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Inventory Control Models with Motivational Policies

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Preface

In many competitive markets, sellers in order to increase their sales usually use motivational policies to encourage the customers to buy more; moreover, these kinds of strategies are used as competitive tools. Quantity discounts such as all-unit, incremental, and freight discounts are usually used as some famous motivational policies. Delayed payment or trade credit policies are also being applied by many suppliers or wholesalers to increase the sales. Advance payment or prepayment schemes are used by some firms for special products, and in this way, the customers are encouraged to pay a prepayment and buy the product. Special sales are offered by the retailer to increase the sale level or decrease the inventory level. So reducing the selling cost persuades people to not give up the suggested opportunity. Finally, announced price increase persuades the customer to buy the product at the current price and not to pay more in future to receive the same product. So the main aim of this book is to introduce all motivational policies through which the seller can increase his/her sale. An overview of these *Motivational Policies* is presented in Chap. 1.

One of the traditional motivational policies applied by the supplier or wholesaler is a group of quantity discounts presented in Chap. 2. In this policy, the purchasing cost depends on the order quantity; in fact, the buyer can use discount if he/she buys a large number of products. In all-units discount, as the most famous one, the unit purchasing cost is the same for all ordered products, while in the incremental discount, the policy is different. In incremental discount policy as the quantity per order increases, the unit purchasing cost declines incrementally on additional units purchased as opposed to on all the units purchased. This topic can be studied and presented in several inventory control models including both deterministic and stochastic ones. Moreover, considering cases in which shortage is permitted or not or even providing material for deteriorating items are streams of this section.

Delayed payment or trade credit strategy is one of the famous motivational policies that is applied to encourage the buyers are presented in Chap. 3. Several types of this policy such as simple delayed payment, linked to order delayed payment, partial delayed payment, or combinations of mentioned policies are suggested by the wholesalers and suppliers to the customers. In this policy, the interest earned or should be paid by the wholesaler and buyer has a significant impact on the profits of both sides. This topic can be investigated when the shortage is permitted or not allowed and also both stochastic and deterministic inventory control systems can be presented. Applying delayed payment in inventory control system of deteriorating and perishable items is another stream.

Unlike delayed payment, in advance payment policy, the purchasing cost should be paid in advance before the delivery time. Similar to the delayed payment policy, several kinds of advanced payment policies can be requested from the wholesaler or supplier. Partial advanced payment, linked to order advanced payment, multiple advanced payments, etc., are different types of this policy. Here, similar to the delayed payment policy, the interests earned or should be paid by the wholesaler and the buyer will have a significant impact on the profits of both sides. Similar to the previous cases, the mathematical models of applying this policy in both stochastic and deterministic inventory control systems for regular and deteriorating items can be presented. The related materials are presented in Chap. 4.

In practice, suppliers sometimes offer special sale prices to stimulate sales or decrease inventories of certain items. Not only this policy is useful for the seller to decrease the inventory level of some stock items but also provide the benefit of buying items at a lower price for the customers simultaneously. Special sale in both deterministic and stochastic inventory systems for simple and deteriorating items when the shortage is permitted or is not allowed will be provided. Chapter 5 includes several mathematical models of special sale as the fourth motivational policy.

On the other hand, suppliers sometimes use announced price increase policy to attract the customer to stimulate sales or decrease inventories. In this way, they announce the customers that the unit selling price will increase at a specific time in future. So announced price increase as a motivational policy can be useful for both sides, because it encourages the customers to place orders before facing price increases. This section includes both stochastic and deterministic models, covering all shortage cases (whether shortage is permitted or not), models for perishable or common products as provided in the final chapter.

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Ata Allah Taleizadeh received his B.Sc. degree in Industrial Engineering from the Azad University of Qazvin, Qazvin, Iran, and M.Sc. and Ph.D. degrees from the Iran University of Science and Technology, Tehran, Iran, both in Industrial Engineering. He is currently an Associate Professor of Industrial Engineering, University of Tehran, Iran. He has published extensively in reputable and leading journals such as *European Journal of Operational Research, OMEGA: International Journal of Management Sciences, IEEE Transactions on Systems, Man, and Cybernetics: Systems, International Journal of Production Economics, Annals of Operation Research, Service Science*, etc. His current research interests include inventory control and production planning, pricing and revenue optimization, and game theory. Prof. Taleizadeh serves as an editor/associate editor/editorial board member for a number of international journals such as *International Journal of Systems Sciences, Journal of Industrial Engineering, International Journal of Systems Sciences: Logistic & Operation Research, International Journal of Applied and Computational Mathematics, International Journal of Industrial Engineering: Theory, Applications and Practice,* and *IIE Transactions on Healthcare Systems Engineering.*

