ICME-13 Monographs

Denisse R. Thompson
Megan Burton
Annalisa Cusi
David Wright *Editors*

Classroom Assessment in Mathematics

Perspectives from Around the Globe

Springer

ICME-13 Monographs

Series editor

Gabriele Kaiser, Faculty of Education, Didactics of Mathematics, Universität Hamburg, Hamburg, Germany

Each volume in the series presents state-of-the art research on a particular topic in mathematics education and reflects the international debate as broadly as possible, while also incorporating insights into lesser-known areas of the discussion. Each volume is based on the discussions and presentations during the ICME-13 Congress and includes the best papers from one of the ICME-13 Topical Study Groups or Discussion Groups.

More information about this series at http://www.springer.com/series/15585

Denisse R. Thompson · Megan Burton
Annalisa Cusi · David Wright
Editors

Classroom Assessment in Mathematics

Perspectives from Around the Globe

Springer

Editors
Denisse R. Thompson
Department of Teaching and Learning
University of South Florida
Tampa, FL
USA

Megan Burton
Auburn University
Auburn, AL
USA

Annalisa Cusi
University of Turin
Turin
Italy

David Wright
Research Center for Learning and Teaching
Newcastle University
Newcastle upon Tyne
UK

ISSN 2520-8322 ISSN 2520-8330 (electronic)
ICME-13 Monographs
ISBN 978-3-319-73747-8 ISBN 978-3-319-73748-5 (eBook)
https://doi.org/10.1007/978-3-319-73748-5

Library of Congress Control Number: 2017964250

© Springer International Publishing AG 2018
This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.
The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.
The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

This monograph is an extension of the discussions and presentations shared in Topic Study Group (TSG) 40 on Classroom Assessment for Mathematics Learning that occurred during the 13th International Congress on Mathematical Education (ICME-13) held in Hamburg, Germany, in July 2016. Co-chairs of that Topic Study Group were Karin Brodie (South Africa) and Denisse R. Thompson (United States), with committee members Leonora Díaz Moreno (Chile), Nathalie Sayac (France), Stanislaw Schukajlow (Germany), and IPC liaison Elaine Simmt (Canada).

The goal of TSG 40 was "to share and build research relating to assessment *for* and *as* learning in mathematics classrooms" (http://www.icme13.org/files/tsg/TSG_40.pdf). With this goal in mind, contributions (papers, oral presentations, and posters) were solicited "relating to teaching mathematics in classrooms as well as practices in teacher education and professional development that address issues on assessment for learning and teaching mathematics. How teachers learn to assess for learning and how their learning is enacted is key to developing assessment for learning that enables teachers to gain insight into students' thinking to guide further instruction" (http://www.icme13.org/files/tsg/TSG_40.pdf).

Seven themes were identified as specific areas where contributions might be developed that would fit within the broad aims of the TSG:

- "The enactment of classroom practices that reflect current thinking in assessment for learning or assessment as learning in mathematics (for example, giving feedback, developing classroom conversations, peer or self-assessment);
- The development of pre-service and in-service teachers' professional knowledge or practices related to assessment for learning mathematics;
- The enactment of practices in teacher education and professional development that reflect current thinking relative to assessment for learning and assessment as learning;
- The development of assessment tasks that reflect the complexity of mathematical thinking, problem solving, and other important mathematical competencies;
- The design of alternative modes of assessment for learning (e.g., online, investigations, forms of formative assessment);

- The development of assessment practices that support equity or enhance access to the learning of mathematics;
- The enactment of practices to ensure that curriculum, instruction, and classroom assessment are well aligned." (http://www.icme13.org/files/tsg/TSG_40.pdf)

Not all themes were equally represented across the contributions. Within the sessions assigned to various aspects of the TSG during ICME, 12 papers, 12 oral presentations, and 14 posters were presented. Two of the papers were presented as part of a joint session with TSG 39 on Large-Scale Assessment and Testing in Mathematics Education. More information about the overall structure of the various sessions can be found in the *Proceedings of ICME-13*.

