David E. Rowe

A Richer Picture of Mathematics

The Göttingen Tradition and Beyond

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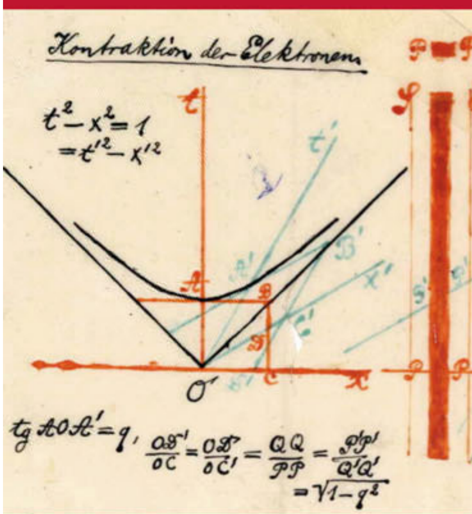
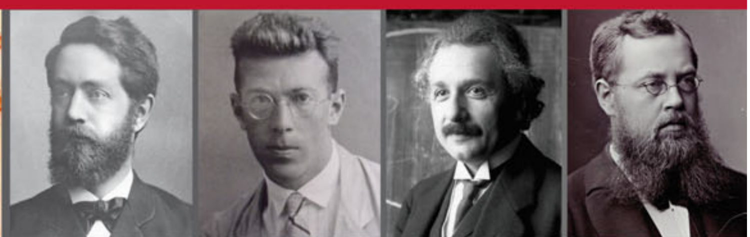
For Hilde and Andy

A RICHER PICTURE OF MATHEMATICS

A symposium in honor of David E. Rowe





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Foreword

"A Richer Picture of Mathematics" was the title of a symposium held in honor of David Rowe in May 2016 at the University of Mainz, his academic home for 25 years, as a sign of gratitude from his longtime friends and colleagues. The title conveyed the impetus and spirit of David's various and many-faceted contributions to the history of mathematics: Penetrating historical perspective paired with a sensitivity for the relevant broader context of mathematics as a human activity, an immensely broad knowledge and care for historical detail, and attention to remote and sometimes surprising but always revealing cross references.

The title also seems to us to capture particularly well David's regular contributions to the *Mathematical Intelligencer* over the long time of his association with this journal. Taken together, these articles display a surprising (perhaps unintended) coherence and a charming seriousness. They cover topics ranging from ancient Greek mathematics to modern relativistic cosmology. They were written for a broad readership of open-minded and curious but mathematically trained and educated readers. They are collected here, augmented by two other of David's papers and set in context by a foreword and new introductions. This volume is indeed "a richer picture of mathematics."

Northampton, MA — Marjorie Senechal
Mainz, Germany — Tilman Sauer

Preface

This is a book that explores how mathematics was made, focusing on the period from 1870 to 1933 and on a particular German university with a rich history all its own. Relatively little of that mathematics will be explained in these pages, but ample references to relevant studies will appear along the way. My intention is not so much to add to that scholarly literature as to shed light on the social and political contexts in which this intellectual activity took place. Or, to put a twist on a well-known book title, instead of asking, "*what is mathematics?"*, I wish to explore a different question, namely "how was mathematics made?" The reader should note the verb tense here: this is a book about the past, and as a historian, I make no prescriptive claims. Clearly, mathematicians can, and often do, voice their opinions about how mathematics should be made, or about the relative importance of the people and ideas discussed in the pages that follow, but that is not my purpose here. Rather, my aim in presenting this series of vignettes centered on the Göttingen mathematical tradition is to illustrate some important facets of mathematical activity that have received far too little attention in the historical literature. It is my hope that by bundling these stories together a richer picture of mathematical developments emerges, one that will be suggestive for those who might wish to add other dimensions to those described here.

Although the central actors in this book were not always situated in Göttingen, I have attempted to show how each was drawn, so to speak, into that university's intellectual force field. Over the course of time, the strength of that field grew steadily until it eventually came to dominate all others within the German mathematical community. At the same time, Göttingen's leading representatives, acting in concert with the Prussian Ministry of Education, succeeded in transforming this small university town into one of the world's leading mathematical research centers. This larger story has been told, of course, before; what I offer here is a collection of shorter tales, many based on archival and primary sources, while drawing on a wide range of secondary literature. Taken as a whole, these essays point to several new dimensions that have been overlooked in earlier accounts. In particular, I believe they provide a basis for better understanding how Göttingen could have attracted such an extraordinary array of talented individuals. A number of them played central roles in creating a new kind of mathematical culture during the first decades of the twentieth century.