Chapter 1
Introduction

In many competitive markets, seller in order to increase their sales usually use motivational policies to encourage the customers to buy more and also these kind of strategies is used as competitive tools. Quantity discounts such as all-unit and incremental discounts are usually used as some famous motivational policies. Delayed payment or trade credit policies are also being applied by many suppliers or wholesalers to increase the sales. Advance payment or prepayment schemes are used by some firms for special products and in this way the customers are encouraged to pay a prepayment and buy the product. Special sales are offered from the retailer to increase the sale level or decrease the inventory level. So reducing the selling cost persuades people to not give up the suggested opportunity. Finally announced price increase persuades customer to buy the product at current price and do not pay more in future to receive the same product. So the main aim of this book is introduce and mathematically modelling all motivational policies through which seller can increase his/her sale.

1.1 Quantity Discounts

One of the policies which companies incorporate to increase sales is discounting as the order size gets larger; that is the price would decrease as the order quantity increases. In the price discount model, the optimal order quantity is determined according to different product prices. In this context, it is assumed that the price per unit of the product is related to the volume of the purchase (i.e. Q). To put it in a nutshell, quantity discounts project the relation between the unit price and the amount of each order. The vendors offer a situation where for different quantity thresholds, various selling prices are provided. That is to say, the amount purchased at a certain level will include a drop in price. Discounting policies do not confine the idea to price reductions and they could sometimes involve price increases as well. These are called anti-hoarding models. The purpose of these discounting schemes is

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to encourage buyers to buy more as one time and large quantities. Price discount provide the buyers with lower ordering costs. On the other hand, the related inventory holding cost will increase due to increased inventory levels.

The second chapter of this book is assigned to discount policies. In this chapter, three types of discounting schemes are surveyed as follows:

1. *All-unit quantity discount model:* In this case, the discounting is applied for all products in the same way. In other words, all the units are purchased at a unit price.
2. *Additive, incremental, imperceptible discounts model:* In this case, discounting is applied to each range individually. In other words, all the units are not purchased at a unit price and based on the values within the discounted range, the discount is determined for the product unit.
3. *Freight discount model:* As two previous discounts are offered for purchasing, they can also be considered in shipping.

The aim is to familiarize the readers with various discount models along with the numerical examples presented, in following.

In the first part of this chapter, all-unit quantity discount model will be discussed. This model will be studied in different inventory frameworks including no shortage, shortage, partial shortage and deteriorating products. The second section is developed for incremental discount covering no shortage, shortage and partial shortage. Finally in the third section, freight discount is offered without shortage. In each section, firstly, the related literature body of the problem is reviewed and after introducing the modeling parameters, a solution procedure for each model is presented.

1.2 Delayed Payment

When the supplier delivered goods to their customers, often do not require to be paid immediately and offer credit terms allowing the buyers to delay the payment. This is known as trade-credit policy. The term is a beneficial implement for the customers since they are not obliged to pay the sales immediately after receiving the orders; but instead, they are allowed to delay the payment procedure until a specified permitted period. In diverse variety of commercial transactions, an appointed time for delaying the payments is offered or accepted by the seller. It is worth noting that during the time period that the account is not settled, generated sales could be deposited in an interest bearing account. So the policy can be regarded as a discounting scheme which has the potential to amplify the order size. These kinds of impacts are not explicitly incorporated in the classical formulas of economic order quantities (EOQ). In 1985, Goyal developed an Economic order quantity (EOQ) model under conditions of permissible delay in payments.

