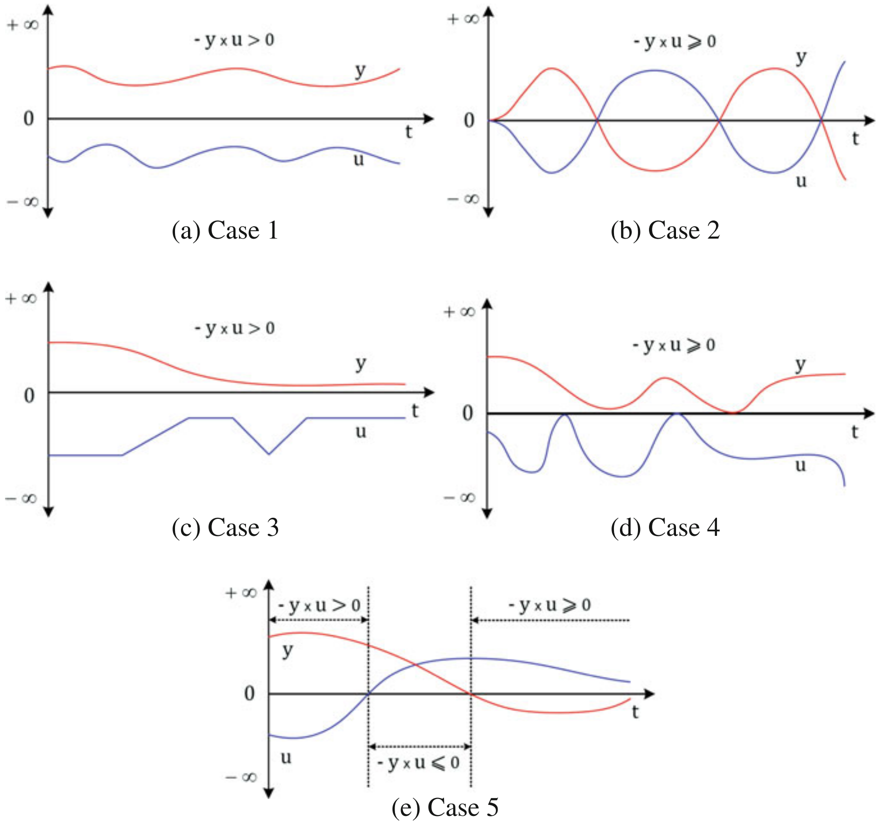


Fig. 1.4 The meaning of $-y^T u \geqslant 0$ in a feedback control system

in the time domain does not necessarily convey a stability or convergence message. The five cases of $y(t)$ and $u(t)$ trajectories shown in Fig. 1.4 illustrate the typical examples of $-y^T u \geqslant 0$, which suggest irrelevant attractiveness of the related controlled system. In what follows, the time index $k \in \mathbb{Z}^+$ is used for discrete-time systems.

1.2.1 Positive Supply Power

The discussion starts with a special case of relationship between the input and output of an *absolute energy-passive* (AP) signal system, such as the input and output of a feedback control system. We will show that the concept of *energy-passivity* can be adopted to illustrate the connotation of dissipating the *abstracted energy* around the steady states, and the *energy-dissipativity* may imply a converging output and input pair or an attractive controlled system.

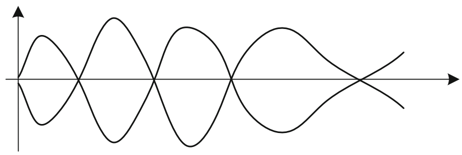


Fig. 1.5 An absolute energy-passive (AP) motion

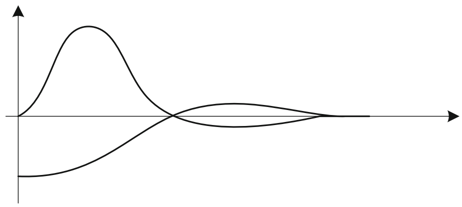


Fig. 1.6 A perfect one-overshoot stable (POS) motion

An input and output signal system is said to be absolute energy-passive (AP) *if and only if both of its input and output trajectories, $u_{(k)}$ and $y_{(k)}$, respectively, always pass through their corresponding steady states* (*or set point*) *at the same time instants.*

For clarity, the output and input steady states can be assumed to be zero ($y_{ss} \equiv 0$, $u_{ss} \equiv 0$), i.e. normalised to zero steady states. An AP motion is not necessarily attractive or convergent. Figure 1.5 depicts a non-attractive AP motion. For signal systems that are AP, the following discrete-time inequality holds true for all $k \geqslant 0$:

$$p_{(k)} := (y_{(k)} - y_{ss})^T (u_{ss} - u_{(k)}) \geqslant 0. \tag{1.1}$$

The inequality (1.1) is called *instant absolute energy-passive inequality* (iAPi) herein. An AP signal system is nothing else than a special case of passive systems and a subclass of zero input–output phase shift systems.