A brief word about the structure of the book: its six parts reflect thematic aspects of the overall story, whereas the individual essays largely recount episodes related to these six themes. These essays were originally written over a period of more than three decades, a circumstance that helps account for their rather uneven character. I have arranged them largely in accordance with chronological developments, but also in order to bring out different thematic elements. While the main threads running through each section are described in the introductions to each, the essays themselves were written as self-standing pieces that can be read out of order or even at random. There is no grand narrative as such; the six introductions mainly have the purpose of drawing out the underlying themes rather than providing a road map that shows how all the pieces fit together. With two exceptions, the essays in this book represent slightly revised versions of earlier ones that were published over the course of three decades in *The Mathematical Intelligencer*, a magazine with an interesting and novel history.

From its inception in the late 1970s, *MI* has served both to inform and to entertain its readers with topical writings pertaining to present-day mathematical culture, but also with an eye directed toward the past. Historical themes and issues can be found in practically every issue of *MI*, which started as a typewritten newsletter, but eventually grew into a full-fledged magazine familiar to mathematicians around the world. While a few of my contributions to *MI* found here were written during the 1980s, most come from the 15 years when I edited the column "Years Ago." Like many other regular features of the magazine, this column has evolved over time. It was first introduced by Jeremy Gray under the heading "50 and 100 Years Ago," a rubric that had long been used in the British journal *Nature.* Jeremy and others later transformed the column into a platform for historical essays and open-ended reflections on mathematical themes from past eras.

My own long association with *MI* started in a small way. During the 1980s, I was a fellow of the Alexander von Humboldt Foundation, which gave me the chance to delve into the rich archival sources in Göttingen, in particular the papers of Felix Klein and David Hilbert. As it happened, John Ewing, who was editor of *MI* at that time, was visiting Göttingen as a guest of the institute's *Sonderforschungsbereich.* So he and I sometimes had occasion to chat about various things, including my historical interests. John clearly recognized the importance of *MI* as a new type of venue for mathematicians seeking to air their opinions about more general concerns. His editorials from those years are still well worth reading today. As for what came later from my pen, I have to admit that this collection exposes me to the charge that I never managed to escape the pull of Göttingen and its famous mathematical tradition. So let me take this opportunity to thank Hans Becker, Helmut Rohlfing, and the many helpful staff members in the *Handschriftenabteilung* of the Göttingen library, past and present, who offered me their kind assistance over the years.

A Richer Picture implies that the reader probably already has a picture of many of the people and topics discussed in this book, as the essays in it were written with such an audience in mind. It should also be remembered that *The Mathematical Intelligencer* is a magazine, not a scholarly journal. Its mission from the beginning has been both to inform and to entertain. So I tried over the years to write about historical things that I happen to know about in such a way as to interest historically minded mathematicians. Some may nevertheless wonder: how could I write about mathematics in Göttingen and say so little about Riemann? Or about relativity, without saying more about Einstein's brilliant ideas? Of course, I am well aware of these and other glaring gaps. This volume does not pretend to give a comprehensive picture of all the many significant themes that run through the Göttingen tradition. In some of the introductions, I have taken the opportunity to address missing parts of the story, and for Part II I added new material based on two essays published elsewhere to fill in parts of Felix Klein's early career. I have also tried to give ample attention to the scholarly literature, including recent publications, for those who might want to learn about matters that go well beyond what one finds in these essays. The visual images of mathematical objects in the book were greatly enhanced by the talents of Oliver Labs, with whom I have worked closely in recent years, a collaboration I have greatly enjoyed. For the cover design, I am grateful to his wife, the graphic designer Tanja Labs, whose talents I and others have long admired.