The traditional economic order quantity (EOQ) model assumes that the retailer must pay for the items as soon as the items are received. Indeed, goods are rarely paid for immediately after the items are received in retailer's system. In mercantile deals, nearly all the firms to some extent rely on trade credit as a source of short-term funds. In fact, small firms generally use trade credit more extensively than large firms. When monetary policy is tight and credit is difficult to obtain, small firms tend to increase their reliance on trade credit. That is, during periods of tight money, small firms that are unable to obtain sufficient funds through normal channels may obtain financing indirectly from large suppliers by "stretching" their payment periods and extending accounts payable. Large firms often are willing to finance their smaller customers in this manner in order to preserve their markets. Ordinarily the forms of trade credit are open account, promissory note, and trade acceptance (e.g., see Solomon and Pringle 1980). As to a retailer conducting business with foreign suppliers, it must pay attention to the exchange rates between foreign currencies and its own currency, and the effects of fluctuating currency values in its financial analysis.

During the period, he may sell the goods, accumulate revenues on the sales and earn interest on that revenue. Thus it makes economic sense for the customer to delay the payment of the replenishment account up to the last day of the settlement period allowed by the supplier on the producer. Similarly for supplier, it helps to attract new customer as it can be considered some sort of loan. Furthermore, it helps in the bulk sale of goods and the existence of credit period serves to reduce the cost of holding stock to the user, because it reduces the amount of capital invested in stock for the duration of the credit period. So it could be claimed that the policy is beneficial for both sides of the transaction including seller and buyer.

The delayed payment strategy can be performed under three different styles including

1. *Whole delayed payment:* In this method total purchasing cost will be paid by delay based on initial agreement between parties.
2. *Partial delayed payment:* Partial trade credit is a reformed type of the prescribed policy. In this case, the buyer is allowed to delay a part of its ordered items. That is to say, the buyer has to pay a part of its ordered items immediately after they are received and the rest of the sales have the opportunity to be paid after the provided delayed length.
3. *Linked to order delayed payment:* Trade credit can also get a different structure by linking the policy to the order quantity. Clearly speaking in this case, if the ordered quantity achieves a predetermined threshold, the seller offers the fully trade credit policy, otherwise two conditions are possible: no trade credit or partial trade credit.

In the third chapter of this book different schemes of delayed payment policy are presented and both solution method and numerical examples are provided.

1.3 Advanced Payments

The optimal order quantity can influentially be affected by payment timing and the reaction of customers when the vendor runs out of stock. When an order is placed, the time of the payment can take three possible points:

1. *Prepayment:* At the time the order is placed or prior to delivery.
2. *Instant payment:* At the time of delivery.
3. *Delayed payment:* At some time after delivery.

In some cases the timing of payments may be a combination of two or even all three of these possibilities, with part of the cost due in advance, more due at the time of delivery, and the rest due at a later date. Using the classic EOQ model as a framework, Bregman (1992) analyzed the effects of the timing of disbursements on the order quantity, showing that large timing differences may significantly affect order quantities. In a comment on Bregman's paper, Lau and Lau (1993) found that, although the effect on the order quantity may be large, the impact on the overall cost may be relatively small. In the competitive environment of business, it is normally observed that a wholesaler requires some payment when an order from a retailer is placed. Further, there are situations in which if a retailer gives an extra advance payment (AP), then he may get some price discount at the time of final payment (e.g. brick and tile factories announce such an offer at the beginning of the season). Generally speaking, it could be claimed that the main purpose of AP is either financing the procurement cost of material or controlling the risk of cancelling the orders. However, by paying a certain percentage of the total purchase cost per cycle as an advance payment to the wholesaler, the retailer sacrifices the interest on the amount of money paid as AP.

Financial exchange of purchasing costs between buyers and sellers can follow one of the three prevalent policies in the literature (i.e., instant payment, prepayment, and delayed payment) or can be based on a hybrid strategy of the mentioned policies. The policy employed for classical EOQ model (Harris 1913) is a traditional way for the aforementioned purpose (i.e. instant payment), in which, when a buyer receives an order, he pays for the purchased items. In another scenario, the sellers may ask the buyers to pay the purchasing cost, prior to order delivery. This situation is called full prepayment, if all purchasing cost is requested by the seller. On the other hand, if only a portion of the cost is prepaid, we face a partial prepayment case. This situation may also be requested by the buyers in order to get a discount from the sellers. In the other scenario, the sellers provide this opportunity for the buyers to pay the purchasing cost after a predetermined amount of time. Such a policy is usually adopted as a marketing strategy in a competitive market (Teng et al. 2014; Zhang et al. 2014). Indeed, the sellers adopt this strategy to attract buyers and consequently increase their sales and decrease inventory levels (Teng et al. 2013; Wu and Chan 2014). This is because in this policy the buyers has the opportunity to accumulate revenue and earn interest during the allowed period (Chen et al. 2014). Moreover, delayed payment policy (which is also called trade credit) can be employed as an alternative to price discount, although it may impose the risk of the inability of the buyer to pay off its debt at the given time, in addition to the capital cost. In practice, the sellers do not charge interest from buyers during the permissible delayed payment; but beyond the allowed time to pay off the debt, the remaining amount should be accompanied with an interest (Teng et al. 2013). Delayed payment can take place in two forms of full delayed and partial delayed payments. In the first, all purchasing cost is paid after the given period, while in the second, a fraction of the payment must be paid at the delivery moment and the remaining is paid during the allowed time. In this book, the fourth chapter is assigned to the advanced payment strategies and several mathematical models of inventory systems in which advanced payment is incorporated are presented.