Development of this Volume

At the conclusion of the Congress, three Topic Study Group participants (Megan Burton, Annalisa Cusi, and David Wright) joined with one of the co-chairs (Denisse R. Thompson) to serve as the editorial panel responsible for overseeing the development of this post-Congress monograph. Given the relatively small number of overall contributions to the various themes of the TSG, a general call was made to authors of papers, oral presentations, or posters to expand their original contribution for the monograph. Fifteen of the original contributors chose to make that investment of time and submitted a revised and expanded version of their presentation made in Hamburg.

All submissions underwent a review process in which they were reviewed by two members of the monograph's editorial panel as well as one other potential contributing author. Reviews were returned to authors with guidelines and suggestions for revisions needed to strengthen the paper and make it acceptable for publication in the monograph. Two authors chose not to make the requested revisions, primarily because their research was not far enough along to enable the requested revisions to be made. Revised papers were reviewed again by members of the monograph's editorial panel and edited as needed.

Structure of the Volume

The remaining contributions, together with an introductory paper and a concluding paper, provide insight into various assessment practices from educators and researchers around the globe. Under no circumstances would we claim that the papers in this volume provide a complete picture of assessment practices in various countries. It is not even clear that they provide a representative picture of the types of practices that teachers use around the globe. Rather, they provide a glimpse into possible assessment practices, the types of information or data to be collected from

those practices, and the potential for that information or data to inform further instruction.

The authors of the papers hail from eleven different countries. Thus, the papers provide a glimpse into the extent to which issues surrounding classroom assessment, particularly formative assessment, are increasingly important regardless of the schooling or cultural context.

The papers expanded from conference contributions have been grouped into four main categories:

- Three papers provide examples of classroom assessment in action. The paper by Swan and Foster focuses on designing curriculum and assessment lessons to encourage communication and problem-solving. Pai considers how teachers deal with in-the-moment assessment actions, and Straumberger investigates how self-assessment might be used by students to enhance their own individual mathematics practice.
- Four papers illustrate how technology can be used as a tool to facilitate formative classroom assessment, regardless of schooling level. For instance, the paper by Downton focuses on how digital flip cameras can help primary teachers capture assessment practices of young children so they can explore them in more detail with the children. The paper by Cusi and colleagues uses software, tablets, and interactive whiteboards to help teachers enhance formative assessment when working with fifth-grade children to engage them in problem-solving and explaining their reasoning. At the other end of schooling, the paper by Nagari-Haddif and Yerushalmy considers how online assessments can be used to understand the thinking of high school students in calculus, and the paper by Platz and colleagues uses an e-proof environment within tertiary education to assist students in developing their skills in constructing mathematical proofs.
- Two papers focus on how statistical models might assist with formative assessment; both were originally presented in the joint session with TSG 39 on large-scale assessment. Using Rasch modeling or Cognitive Diagnostic Assessment, the authors of the two papers explore how assessments related to problem-solving or concept development can inform teachers so that appropriate instructional interventions could occur.
- The final four papers address different perspectives to engage teachers in formative assessment. Sayac investigates the assessment practices of French primary teachers, under the assumption that one needs to understand teachers' assessment practices to develop professional development to enhance those practices. Andrade-Molina and Moreno illustrate how national curricular guides, together with national assessments, can send mixed messages to teachers about the nature of learning and the types of classroom assessments that can support that learning. Burton and colleagues describe five different pedagogical approaches used in preparing teachers and in professional development settings and how teacher educators might highlight the formative assessment practices that are naturally linked to the instruction within those pedagogical approaches.

Wright and colleagues share the work of a large-scale assessment project within Europe that has developed a toolkit to assist teachers as they work to integrate formative assessment into their regular classroom instruction.

The volume's introductory paper attempts to set the stage for the importance of classroom assessment by contrasting formative and summative assessment practices. Members of the two ICME-13 TSGs on assessment joined together to prepare a Topical Survey, *Assessment in Mathematics Education: Large-Scale Assessment and Classroom Assessment* (Suurtamm et al. 2016), that represents an overview of the state of assessment and the interactions of classroom and large-scale assessment as of Spring 2016. Rather than repeat the information in the introductory chapter, readers are referred to that volume for research issues related to (1) purposes, traditions, and principles of assessment; (2) design of assessment tasks; (3) classroom assessment in action; (4) interactions of large-scale and classroom assessment; and (5) enhancing sound assessment knowledge and practices. Each of the five sections in that volume concludes with a list of questions for possible future work.