The title chosen for this book was actually an idea I owe to Tilman Sauer, though it reflects very well my general views as an historian. That outlook can be put succinctly by citing the words of a famous philosopher from the first half of the twentieth century, the Spanish-born man of letters and Harvard don, George Santayana, who wrote many famous aphorisms. One of them – perhaps more appropriate than ever for the present century – maintains that "those who neglect history are condemned to repeat its mistakes" (or words to that effect). No one should want to dispute the wisdom of that saying, but I would offer a kind of inversion that runs like this: "those who think that history repeats itself – whether as tragedy or farce – are condemned to misunderstand it." A corollary to that claim would suggest that engagement with the past is its own reward; we only demean that activity by insisting that we should study history to learn what went wrong or to feel morally superior to those whose mistakes are so self-evident to us today. The essays presented here reflect my own efforts to learn about mathematics in Germany over many years during which I tried to learn from my own mistakes, some of which surface in the pages that follow.

Of course, so much went wrong in Germany that this theme was bound to be inescapable for historians, many of whom regard the Holocaust as the great looming moral problem of the last century. Quite a number of those who have studied this dark chapter in European history have done so exceedingly well. In the introduction to Part V, I briefly describe some of the key events bearing on mathematics in the Weimar and early Nazi eras by drawing on some of that literature. For this volume, I have included among the reference works several books and articles that offer background information on larger developments in German history relevant for understanding what I have written about. Here I would like to mention just two books that I have personally found particularly insightful. The first is *The Germans*, a collection of essays by Gordon A. Craig that first came out in 1982 when I was studying late modern European history at the CUNY Graduate Center. Craig was long considered the dean of American authorities on modern German history, a reputation gained in part from his volume *Germany, 1866–1945* published in the Oxford History of Modern Europe series. His *The Germans*, a kind of sequel, still makes delightful reading today. Particularly relevant are the five chapters entitled: Germans and Jews, Professors and Students, Romantics, Soldiers, and Berlin: Athens on the Spree and City of Crisis.

The second book was another best-seller, *The Pity of It All: A Portrait of the German-Jewish Epoch, 1743–1933,* written by the Israeli journalist and writer Amos Elon. Many historians have doubted that the much-vaunted notion of a German-Jewish symbiosis ever had any chance of succeeding, but none would deny that for many Jews this was a very real dream. Elon begins his account with the arrival of fourteen-year-old Moses Mendelssohn at an entrance gate to Berlin. Mendelssohn's rise to prominence as a philosopher and proponent of the German Enlightenment served as a symbol for German Jewry throughout the nineteenth century. His friend, the playwright Gotthold Ephraim Lessing, immortalized him in *Nathan der Weise*, first performed in Berlin in 1783, three years before Mendelssohn's death. In 1763, he won a prize competition offered by the Berlin Academy for his essay "On Evidence in the Metaphysical Sciences," in which he applied mathematical proofs to metaphysics (Immanuel Kant finished second). Mendelssohn's larger importance for mathematics in Germany was highlighted in the travelling exhibition "Transcending Tradition: Jewish Mathematicians in German-Speaking Academic Culture." This exhibition appeared at several venues around the world, including the Jewish Museum in Berlin, where it opened in July 2016 on the occasion of the VIIth European Congress of Mathematics.

Over the years that these essays were written, I have benefitted from the encouragement of a great many people, both within academia and outside it. Some of their names recur throughout this volume, but I should begin by thanking Leonard Rubin, my mathematical mentor at the University of Oklahoma, who taught me what doing mathematics was all about and then forgave me for leaving the field to do history. The inspiration to make that shift first came from members of the history of science department in Norman: Ken Taylor, Mary Jo Nye, and above all Sabetai Unguru, who opened my eyes to ancient Greek mathematics. Realizing that I would never learn ancient Greek, I tried German, and with my wife's help succeeded, at least to some extent. Our first visit to Germany in the summer of 1980 was what first put an unlikely idea into my head: why not try to study the history of mathematics in Göttingen? That thought soon led to a letter from Joe Dauben, inviting me to study with him in New York. Without his help and support, I'm sure nothing would have come of what was then only a dreamy idea.