1.4 Special Sales

Since the basic economic order quantity model (EOQ) was introduced by Harris (1913), there have been many extensions which either have relaxed one or more of its assumptions or added additional features to the model. A constant unit purchase cost is one of the main assumptions in the classic economic order quantity model. In practice, suppliers sometimes offer special sale prices to stimulate sales or decrease inventories of certain items. The two possible decisions by the purchaser are: (1) place a special order or (2) continue with the normal order quantity. If a special order is decided to be placed, the problem is to determine its optimum size. In the industrial or business environment, many items are procured or replenished. Frequently, the supplier offers a unit price reduction that is available for a short duration. The suppliers declare a temporary price reduction in advance and the purchasing industry or trading firm usually procures a large quantity in order to avail itself of the cost benefit. This special order quantity is more than their usual ordering size. General purpose of this inventory planning model is to determine (a) the time of placing an order, and (b) the order size, seeking to minimize total cost of the system.

However, different extensions are made on the basic problem by adding further decision variables to the model or changing the objective function of the problem. Periodic review inventory models constitute a distinct area in inventory management literature. One fundamental assumption about periodic review inventory systems is that the review periods are of fixed length. In practice, however, the review periods may take variable lengths. For several reasons this fact implies variations in supply time and consumption, delays in transportations and so forth. This setting represents the real life cases where a supplier visits a retailer with random inter-arrival times and the retailer replenishes his inventories based on a replenish up-to-level inventory control policy. Readers should note that "review interval" (or review period) and "replenishment interval" are not exactly the same. To be precise, a review of inventory is only an opportunity for replenishment. So for example, with fixed review intervals, replenishment does not necessarily take place at fixed intervals due to variating lead-time. On this basis, many scholars use the term "replenishment" only when referencing to actual replenishment decisions, and elsewhere using "review". However, in a case that replenishment is done just when the agent of the supplier visits a retailer, one can use both terms alternatively without any problem. In the fifth chapter, the different types of incentory control systems with special sales are presented.

1.5 Known Price Increase

When a supplier announces either a temporary reduction or a permanent increase in the unit purchasing cost of an item, the buyer can generally decrease his total purchasing cost by placing a larger-than-normal special order. The decision about whether and how much to increase the order quantity which has been studied by a number of other researchers, is to be investigated in a system where the unsatisfied demand will be backlogged or when shortage is not permitted. Then, the decision problem is to determine the optimal quantity to order before a price increase by recognizing the imminent price increase and partial backordering. In an inflationary environment, it is quite usual that a wholesaler or a manufacturer increases the price of his goods. Moreover, quality improvements which impose additional invested costs to the system, make the sellers increase their selling price to recover the investment and earn profit.

Therefore, there are frequent situations in which the wholesaler or the manufacturer increases their prices at a specific time. This increase in prices influences the cost of the retailer and/or the customers resulting in some changes in their profit and order quantity as well. Furthermore, the time of delivery of the goods or replenishment intervals to a buyer is usually assumed to be deterministic in traditional inventory systems; but in the real world this time may not be deterministic and the buyer cannot exactly predict when the suppliers will deliver the materials. This time is thus probabilistic in salesman-based selling systems. In this selling strategy the manufacturer's salesman calls on customers to receive the orders. This situation addresses a salesman-based selling system wherein the supplier announces that the selling price of his goods will increase in specific time in future, so the supplier encourages and lets the buyer place a special order before the price hike. The background research is mainly investigated from two related perspectives: known price increase and stochastic replenishment intervals. According to the above description, in the last chapter author tries to disccuss about different deterministic and stochastic inventory systems under known price increase.