The concluding paper in this volume looks across the various papers to consider what lessons can be learned from the various models of assessment practices and to consider how those lessons might suggest future areas of research. The fact that the papers are authored by researchers in many countries highlights the importance of cross-national and cross-cultural research studies so that we can learn from each other.

Potential Audience for This Book

This volume is applicable to a wide audience. Classroom teachers might read the volume for ideas about research initiatives and practices in other parts of the world that can be applied to their own context. Researchers might use the volume to contemplate areas for additional research. Mathematics teacher educators and professional development providers might use the volume, perhaps in conjunction with the Topical Survey on Assessment, as a supplement in a course in the preparation of teachers or the enhancement of teachers' instructional practice.

Acknowledgements

We thank all those authors who wrote and reviewed manuscripts for this publication. In addition, we appreciate the encouragement of Gabriele Kaiser, series editor and convener of ICME-13, to support the sharing of contributions from ICME with

the global mathematics education community, and especially with those unable to travel to Hamburg in 2016.

Tampa, USA — Denisse R. Thompson
Auburn, USA — Megan Burton
Turin, Italy — Annalisa Cusi
Newcastle upon Tyne, UK — David Wright

Reference

Suurtamm, C., Thompson, D. R., Kim, R. Y., Moreno, L. D., Sayac, N., Schukajlow, S., Silver, E., Ufer, S., & Vos, P. (2016). *Assessment in mathematics education: Large-scale assessment and classroom assessment*. (ICME-13 Topical Surveys.) SpringerOpen.

Contents

Editors and Contributors

About the Editors

Denisse R. Thompson is Professor Emeritus of Mathematics Education at the University of South Florida in the U.S., having retired in 2015 after 24.5 years on the faculty. Her research interests include curriculum development and evaluation, with over 30 years of involvement with the University of Chicago School Mathematics Project. She is also interested in mathematical literacy, in the use of children's literature in the teaching of mathematics, and in issues related to assessment in mathematics education. She served as co-chair of Topic Study Group 40 on classroom assessment at ICME-13. In addition, she is a co-editor of the series *Research in Mathematics Education*, published by Information Age Publishing.

Megan Burton is an Associate Professor and the Elementary Education Program Coordinator at Auburn University, Alabama (USA). She teaches and advises undergraduate and graduate students in elementary education and conducts research related to elementary mathematics education, with focus on elementary teacher change, inclusion, and rural education. As a former elementary teacher with experience in inclusion and English Language Learners, she is committed to classrooms that allow all students to encounter strong mathematics instruction in meaningful, differentiated ways.

Annalisa Cusi graduated in Mathematics at Modena and Reggio Emilia University in 2001, where she obtained a Ph.D. in Mathematics in 2009. She has been teaching mathematics and physics in upper secondary school since 2001. She worked as a Research Fellow at the University of Turin from 2014 to 2016 within the European Project FaSMEd. Her main research interests are innovation in the didactics of algebra; the analysis of teaching/learning processes, with a focus on the role played by the teacher; methods to promote early algebraic thinking in young students; teacher professional development; and formative assessment processes in mathematics.

David Wright is Senior Research Associate: Research Centre for Learning and Teaching Newcastle University (United Kingdom) (now retired). He has 15 years of experience in teaching mathematics at secondary, further, and higher education as an Associate Lecturer in the Open University. He was Subject Officer for Mathematics for the British Educational Communications and Technology Agency (Becta) for 4 years and 10 years in initial teacher education and research at Newcastle University. He is the Scientific Director of the European Union research project: Formative Assessment in Science and Mathematics Education (FaSMEd).

