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Sine Leergaard Wiggers
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Structural Stability and Vibration

An Integrated Introduction by Analytical and Numerical Methods

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To Hanne, Søren, Sara, and Sofia

Preface

This book is based on lecture notes for a postgraduate course at University of Southern Denmark (SDU). For us, the integrated teaching of stability and vibration justifies the efforts needed to join many different subjects in a presentation on the introductory level.

Basic understanding of the content of the course, named Stability and Vibration, can be obtained from the derived formulas and graphical presentations of numerical results that give an overview of parameter dependence. Students may reproduce these graphs with their personal MATLAB programs and obtain further results and a deeper insight. Formulas for instability modes and vibrational modes should be used to write interactive and dynamic programs to get a deeper insight into the subject of the course.

The chosen presentation to a large extent in non-dimensional quantities may at first seem disturbing, but it gives more generality and is hopefully appreciated after some time. It should be added that dimensional check of formulas is still possible.

A number of different notes written in Danish are behind this book. Critics of the present version and suggestions to improve the book are most welcome on email to slp@iti.sdu.dk.

Denmark
September 2017

Sine Leergaard Wiggers
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Acronyms

Traditional notations are normally preferred. For non-dimensional quantities, the use of the Greek letters is preferred, but with exceptions as for non-dimensional boundary stiffnesses. For dimensional quantities, the use of the Latin letters is preferred. Matrix notation is used for linear algebra with [ ] for rectangular and quadratic matrices, $\{\}$ for column matrices and $\{\}^T$ for row vectors, i.e., T as upper index for transpose.

Notations

Latin notations, mainly dimensional quantities

A	Cross-sectional area
A_{min}	Minimum allowable cross-sectional area
$B, \bar{B}, \tilde{B}$	Berry functions
C	Lower index for critical quantity
C, C_1, C_2, C_3, C_4	Constants in a solution
c	Cross-sectional position for largest axial stress in (11.5)
D	Determinant and characteristic function
$D_1, D_2, D_3, D_4, D_5, D_6, D_7$	Sub-functions of D
D, d	Diameters of a circular cross section
E, E_t, E_s, E_r	Moduli of elasticity, specifically, Young's, tangent, secant, and reduced
e	Eccentricity length
F	Function for differentiation of finite integral in (2.4)
$f, f_{i,j}$	Flexibility components

G	Shear modulus of elasticity
g	Acceleration of gravity
$g, \bar{g}$	Parameters defined by (8.12)
h	Height
I, i	Cross-sectional moments of inertia
i	Imaginary unit = $\sqrt{-1}$
K	Stiffness for translational spring at boundary
k, k_C	Non-dimensional K and critical value of this
L, l	Lengths of a beam
$\tilde{L}$	Deformed length of a bar
l_e	Effective length in buckling or in vibration
M	Moment, internal or external load
M_0, M_1	Moments at the boundary of a beam-column
M_A, M_{AB}	Moment at a point A specifically of beam-column between A, B
m	Distributed moment per length
N	Axial column force, positive in tension
N_C	critical axial column force
N_{AB}	Axial column force in beam-column between A, B
n	Number, say 0, 1, 2,...
n_C	Number of buckles in critical state
P, P_C	Axial column compressive force, and critical value of this
P_p	Compressive force resulting in compressive stress σ_p
p	Volume force
$P(z)$, $\tilde{P}(z)$	Complex polynomial and its conjugated, see (15.18)
$Q(z)$	Real coefficient part of a complex polynomial
$R(z)$	Imaginary coefficient part of a complex polynomial
Q	Beam external force in transverse direction
q	Distributed beam external force in transverse direction per length
r	Stiffness per length
S_0, S_1	Stiffness for rotational springs at boundary ends 0, 1
s_0, s_1	Non-dimensional S_0, S_1
$s, s_{i,j}$	Stiffness components
$(s_0)_C$	Critical non-dimensional stiffness at 0
s	Length position at a curved beam-column
T	Internal shear force for a beam-column
$T_0.T_1$	Shear forces at boundary ends 0 and 1
t	Time parameter
x	Position parameter in the length direction of a beam-column

y	Displacement in the transverse direction, see Fig. 2.1, i.e., $y = y(x)$
z	Polynomial complex parameter

Greek notations, mainly non-dimensional quantities

α	Stability parameter in the exponential time function $e^{(\alpha + i\omega)t}$
α	Factor in trigonometric functions
α	Value of angle in the Elastica
α	Parameter defined by (6.7)
β	Parameter defined by (6.7)
Γ	Slenderness ratio
γ	Parameter for non-conservative load
γ	Shear strain
Δ	Small but finite incrementation prefix
δ	Virtual prefix
δ	Displacement at a specific point
∂	Partial prefix
$\varepsilon_{i,j}$	Strain component
$\varepsilon, \varepsilon_C$	Axial strain and critical value of this
ζ	Factor in (9.21) for mass moment of inertia
η	Non-dimensional position parameter for load
$\eta_{i,j}$	Green-Lagrange strain component
θ	Angle of cross-sectional rotation (radians positive anticlockwise)
θ_0, θ_1	Angles for end rotation at 0 and 1
κ	Curvature of a beam
Λ	Modified eigenvalue in (11.25)
$\lambda, \bar{\lambda}, \lambda_C$	Non-dimensional column loads and critical value
μ	Cross-sectional parameter for maximum shear stress
μ_E	Factor of safety in the Euler range
v	Poisson's ratio
$\xi, \bar{\xi}$	Non-dimensional position parameter $\xi = x/L$ and specifically integration parameter
ρ	Density parameter
ρ	Non-dimensional inertia radius
$\sigma_{i,j}$	Stress component
σ, σ_C	Axial stress and critical value of this
σ_p	Compressive stress at proportionality limit
σ_y	Compressive yield stress
τ	Shear stress
ϕ, ϕ_C	Non-dimensional squared eigenfrequency and its critical value