To my own amazement, after two years at the CUNY Graduate Center, I was on my way to Göttingen with a fellowship from the Alexander von Humboldt Foundation. My advisor in Germany was Herbert Mehrtens, one of the first historians to explore the dark secrets of mathematics during the Nazi period. We barely saw each other, though, since he was then working at the Technische Universität in Berlin, where he later habilitated. In 1990, he published *Moderne – Sprache – Mathematik*, a book that created quite a stir at the time. Unfortunately, it went out of print some years ago without ever having been translated into English. During my two years as a Humboldt fellow I got to know several other historians of mathematics with whom I've been in contact ever since, among them: Umberto Bottazzini, Jeremy Gray, Jesper Lützen, John McCleary, Walter Purkert, Norbert Schappacher, and Gert Schubring. Afterward my circle of colleagues and friends widened considerably. So along with the above group, I'd like to thank Tom Archibald, June Barrow-Green, Jed Buchwald, Leo Corry, Michael Eckert, José Ferreirós, Livia Giacardi, Catherine Goldstein, Ulf Hashagen, Tom Hawkins, Jens Høyrup, Tinne Kjeldsen, Karen Parshall, Jim Ritter, Tilman Sauer, Erhard Scholz, Reinhard Siegmund-Schultze, Rossana Tazzioli, Renate Tobies, Klaus Volkert, Scott Walter, and several unnamed others who have helped inspire my work.

An especially important friendship for my wife and me was the one that developed with Walter Purkert, who took a leave of absence from the Karl Sudhoff Institute in Leipzig to spend a semester teaching with me at Pace University – not in New York City, but in the idyllic atmosphere of Westchester County. Marty Kotler, who as department chair at Pace was always highly supportive, helped make that invitation possible. Walter's visit also gave John McCleary and me the incentive to co-organize a memorable conference at Vassar College. That event, which brought together several of the aforementioned historians from Europe and North America, took place during a scorching hot week in the summer of 1988. It led to two volumes of essays that John and I published with the help of Klaus Peters, who was then at Academic Press in Boston.

The following summer we were in Göttingen again, but also spent a month in Leipzig, thanks to an invitation arranged by Walter. That was in August 1989, just before the famous Monday demonstrations that began at the Nikolaikirche. The atmosphere in the city was noticeably tense, though some people were very eager to meet and talk with us the moment they realized we were Americans. One could see younger folks standing in long lines, hoping to get visas for Hungary, which had recently opened its border to Austria. In Leipzig, it was easy to watch reports of such things on West German television and, of course, there was much talk about *Glasnost* and *Perestroika*, the sweeping reforms Michael Gorbachev was calling for just as the GDR was preparing to celebrate it fortieth anniversary. His state visit in early October put renewed pressure on the East German government, setting off the October demonstrations that led to the fall of the Berlin Wall in early November. Having gained an outsider's sense of life in the former GDR during the late 1980s, coupled with the experience of living in western Germany during the early 1990s, I am still inclined to view the reunification as a missed opportunity, though the sands of time have by now left that brief window of promise covered beyond all recognition.

In August of 1990, I had the opportunity to speak at a special symposium on the history of mathematics held in Tokyo and organized by Sasaki Chikara along with several other Japanese historians. The larger occasion was the Twenty-First International Congress of Mathematicians that took place in Kyoto one week earlier. This was the first ICM ever held outside the Western hemisphere and a truly memorable event. At the symposium, I spoke about "The Philosophical Views of Klein and Hilbert," whereas Sasaki presented an overview showing how Japanese mathematics had gradually become westernized during the latter half of the nineteenth century. I only realized then that his story and mine were closely related. From a purely intellectual standpoint, the great number theorist Takagi Teiji stands out as the key figure in this regard, and so it was fitting that two of the speakers at the symposium, Takase Masahito and Miyake Katsuya, gave summary accounts of developments in number theory that both preceded and succeeded Takagi's fundamental contributions to the field.[1] Readers who would like to find out how Hilbert enters into this story – in particular by way of his famous Paris problems – can find a stirring account of this in Ben H. Yandell's *The Honors Class: Hilbert's Problems and their Solvers.*

Although we often spent summer months in Göttingen, the thought that I might someday be offered a professorship at a German university rarely crossed my mind, and then only as a remote possibility. So I was both surprised and delighted to receive such an offer from the Johannes Gutenberg University in Mainz, where I began teaching history of mathematics in the spring of 1992. The adjustment wasn't easy for any of us, but probably it was hardest for my wife, Hilde, and our five-year-old son, Andy. We tend to think back on the good times now, but both of them had to deal with a lot of discouraging things. Luckily, we managed to get through difficult episodes together, for which I'm extremely grateful to them. Adversity, too, can make the heart grow fonder.