Contributors

Melissa Andrade-Molina Faculty of Engineering and Science, Aalborg University, Aalborg, Denmark

Wanda Audrict TLLR, Inc, Stone Mountain, GA, USA

Jonathan D. Bostic Bowling Green State University, Bowling Green, OH, USA

Megan Burton Auburn University, Auburn, AL, USA

Jill Clark Research Centre for Learning and Teaching, Newcastle University, Newcastle upon Tyne, UK

Annalisa Cusi Department of Philosophy and Education, University of Turin, Turin, Italy

Ann Downton Faculty of Education, Monash University, Surrey Hills, VIC, Australia

Leonora Díaz Moreno Department of Mathematics, Faculty of Science, Valparaiso University, Valparaiso, Chile

Colin Foster School of Education, University of Leicester, Leicester, UK

Miriam Krieger Institute for Mathematical, Natural Sciences and Technology Education, University of Flensburg, Flensburg, Germany

Chap Sam Lim School of Educational Studies, Universiti Sains Malaysia, Gelugor, Penang, Malaysia

Valerie L. Mills Oakland Schools, Waterford, MI, USA

Francesca Morselli Department of Mathematics, University of Genova, Genoa, Italy

Galit Nagari-Haddif Faculty of Education, University of Haifa, Haifa, Israel

Engelbert Niehaus Institute of Mathematics, University of Koblenz-Landau, Landau, Germany

Jimmy Pai University of Ottawa, Ottawa-Carleton District School Board, Ottawa, ON, Canada

Marjorie Petit Ongoing Assessment Project, Moretown, VT, USA

Melanie Platz Institute for Mathematics Education, University of Siegen, Siegen, Germany

Cristina Sabena Department of Philosophy and Education, University of Turin, Turin, Italy

Nathalie Sayac Laboratoire de Didactique André Revuz, Université Paris-Est Créteil, Livry-Gargan, France

Carolyn Jia Ling Sia School of Educational Studies, Universiti Sains Malaysia, Gelugor, Penang, Malaysia

Edward A. Silver University of Michigan, Ann Arbor, MI, USA

Toni A. Sondergeld School of Education, Drexel University, Philadelphia, PA, USA

Waldemar Straumberger University of Bielefeld, Bielefeld, Germany

Marilyn E. Strutchens Auburn University, Auburn, AL, USA

Malcolm Swan School of Education, Centre for Research in Mathematics Education, University of Nottingham, Nottingham, UK

Denisse R. Thompson College of Education, University of South Florida, Tampa, FL, USA

Lucy Tiplady Research Centre for Learning and Teaching, Newcastle University, Newcastle upon Tyne, UK

Kathrin Winter Institute for Mathematical, Natural Sciences and Technology Education, University of Flensburg, Flensburg, Germany

David Wright Research Centre for Teaching and Learning, Newcastle University, Newcastle upon Tyne, UK

Michal Yerushalmy University of Haifa, Haifa, Israel

Part I
Introduction to the Volume

Chapter 1
Formative Assessment: A Critical Component in the Teaching-Learning Process

Denisse R. Thompson, Megan Burton, Annalisa Cusi and David Wright

Abstract This introductory paper to the volume contrasts formative assessment with summative assessment and describes the importance of formative assessment to classroom instruction. In particular, it argues that a task is formative to the extent that data from the task are used to enhance and inform further instruction rather than simply to provide an evaluation of a student or of instruction. The use of design research as a mechanism to develop sound classroom assessment is outlined because a design science framework provides a means to tie together varied exemplars of innovations in assessment. A cycle of task implementation and revision can lead to improved assessment practices.