Chapter 2
Quantity Discounts

2.1 Introduction

One way to increase companies' sales is to discount products' prices for large orders, so that the more orders, the more discounts. In the price discount model, the optimal order quantity for the warehouse is determined according to different product prices. In this context, it is assumed that the price per unit of the product is related to the volume of the purchase or Q. Discounts are the conditions where the unit price depends on the amount of each order. It means that vendors and suppliers of products offer, if the order quantity (Q) increases to certain limits, they sell the total amount of Q at a specified price. In other word, the amount purchased at a certain level will include a drop in price. Of course, we cannot consider reducing the price as a discount since sometimes discount models leads to increasing price, which we consider them as anti-hoarding models. The purpose of these discounts is to encourage buyers to buy more as one time and large quantities. In anti-hoarding models, the goal is to prevent customers from acquiring much of the price, contrary to the goal of discounts. Benefit of business price discount is lowering ordering costs due to reducing ordering times. But on the other hand, holding costs increase due to increased inventory levels. Some related work can be found in Lee et al. (2013); Taleizadeh et al. (2008a, b, 2009b, 2010a, b, c, 2011a, b, c, 2012a, b, 2013c, d, e); Taleizadeh (2017b) in which different types of discounts are modeled and solved.

In this chapter, three types of discount are surveyed as follows.

1. All-unit quantity discount model: In this case, the discount is applied for all products in the same way. In other words, all purchased units are purchased at a unit price.
2. Additive, incremental, imperceptible discounts model: In this case, discount applies to each range individually. In other words, all purchased units are not purchased at a unit price and based on the values within the discounted range, the discount is determined for product unit.
3. Freight discount model: As two previous discounts are offered for purchase, they can also be considered in shipping.

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In this chapter, we go to examine various types of discounts (all-unit, incremental, freight) in an EOQ model. The aim is to familiarize with various discount models along with the numerical examples presented, in following.

In the first part of this chapter, all-unit quantity discount model will be discussed. This model will be studied without shortage, with shortage, with partial shortage, and for deteriorating products. In the second and third sections, incremental discount will be developed without shortage, with shortage, with consideration of partial shortage, and freight discount without shortage, respectively. In each section, firstly, the related literature review is reviewed and after introducing the modeling parameters, a solution algorithm of each model is presented.

2.2 All-Unit Quantity Discount

It has previously been assumed that prices are fixed, but here the costs change with the ordering value. The more order, the lower price. On the other hand, the holding cost of inventory goes up and even the shipping cost may even be higher. So, here, the purpose is to find the optimal order point.

2.2.1 All-Unit Quantity Discount Without Shortage

In this section, a classic inventory control economic order quantity (EOQ) model with all-unit quantity discount and without shortage is developed. In this way, depending on the order quantity, different purchase prices are considered for the buyer. Benton and Park (1996) investigated the size of the stock where at least one of the types of discounts was considered. Weng (1995) expanded models for determining optimal all-unit quantity and incremental discounts policies and examined the impact of these policies on price-sensitive demands. Lee et al. (2011) first modeled the accumulated size problem as a complex numerical programming model then expanded the model to solve the accumulated size problems with quantity discount and shipping costs. Hwang et al. (1990) proposed a model with quantity discount consideration on both price and shipping cost.

Lin and Kroll (1997) developed models for single period newsboy problem with quantity discounts and dual-purpose which is maximizing expected profit provided that the probability of reaching the target profit level is lower than the predetermined risk. Lin and Ho (2011) developed a model to find an optimal point-of-sale strategy for integrated inventory control systems with quantity discounts. Also Lin (2010) presented a new inventory control model for non-perfect products under discounts where buyers are powerful than suppliers.

Wang and Wang (2005) studied optimal discount policies of suppliers which includes both all-unit quantity discount and incremental discount for a group of heterogeneous and independent retailers. Mendoza and Ventura (2008) studied two economic order quantity models, along with shipping costs, under all-unit quantity and incremental discounts.