When I first came to Mainz, I had some doubts about whether this new life was going to work out. The atmosphere in the department was friendly, but very formal and somewhat provincial. I was one of the youngest members of the faculty and the only foreigner. Probably my colleagues were dimly aware that this was not going to be an easy transition for me. I had virtually no experience teaching history of mathematics, nor had I even taken courses at a German university, let alone taught in German. Luckily, the professorship came with a second position, and so I was pleased to bring in Moritz Epple as a post-doc after my first semester. During his years in Mainz, Moritz taught several different types of courses while working on his splendid book, *Die Entstehung der Knotentheorie* (1999). A second stroke of luck came when Volker Remmert replaced Moritz, who went to Bonn on a Heisenberg fellowship. Both of them contributed greatly to the activities of our small group in Mainz, where I soon felt very much at home. Two senior colleagues, Albrecht Pfister and Matthias Kreck, deserve a special note of gratitude for their support during those first years. Among the younger generation that came to Mainz after me, I am especially grateful to Duco van Straten, Volker Bach, Manfred Lehn, Stefan Müller-Stach, and Steffen Fröhlich. Many others in our immediate *Arbeitsgruppe* ought to be named as well, but six who deserve special thanks are Martin Mattheis, Annette Imhausen, Andreas Karachalios, Martina Schneider, Eva Kaufholz-Soldat, and our wonderful secretary, Renate Emerenziani.

Beginning in the mid 1990s, when I was a fellow at MIT's Dibner Institute for the History of Science, I had the opportunity to interact with colleagues at Boston University's Einstein Papers Project. Some of the fruits of those interactions can be found in Part IV of this volume, for which I thank Hubert Goenner, Michel Janssen, Jürgen Renn, Tilman Sauer, Robert Schulmann, and John Stachel. Two historians who deserve special thanks for their support over many years are Reinhard Siegmund-Schultze and Scott Walter, both of whom have been exceedingly generous in sharing their time and expertise. During my tenure as column editor, which ended in 2016, I had the pleasure of working closely with Marjorie Senechal, now editor-in-chief of *MI*. Marjorie was, from the beginning and ever afterward, a great source of support and intellectual stimulation. Her efforts on behalf of the magazine have been tireless and appreciated by everyone involved with its success. As a retirement present, she and Tilman Sauer, my successor in Mainz, came up with the idea of bundling my writings from *MI* into this book. To both of them, as well as to Marc Strauss and Dimana Tzvetkova at Springer-New York, go my heartfelt thanks.

Marjorie has a keen sense for the importance of history and memory in mathematical cultures, particularly when it comes to understanding that mathematics is a human activity: it's about people. A fine example of this can be found in the special thirtieth anniversary issue

[1]Most of the papers from the Tokyo symposium were later published in *The Intersection of History and Mathematics*, ed. Sasaki Chikara, Sugiura Mitsuo, Joseph W. Dauben, Science Networks, vol. 15 (Proceedings of the 1990 Tokyo Symposium on the History of Mathematics), (Basel: Birkhäuser, 1994).

(*MI,* vol. 30(1)) she put together. This contains her interview with Alice and Klaus Peters, who founded *MI* when both were working for Springer. Since *The Mathematical Intelligencer* will soon celebrate its fortieth birthday, those readers with a sense for history will surely appreciate the chance to page through earlier issues of the magazine, which offer so many insights into people, places, and ideas of the recent past. The selection offered here represents only a narrow slice of what one finds in the magazine as a whole, but I am pleased to have had the chance to contribute to its historical dimension.

Mainz, Germany David E. Rowe
August 2017

Contents

Part I

Two Rival Centers: Göttingen vs. Berlin

1 Introduction to Part I

Mathematicians can sometimes be ferociously competitive, making rivalry a central theme in the history of mathematics. Various forms of competitive behavior come easily to mind, but here I am mainly concerned with a specific rivalry between two leading research communities as this evolved during the latter-half of the nineteenth century in Germany. This concern has, in fact, less to do with specific intellectual achievements, important as these were, than with the larger context of professional development that eventually led to a clearly demarcated German mathematical community by the end of that century. Unlike the highly centralized French community, in which Parisian institutions and their members dominated the scene, the German universities were largely autonomous and tended to cultivate knowledge in local settings, some more important than others.