Keywords Design research · Formative assessment · Summative assessment Evaluation

D. R. Thompson (✉)
University of South Florida, Tampa, FL 33620, USA
e-mail: denisse@usf.edu

D. R. Thompson
College of Education, EDU105, Tampa, FL 33620, USA

M. Burton
Auburn University, 5020 Haley Center, Auburn, AL 36849, USA
e-mail: megan.burton@auburn.edu

A. Cusi
University of Turin, Via Tamburini 45, 42122 Reggio Emilia, Italy
e-mail: annalo@tin.it

D. Wright
Research Center for Learning & Teaching, Newcastle University, Newcastle upon Tyne NE1 7RU, UK
e-mail: wrightdavidg@gmail.com

© Springer International Publishing AG 2018

D. R. Thompson et al. (eds.), *Classroom Assessment in Mathematics*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-73748-5_1

1.1 Introduction

For much of the general public, including parents and politicians, assessment is often synonymous with tests. But assessment can and should be much more than just a test. In fact, one way to define *assessment* in mathematics is "as the process of gathering evidence about a student's knowledge of, ability to use, and disposition toward, mathematics and of making inferences from that evidence for a variety of purposes" (National Council of Teachers of Mathematics [NCTM] 1995, p. 3). In contrast, *evaluation* is "the process of determining the worth of, or assigning a value to, something on the basis of careful examination and judgment" (NCTM 1995, p. 3). Tests, then, are a means of evaluation, and evaluation is just one aspect of assessment.

The tension implicit in the previous paragraph reflects the fact that assessment has both formative and summative perspectives. A given assessment task can be either formative or summative, depending on how the information gathered from that task is used. If an assessment task is used for accountability purposes, at the individual student level or to make value judgments about the quality of education in a school or country, then that assessment task is *summative*; most large-scale external assessments or classrooms assessments used at the end of a unit of study fit within this category. However, when assessment tasks are used to collect insight into students' thinking that can inform the teacher or the students about their learning which is then used to guide further instruction, the assessment task is *formative*; tasks and activities that help move students' thinking forward and help guide teachers as they make instructional decisions fit within this side of the assessment coin.

Too often, assessment is viewed as something that occurs at the end of a unit of study or a specific time period. However, assessment "that enhances mathematics learning becomes a routine part of ongoing classroom activity rather than an interruption. … [and is] an integral part of instruction that encourages and supports further learning" (NCTM 1995, p. 13). The papers in this volume take this view of assessment—as an ongoing and integral part of instruction to enhance the learning of students.

1.2 The Role of Formative Assessment in the Classroom

Black and Wiliam (2009) describe formative assessment in terms of decisions made based on the assessment rather than on the actual collection of information from the assessment. Assessment is formative

> to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded than the decisions they would have taken in the absence of the evidence that was elicited. (p. 9)

As noted by Wiliam, this definition means that formative assessment necessitates "that one is clear about what it is that students are to learn, but it does not impose a particular view of the mathematics curriculum, nor does it entail any particular view of what happens when learning takes place" (2015, p. 250). That is, a determination of the nature of an assessment depends on how information from that assessment is used. A given task, even an end-of-unit test, could be formative if it is used to guide instruction or help teachers determine how to move students' learning forward, but could be summative if it is used solely to provide a grade.

The definition of formative assessment posited by Black and Wiliam poses a challenge for teachers, educators, and researchers. To gain the type of information needed to make effective instructional decisions, cognitively demanding tasks are needed that focus on conceptual understanding rather than just surface knowledge. Identifying and developing such tasks is not only a challenge for teachers, but is also a challenge for students who are asked to think mathematically in ways that involve more than just procedures and to explain their thinking in multiple ways—via pictures, words, symbols, or in some other format. Students and their teachers need many opportunities to engage with such tasks to develop an appreciation for the extent to which they can facilitate the learning process.

Over the last three decades, in particular, there has been a recognition around the globe of the need to engage many more students in mathematics, and to ensure that all students have an opportunity to be successful. As a consequence, mathematics educators in many countries have emphasized the importance of a student-centered classroom rather than just a teacher-centered or teacher-directed one. Formative assessment is a critical component of shifting to a student-centered perspective because it places the student at the center of the assessment process, through having students assess their own learning as well as supporting the learning of classmates. Black and Wiliam stress that, together with the teacher and the learner himself, fundamental agents in the assessment processes are the peers. Peers can challenge learners to reflect on their own thinking, helping them "to make unconscious processes overt and explicit and so making these more available for future use" (2009, p. 19). As Leinwand and colleagues note, "an important goal of assessment should be to make students effective self-assessors, teaching them how to recognize the strengths and weaknesses of past performance and use them to improve their future work" (2014, p. 95). Through both self-assessment and peer assessment of present and past performance, students become the center of the instruction and assessment cycle.