Munson and Hu (2010) examined four different conditions for centralized and decentralized purchasing and pricing systems with a local distribution system that includes both all-unit quantity discount and incremental discount. Meena and Sarmah (2013) reviewed the issue of assigning a producer or buyer to the suppliers under the risk of supply disruptions and discounts. Goossens et al. (2007) surveyed a profit problem in which a buyer purchases various products from suppliers to benefit from all-unit quantity discount.

Schotanus et al. (2009) examined the circumstances in which a buyer faces the discounts of a vendor. They provided an empirical and analytical foundation for the generalized discount function, which is applicable for the main functions of all types of discounts. Feess and Wohlschlegel (2010) examined the issue of all-unit quantity discount and surplus deviations. Zhou (2007) studied pricing policies for a manufacturer and a retailer under a quantity discount.

In 2009, Mirmohammadi et al. (2009) used an optimal algorithm based on the branch and bound approach to determine the accumulated size for an item in MRP under all-unit quantity discount, zero delivery time, and fixed ordering cost. Mahdavi Mazdeh et al. (2015) examined the problem of the accumulated size of a single dynamic case with the supplier selection and quantity discount. They divided the issue into two different cases. In the first case, the discount is not considered, but in the second case, both all-unit quantity discount and incremental discount are considered.

Gurnani (2001) developed a quantity discount pricing model with different ordering structures in a supply chain including a supplier and heterogeneous purchasers. Nguyen et al. (2014) studied integrated quantity discount and vehicle routing issues. Sheen and Tsao (2007) examined how coordinating channels are obtained using commercial credentials and how commercial credit is affected by quantity discount on shipping costs. They showed that the profit margin increases for both issues when the credit period is maintained in an appropriate range.

Zissis et al. (2015) studied supply chain issues under separate asymmetric information and quantity discounted. Manerba and Mansini (2014) presented a meta-heuristic method to solve the complex supplier selection problem that requires selection a subset of suppliers, while minimizing purchase costs and meeting demands. This issue is also known as the all-unit quantity discount problem.

Notations and Assumptions

It should be noted that in this model, the demand for the product is specified and deterministic. Other assumptions are discussed below.

1. All parameters of the model are deterministic.
2. The inventory system is a single product and without limitation.

3. Delivery time is zero.
4. Time horizon is infinite.
5. Demand is constant over time and is equal to D.

The symbols used in modeling the problem are as follows:

Parameters:	
A:	Ordering cost per order
D:	Demand rate per unit time
C_J:	Purchasing cost per unit item in jth interval
i:	Interest rate per item per unit time; (which holding cost per item per unit time is $C_J i$)
j:	Indexes of discount categories based on a price; $j = 0, 1, 2, \ldots, n$
q_j:	Break point of discount; $j = 0, 1, 2, \ldots, n$
$R(Q)$:	Total purchasing cost
Q_{wj}:	Order quantity per order without discount
Decision variable:	
Q:	Order quantity per order
Other variables:	
CTC:	Total cost per period

Modelling

In this model, the discount is given to the total purchased units and in fact the purchase price of all items is the same. The total discount costs are as follows (Table 2.1):

As mentioned earlier, under the discount, increasing the order quantity per unit reduces price per unit product. It means that:

$$C_0 > C_1 > \ldots > C_{n.}$$

Purchasing cost diagram are as follows:

Table 2.1 Purchasing cost under all-unit quantity discount

Range number	Discount range	Price per item	Total purchasing cost
0	$q_0 \leq Q < q_1$	C_0	$C_0 Q$
1	$q_1 \leq Q < q_2$	C_1	$C_1 Q$
2	$q_2 \leq Q < q_3$	C_2	$C_2 Q$
.	.	.	.
.	.	.	.
J	$q_j \leq Q < q_{j+1}$	C_j	$C_j Q$
.	.	.	.
.	.	.	.
N	$q_n \leq Q < q_{n+1}$	C_n	$C_n Q$