The absolute energy-passivity is apparently difficult to be obtained from a feedback control system. From the feedback control and circuit theory point of view, we are familiar with the overshoot of under-damped control systems. Therefore, the so-called *perfect one-overshoot stable* (POS) motion is introduced next.

Figure 1.6 depicts a typical POS motion. While AP motions are not necessarily attractiveness or convergence, the POS motion is a special case of stabilised and well-performed control systems.

The critically damped controlled system is perfectly one-overshoot stable, or POS, *if and only if it is* AP *and the output and input motions are attracted to their steady states after exactly one overshoot.*

Similar to AP signal systems, it is not easy to obtain a POS behaviour from a control design problem. We usually have a stabilised controlled system that also has an overshoot, but does not have the AP property; i.e., the input and output trajectories do not cross their steady states at the same time instants, as shown in Fig. 1.7. This is the time response of a typical critically damped system in the feedback control literature. One can easily verify that the sum of $p_{(k)}$ over the time intervals of T_1 and

T_3 is greater than that over T_2 for some stabilised controlled systems. Further, the critically damped system will have the AP property when $T_2 \rightarrow 0$. We now link a stabilised controlled system with the energy-passivity defined below.

For an energy-passive and stabilised controlled system, the accumulative sum of $p_{(k)}$ over time is always non-negative, i.e.

$$\sum_{\kappa=0}^{k}(y_{(\kappa)} - y_{ss})^T(u_{ss} - u_{(\kappa)}) \geqslant 0 \ \ \forall k \geqslant 0. \tag{1.2}$$

Alternatively, it can be said that, for an energy-passive and stabilised controlled system, the sum $s_{(k)}$ of $p_{(k)}$ where $p_{(k)} \geqslant 0$, denoted as $s^+_{(k)}$, is greater than or equal to the absolute value of the sum $s_{(k)}$ of $p_{(k)}$ where $p_{(k)} < 0$, denoted as $s^-_{(k)}$, i.e.

$$s^+_{(k)} \geqslant -s^-_{(k)}. \tag{1.3}$$

An example for the trajectories of a more general case of an energy-passive and stabilised controlled system is shown in Fig. 1.8. In this figure, the sum of $p_{(k)}$ from odd intervals $T_1,\ T_3,\ T_5, \ldots (= s^+_{(k)})$ is greater than the sum of $p_{(k)}$ from even intervals $T_2,\ T_4,\ T_6, \ldots\ (=\ s^-_{(k)})$. The even time intervals represent the phase differences between the input and output trajectories. It can be seen that this type of trajectories is achievable from a practical control design problem and that if $T_{\text{even}} \rightarrow 0$, the controlled system will become AP or POS, which is impractical for a control design problem.

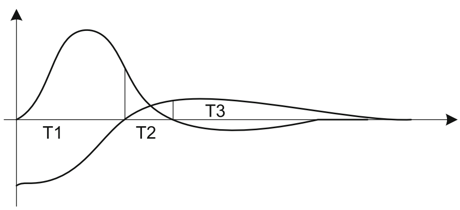


Fig. 1.7 POS and energy-passive, but not AP

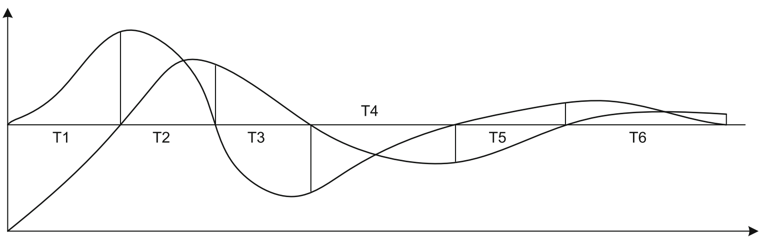


Fig. 1.8 Energy-passive and stabilised controlled system—an example of input and output trajectories