1.3 Design Research in Classroom Assessment

The report *Knowing What Students Know* (Pellegrino et al. 2001) identifies progress in the science of designing assessments as a key factor in enhancing classroom assessment. The report provides a range of assessment examples and steers the analysis of them towards a science of design:

> while it is important to carefully analyze each of the examples as a separate instance of innovative design, they also need to be analyzed as a collective set of instances within a complex 'design space.' The latter can be thought of as a multivariate environment expressing the important features that make specific instances simultaneously similar and different. (Pellegrino et al. 2001, p. 304)

Developments in design science in recent years (Barab and Squire 2004; Bereiter 2002; Burkhardt 2006; Cobb et al. 2003; DBRC 2003; Kelly 2003; van den Akker et al. 2006) provide a clearer view of what might be required for the design of effective assessments. The principles of design research can be described as:

> a formative approach in which a product or process (or 'tool') is envisaged, designed, developed and refined through cycles of enactment, observation, analysis and redesign, with systematic feedback from end-users. Educational theory is used to inform the design and refinement of the tools, and is itself refined during the research process. Its goals are to create innovative tools for others to use, to describe and explain how these tools function, account for the range of implementations that occur, and develop principles and theories that may guide future designs. Ultimately, the goal is transformative; we seek to create new teaching and learning possibilities and study their impact on end-users. (Wright et al. 2017, this volume as adapted from Swan 2014)

Examples within the papers in this volume provide windows into the different perspectives of the design process as researchers attempt to develop innovations in assessment occupying the complex design space identified in *Knowing What Students Know*. Teaching itself has also been characterized as a design science (Laurillard 2012) with technology and assessment playing crucial roles in improving practice. Hence, design research appears to provide a guiding framework for the development of assessment tasks and resources and might be adopted as a strategic approach for further research into assessment practices. A design framework provides one means to tie together different papers in this volume with their varied perspectives on formative assessment. As teachers take small steps in changing their assessment practice, reflect on the benefits and challenges of those changes, and then try again, they are actually engaging in aspects of design science (Suurtamm et al. 2016).

1.4 The Ongoing Nature of Formative Assessment

As noted in Suurtamm et al. (2016), the current climate in mathematics education encourages teachers to focus students' learning on both content and process and to ensure that students have robust mathematical proficiency consisting of appropriate skill proficiency, understanding of concepts, ability to reason, and productive attitudes towards learning mathematics. Research with Canadian teachers as well as with Finnish teachers has found that a focus on the use of formative assessment has encouraged teachers to view assessment as a social practice that becomes a natural part of the daily life of the classroom. As teachers move toward ongoing assessment practices that engage students in demonstrating robust mathematical proficiency,

they often face a number of dilemmas: conceptual dilemmas relate to viewing assessment as more than an end-of-unit result; pedagogical dilemmas focus on how to develop and implement ongoing assessment opportunities; cultural dilemmas address challenges faced by teachers and students when assessment practices change from the established practices in a schooling environment; and political dilemmas arise as teachers' assessment practices interact with district or national assessment practices (Suurtamm and Koch 2014). Although not characterized as such, the papers in this volume reflect various ways in which teachers and researchers have addressed one or more of these dilemmas.

References

Barab, S., & Squire, K. (2004). Design-based research: Putting a stake in the ground. *The Journal of the Learning Sciences, 13*(1), 1–14.

Bereiter, C. (2002). Design research for sustained innovation. *Cognitive Studies, Bulletin of the Japanese Cognitive Science Society, 9*(3), 321–327.

Black, P., & Wiliam, D. (2009). Developing the theory of formative assessment. *Educational Assessment, Evaluation and Accountability, 21*(1), 5–31.

Burkhardt, H. (2006). From design research to large-scale impact: Engineering research in education. In J. van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational design research* (pp. 121–150). London, UK: Routledge.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9–13.