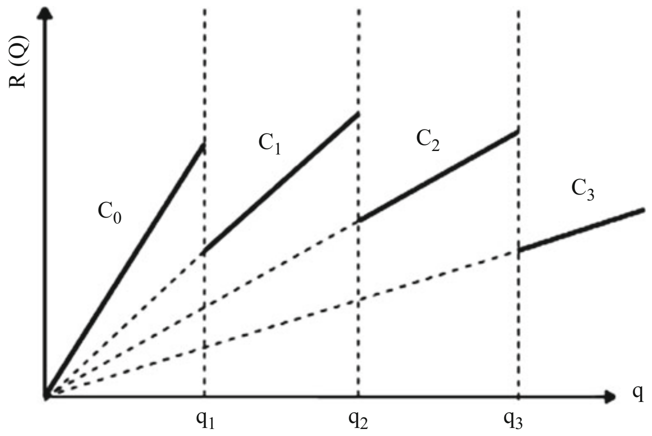

Fig. 2.1 Purchasing cost diagram under all-unit quantity discount

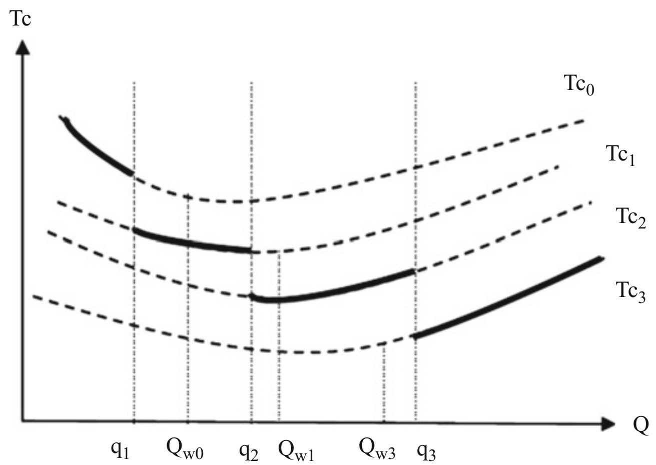

Fig. 2.2 Annual purchasing cost diagram under all-unit quantity discount

This diagram is discrete and extends all lines from the origin. The slope of the lines is also equal to the purchase price. The total annual cost diagram has the same discrete-purchasing diagram as follows:

Total cost function in this system is as follows:

$$CTC = A\frac{D}{Q} + i\,C_j\frac{Q}{2} + C_jD \qquad q_j \le Q < q_{j+1} \tag{2.1}$$

These functions are available in bold places.

Table 2.2 Price per different order size

Order size	0–499	500–2499	2500–4999	5000 or more
Price	5	4.75	4.6	4.5

Solution Algorithm

1. Solve the model as backward. It means that start with the lowest possible price and calculate the value of Q_{wj}.
2. If Q_{wj} is obtained from C_j, stop at the desired distance, i.e., the allowed distance C_j. Otherwise, go to a previous cost, that is, C_{j-1}, which is higher than C_j.
3. Whenever Q_{wj} is at the allowed distance, determine the total cost. Then calculate the same cost for the right break point at that point, i.e., for the order q_{j+1}. Each of these costs be lower would be the answer of the problem.

Example 2.1. All-unit Quantity Discount without Shortage A seller offers the following price scheme to buy the item. This price is for all purchased items (Table 2.2):

The consumption of products per year is 2500 units and holding cost is 0.1 times the average monetary inventory of warehouse. Ordering costs per order is 100 $. Determine the economic order quantity and related costs.

Solution

$$Q_w = \sqrt{\frac{2AD}{ic}}$$

We start to solve as backward:

$Q_{w4} = \sqrt{\frac{2^*100^*2500}{0.1^*4.5}} = 1054$	Not acceptable because it is not placed in the allowed range.
$Q_{w3} = \sqrt{\frac{2^*100^*2500}{0.1^*4.6}} = 1043$	Not acceptable because it is not placed in the allowed range.
$Q_{w2} = \sqrt{\frac{2^*100^*2500}{0.1^*4.75}} = 1026$	It is acceptable because it is placed in allowed range.

So, calculate the cost for this amount:

$$CTC = 100 * \frac{2500}{1026} + 0.1 * 4.75 * 1026.2 + 2500 * 4.75 = 12362.32$$

Now, examine the right break point. i.e., 2500:

$$CTC = 100 * \frac{2500}{2500} + 0.1 * 4.6 * \frac{2500}{2} + 2500 * 4.6 = 12175$$

Since the total cost for Q = 2500 is lower. Then the optimal order quantity is 2500.