ϕ	Non-dimensional for magnetic attraction or for Winkler support
ϕ, ψ	Angles in frame examples
ω^2, ω_C^2	Squared eigenfrequency and its critical value

Matrix notations, mainly for numerical formulations

$\{A\}, \{\tilde{A}\}$	Static and dynamic load vector
$[B_t]$	Strain–displacement relation
$[C]$	Damping matrix, with damping coefficients $c_{i,j}$
$\{D\}, \{\tilde{D}\}$	Static and dynamic displacement vector
$\{\Delta_r\}$	Eigen vector for mode r
$[F]$	Flexibility matrix, with flexibility coefficients $f_{i,j}$
$[L_s]$	Constitutive secant matrix, with coefficients $(l_{i,j}), s$
$[L_t]$	Constitutive tangent matrix, with coefficients $(l_{i,j}), t$
$[M]$	Mass matrix, with mass coefficients $m_{i,j}$
$\{0\}$	Null vector, where all component are zero
$\{R\}$	Residual vector, for Newton–Raphson iterations
$[S]$	Stiffness matrix, with stiffness coefficients $s_{i,j}$
$[S_0]$	Initial stiffness matrix for linear elasticity
$[S_s]$	Secant stiffness matrix
$[S_\gamma]$	Displacement gradient stiffness matrix
$[S_\sigma]$	Stress stiffness matrix
$[S_t]$	Tangent stiffness matrix $[S_\gamma] + [S_\sigma]$

Special symbols and abbreviations

BC	Boundary Condition(s)
DE	Differential Equation(s)
dof	Degree of freedom
FE	Finite Element (method)
d	Prefix for differential
$'$	Partial differentiation with respect to non-dimensional position coordinate
$\cdot$	Partial differentiation with respect to time
$\|\ \|$	Determinant or norm value
$:=$	By definition
$\sim$	Indicate dynamic dependence or alternative quantity

Chapter 1
Introduction

This small book covers the subjects of stability, of vibration, and of dynamic stability, in the first chapters restricted to 2D beam problems. The book is written for a course named stability and vibration, and the intention is to communicate the similarities and interactions of the three subjects that are more traditionally treated separately.

Another non-traditional aspect is the focus on spring supports with graphical results to clearly understand the importance of support modeling. Initial analysis of continuous models is performed by a solution of differential equations, but with finite element (FE) analysis, the generality is rather unlimited.

Why this book?

A larger number of books are written on stability, and an even larger number are written on vibration. References to such books are listed in the list of references at the end of the present book. The two practical important subjects of stability and vibration are seldom treated as an integrated subject, although closely related. The book by Ziegler (1968) is well suited for teaching advanced subjects and contains a clear classification of our different physical problems. The book by Panovko and Gubanova (1964) supports the good tradition of studying very simplified models of physical problems, in order to focus on the behavior in question. A recent book on advanced vibrations and stability theory is by Thomsen (2003). This book focuses on nonlinear theory with chaos theory and special high-frequency effects.

Equal focus on stability and vibration

To integrate stability and vibration on the introductory level is a primary goal of the present book. A second goal of the book is to focus also on non-classical boundary cases for beam problems, i.e., to extend the cases termed Euler cases. Linear elastic rotational springs as well as translational springs are modeled from an analytical point of view by extended use of well-known functions, and the support spring stiffnesses are treated as parameters. From this analysis, a number of functional relations are obtained:

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Extension to linear elastic supports

- Stability as a function of boundary conditions (BC)
- Eigenfrequency as a function of BC.
- Eigenfrequency as a function of column force.

Graphical result display

The intention is to graphical display such results, often obtained by explicit expressions. The book is not an alternative to the many written books, but should be seen as a supplement to these.

It is the intention, generally to let these graphical displays dominate, not for reading specific numerical values from the graph but to illustrate the principal relations between stability (column force), eigenfrequency (with focus on the lowest one), and the parameters of the boundary conditions (BC). Computer tools make this possible and influence the final look of the book, hopefully liked by the readers.

Chosen notations and sign

Traditional notations are normally preferred, as seen in the list of symbols. For non-dimensional quantities, the use of the Greek letters is preferred, but with exceptions as for non-dimensional boundary stiffnesses. For dimensional quantities, the use of the Latin letters is preferred. Matrix notation is used for linear algebra with [ ] for rectangular and quadratic matrices, { } for column matrices and $\{\}^T$ for row vectors, i.e., T as upper index for transpose.

Sign decision is in the direction of the axes of a Cartesian coordinate system, for moments (rotations) as well as for forces (displacements). The book is for the continuous models mainly limited to 2D problems and thus moment (rotation) is chosen anticlockwise.

2D-beam-columns

The initial part of the book is analytical oriented with a focus on beam-columns. Later, the general formulation is related to finite element (FE) models, and the structural or continuum models are then generalized.

Finite element models

A structure/continuum may be described by system matrices like mass matrix $[M]$, stiffness matrix $[S]$, and stress stiffness matrix $[S_\sigma]$. Then, in addition to analysis also synthesis aspects can be involved, such as design for vibration and stability. This includes sensitivity analysis that is determination of response gradients as a function of design parameters.

Design for stability/vibration

Additional insight into the subject of stability as well as that of vibration is hopefully obtained by the integrated treatment. It is the intention that each formula should to a large extent be derived to convince the students and other readers.