DBRC [The Design-Based Research Collective]. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher, 32*(1), 5–8.

Kelly, A. (2003). The role of design in educational research. *Educational Researcher, 32*(1), 3–4.

Laurillard, D. (2012). *Teaching as a design science*. London, UK: Routledge.

Leinwand, S., Brahier, D. J., Huinker, D., Berry, R. Q., III, Dillon, F. L., Larson, M. R., et al. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics*. Reston, VA: Author.

Pellegrino, J., Chudowsky, N., & Glaser, R. (Eds.). (2001). *Knowing what students know: The science and design of educational assessment*. Washington, DC: National Research Council, National Academy Press.

Suurtamm, C., & Koch, M. J. (2014). Navigating dilemmas in transforming assessment practices: Experiences of mathematics teachers in Ontario, Canada. *Educational Assessment, Evaluation and Accountability, 26*(3), 263–287.

Suurtamm, C., Thompson, D. R., Kim, R. Y., Moreno, L. D., Sayac, N., Schukajlow, S., et al. (2016). *Assessment in mathematics education: Large-scale assessment and classroom assessment*, ICME-13 Topical Surveys. SpringerOpen.

Swan, M. (2014). Design research in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 148–152). Dordrecht, The Netherlands: Springer.

van den Akker, J., Gravemeijer, K., McKenney, S., & Nieveen, N. (Eds.). (2006). *Educational design research*. London, UK: Routledge.

Wiliam, D. (2015). Assessment: A powerful focus for the improvement of mathematics instruction. In C. Suurtamm & A. Roth McDuffie (Eds.), *Assessment to enhance teaching and learning* (pp. 247–254). Reston, VA: National Council of Teachers of Mathematics.

Wright, D., Clark, J., & Tiplady, L. (2017, this volume). Designing for formative assessment: A toolkit for teachers. In D. R. Thompson, M. Burton, A. Cusi, & D. Wright (Eds.), *Classroom assessment in mathematics: Perspectives from around the globe* (pp. 207–228). Cham, Switzerland: Springer International Publishing AG.

Author Biographies

Denisse R. Thompson is Professor Emeritus of Mathematics Education at the University of South Florida in the U.S., having retired in 2015 after 24.5 years on the faculty. Her research interests include curriculum development and evaluation, with over thirty years of involvement with the University of Chicago School Mathematics Project. She is also interested in mathematical literacy, in the use of children's literature in the teaching of mathematics, and in issues related to assessment in mathematics education. She served as co-chair of Topic Study Group 40 on classroom assessment at ICME 13. In addition, she is a co-editor of the series *Research in Mathematics Education*, published by Information Age Publishing.

Megan E. Burton is an Associate Professor and the elementary education program coordinator at Auburn University, Alabama (USA). She teaches and advises undergraduate and graduate students in elementary education and conducts research related to elementary mathematics education, with focus on elementary teacher change, inclusion, and rural education. As a former elementary teacher with experience in inclusion and English Language Learners, Burton is committed to classrooms that allow all students to encounter strong mathematics instruction in meaningful, differentiated ways.

Annalisa Cusi graduated in Mathematics at Modena and Reggio Emilia University in 2001, where she obtained a Ph.D. in Mathematics in 2009. She's been teaching mathematics and physics in upper secondary school since 2001. She worked as a research fellow at the University of Turin from 2014 to 2016 within the European Project FaSMEd. Her main research interests are innovation in the didactics of algebra; the analysis of teaching/learning processes, with a focus on the role played by the teacher; methods to promote early algebraic thinking in young students; teacher professional development; and formative assessment processes in mathematics.

David Wright is Senior Research Associate: Research Centre for Learning and Teaching, Newcastle University (United Kingdom) (now retired). David has fifteen years' experience teaching mathematics at secondary, further and higher education as an associate lecturer with the Open University. He was Subject Officer for Mathematics for the British Educational Communications and Technology Agency (Becta) for four years and ten years in initial teacher education and research at Newcastle University. He is the Scientific Director of the European Union research project: Formative Assessment in Science and Mathematics Education (FaSMEd).