2.2.2 *All-unit Quantity Discount with Backlogging Shortage*

In this section, a classic inventory control economic order quantity model with all-unit quantity discount and backlogging shortage is presented. San-José and Garcia-Laguna (2003) studied an inventory control model with backlogging shortage in which price per unit depends on the order size. San-José and Garcia-Laguna (2009) also looked at the all-unit quantity discount model and solved Tersain composite model. Mousavi et al. (2014) expanded a multi-product multi-period seasonal inventory control model where inventory costs are defined by all-unit quantity discount and inflation policies. Park (1983) introduced an inventory control model under conditions that the withdrawal period of inventory is fraction b from demand as backlogged and fraction b-1 as lost-sale. Tersine and Barman (1995) expanded a mixed economic order quantity model that could be decomposed into a family of hybrid models.

Notations and Assumptions

The assumptions of the model are as follows.

1. All parameters of the model are deterministic.
2. Inventory system is a single product and unlimited.
3. Delivery time is zero.
4. Time horizon is infinite.
5. Demand is constant over time and is equal to D.
6. Shortage is allowed and is the type of backlogged and the shortage cost per item is constant.
7. Capacity is unlimited.
8. Warehouses are replenished when face shortage.

The parameters of the model are as follows:

Parameters:	
A:	Ordering cost per order
D:	Demand rate per unit time
C_J:	Purchasing cost per unit item in jth interval
i:	Interest rate per item per unit time; (which holding cost per item per unit time is $C_J i$)
j:	Indexes of discount categories based on a price; $j = 0, 1, 2, \ldots, n$
q_j:	Break point of discount; $j = 0, 1, 2, \ldots, n$
$R(Q)$:	Total purchasing cost
Q_{wj}:	Order quantity per order without discount
W_O:	Shortage cost per item
Decision variable:	
b:	Shortage amount per unit time; $0 \leq b \leq q$
Q:	Order quantity per order
Other variables:	
CTC:	Total cost per period

Modelling

San-José and Garcia-Laguna (2003) analyzed an inventory control model with backlogging shortage, shortage cost over time is constant and purchasing cost depends on the amount of accumulated size. This condition arises when a seller offers a fixed cost not to lose selling as compensation to the customer while there is all-unit quantity discount.

Total cost per period is as follows.

$$CTC(Q,b) = \overbrace{\frac{W_O Db}{Q}}^{\text{Shortage cost}} + \overbrace{C_j D}^{\text{Purchasing cost}} + \overbrace{\frac{iC_j(Q-b)^2}{2Q}}^{\text{Holding cost}} + \overbrace{\frac{AD}{Q}}^{\text{Ordering cost}} \tag{2.2}$$

This cost includes the shortage cost, purchasing cost, holding and ordering costs. Also they proposed a solution algorithm regarding the total cost function.

Solution Algorithm

1. First, calculate Q_{wJ} using the following equation:

$$Q_{wJ} = \sqrt{\frac{2\,AD}{ic_J}} \tag{2.3}$$

   (a) If Q_{wJ} is greater than $W_O D/ic_J$, then the cost function at any of the points defined in the region does not reach its lowest value. Objective function tends to $(W_O + C_j)D$ when Q tends to infinity.
   (b) If Q_{wJ} is equal to $W_O D/ic_J$, then there is a minimum value for the cost function at any point of this set:

$$\{(Q,b)\colon Q \geq \max(Q_{wj}, q_{j-1}), b = Q - W_O D/iC_j\} \tag{2.4}$$

   And the cost function will be equal to:

$$CTC(Q,b) = (W_O + C_j)D \tag{2.5}$$

   (c) If Q_{wJ} is smaller than $W_O D/iC_J$ and Q_{wJ} is greater or equal to q_{j-1}, then the minimum total cost with $Q_{wJ} = Q$ and $b = 0$ is equal to $\sqrt{2ADiC_j} + D$.
   (d) Otherwise, go to step two.

2. Q_{wj-1} for $j = j-1, j-2, \ldots$ until the first j satisfies the following equation:

$$Q_{wj} \geq \min\left\{q_{j-1}, \frac{W_O D}{iC_j}\right\} \tag{2.6}$$

   For $i = j+1, j+2, \ldots, J$, calculate $b_i^* = \max\{0, q_{i-1} - W_0 D/ic_i\}$ and $CTC(q_{j-1}, b_i^*)$.