Part II
Examples of Classroom Assessment in Action

Chapter 2
Formative Assessment Lessons

Malcolm Swan and Colin Foster

Abstract Formative assessment is the process by which teachers and students gather evidence of learning and then use this to adapt the way that they teach and learn in the classroom. In this paper, we describe a design-research project in which we integrated formative assessment into mathematics classroom materials. We outline two examples of formative assessment lessons, one concept-based and the other problem-solving, highlighting the important roles within them of pre-assessment, formative feedback questions, and sample work for students to critique.

Keywords Conceptual understanding · Formative assessment · Problem solving Mathematics task design · Teacher professional development

2.1 Introduction

High-quality *formative* classroom assessment has the potential to produce substantial student learning gains (Black et al. 2003; Black and Wiliam 1998, 1999, 2009). We follow Black and Wiliam's definition that:

> Practice in a classroom is *formative* to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions

Malcolm Swan: Deceased 24 April 2017.

This paper is based on a plenary given at the 37th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (Swan 2015).

M. Swan
School of Education, Centre for Research in Mathematics Education,
University of Nottingham, Jubilee Campus, Wollaton Road, Nottingham NG8 1BB, UK

C. Foster (✉)
School of Education, University of Leicester, 21 University Road, Leicester LE1 7RF, UK
e-mail: colin.foster@leicester.ac.uk

© Springer International Publishing AG 2018

D. R. Thompson et al. (eds.), *Classroom Assessment in Mathematics*,
ICME-13 Monographs, https://doi.org/10.1007/978-3-319-73748-5_2

> they would have taken in the absence of the evidence that was elicited. (2009, p. 9, original emphasis)

Designing mathematics lessons that embed high-quality formative assessment practices could lead to better learning in those lessons, and could also play a part in supporting teachers in developing their formative assessment practices more widely in other mathematics lessons.

In 2010, with funding from the Bill and Melinda Gates Foundation, we began the Mathematics Assessment Project (MAP) to support US middle and high schools in implementing the new Common Core State Standards for Mathematics (CCSSM).[1] These standards place a renewed focus on conceptual understanding and on the development of practices[2] (or processes) that should permeate all mathematical activity. In this project, we explored the research question: How can well-designed materials enable teachers to make high-quality formative assessment an integral part of the implemented curriculum in their classrooms, even where linked professional development support is limited or non-existent?

This ambitious goal was motivated by four empirical findings. First, professional development support is, in practice, in most places, sharply limited in quantity and in the quality of its leaders, and currently few have much deep experience of formative assessment. Second, the development of formative assessment expertise through professional development needs a program that lasts at least two years for significant impact (e.g., Wiliam et al. 2004). Third, most mathematics teachers rely on teaching materials, even when on familiar ground; thus, it is unreasonable to expect them to face the greater challenges of "adaptive expertise" (Hatano and Inagaki 1986) within formative assessment without well-engineered support. Finally, it is our experience that teachers, like students, learn strategies best through constructive generalization of principles from specific high-quality experiences. We see these lessons as supporting such experiences—as well as providing a 'protein supplement' to a generally carbohydrate curriculum diet. It was our goal that over time teachers transfer some aspects of these strategies into their existing practice, with or without the professional development support for which the project also developed materials. There is now some evidence of this happening (see Sect. 2.6).

The MAP project developed over 100 formative assessment lessons, called *Classroom Challenges.* Each lesson consists of student resources and an extensive teacher guide.[3] In this paper, we describe the research-based design of these materials and outline two examples, one concept-based and the other focused on

[1]See http://www.corestandards.org/Math/.

[2]The eight CCSSM Standards for Mathematical Practice are: (i) Make sense of problems and persevere in solving them; (ii) Reason abstractly and quantitatively; (iii) Construct viable arguments and critique the reasoning of others; (iv) Model with mathematics; (v) Use appropriate tools strategically; (vi) Attend to precision; (vii) Look for and make use of structure; and (viii) Look for and express regularity in repeated reasoning.

[3]These lessons are available free on the website, http://map.mathshell.org.