Germany's decentralized university system, coupled with the ethos of *Wissenschaft* that pervaded the Prussian educational reforms, created the preconditions for a new "research imperative" that provided the animus for modern research schools.[1] Throughout most of the nineteenth century, these schools typically operated in local environments, but with time small-scale research groups began to interact within more complex organizational networks, thereby stimulating and altering activity within the localized contexts. Mathematicians have often found ways to communicate and even to collaborate without being in close physical proximity. Nevertheless, *intense* cooperative efforts have normally necessitated an environment where direct, unmediated communication can take place. This kind of atmosphere arose quite naturally in the isolated settings of small German university towns. Such collaborative research presupposes suitable working conditions and, in particular, a critical mass of researchers with similar backgrounds and shared interests. A work group may be composed of peers, but often one of the individuals assumes the role of a "charismatic leader,"[2] typically as the academic mentor to the junior members of the group. This type of arrangement – the modern mathematical research school – has persisted in various forms throughout the nineteenth and twentieth centuries (Rowe 2003).

By the mid nineteenth century, two institutions in Germany had emerged as the leading centers for research mathematics: Göttingen and Berlin. Both were newer universities, founded by royal patrons who looked to the future. Göttingen's Georgia Augusta was first established in 1737 under the auspices of the King of Hanover, better known in the English-speaking world as George II, King of Great Britain and Ireland, and Prince-Elector of the Holy Roman Empire. The Georgia Augusta has often been considered the prototype for the modern university, owing to the fact that its philosophical faculty enjoyed the same status as the other three – theology, law, and medicine – these being traditionally regarded as higher faculties. George II delegated the task of launching this enterprise to the Hanoverian Minister Gerlach Adolph Freiherr von Münchhausen, who as the university's first Kurator made the initial appointments to the faculty. A similar arrangement took place in Prussia in 1810. King Friedrich Wilhelm II, whose country had been overrun and then annexed in large part by Napoleon, empowered the educational reformer and linguist Wilhelm von Humboldt to found Berlin University, renamed Humboldt University in 1949 when it reopened in the German Democratic Republic. Its main building, still located on the avenue Unter den Linden, was originally built during the mid-eighteenth century as a palace for Prince Heinrich, the brother of the previous king. Acting in concert with other scholars, Humboldt conceived

[1] On the preconditions, see Turner (1980); for the character of research schools in the natural sciences, see Servos (1993).

[2] The notion of charismatic leadership was made famous by Max Weber, who however denied that it had a rightful place in academic life since he believed that personal authority had to be based on purely scientific qualifications (*Wissenschaft als Beruf*). Weber's view, however, was clearly idealized; even within the field of mathematics charismatic leaders have often exerted an influence far beyond their own achievements, three noteworthy examples being Weierstrass, Klein, and Hilbert.

D.E. Rowe, *A Richer Picture of Mathematics*, https://doi.org/10.1007/978-3-319-67819-1_1

Berlin's university in the spirit of neohumanism, an ideology in many ways opposed to the utilitarianism associated with French science (Rowe 1998).

When mathematicians think of Göttingen, famous names immediately spring to mind, beginning with Gauss, as well as his immediate successors, Dirichlet and Riemann. These, to be sure, are names to be conjured with, but they are not central players in the larger story told here. Still, they are relevant because their ideas continued to inspire later generations of mathematicians who lived and worked in a radically different intellectual milieu whose leaders strongly identified with this older Göttingen mathematical tradition. Indeed, to a considerable extent these modern representatives of Göttingen mathematics created a near cult-like worship of Gauss and Riemann that enabled them to bask in the reflected light of their predecessors past glory. Thus, in a word, they were engaged in constructing an image of the grand tradition to which they belonged. Before addressing that theme, however, I should first say something about what made this particular university such a fertile environment for mathematical creativity. This requires taking a glance further backward into the history of German higher education as it developed over the course of the eighteenth and nineteenth centuries. During that time important reforms were inaugurated at leading German universities, spurred by the example of Göttingen, where teaching and research came to be combined in a fundamentally new way.

Despite many common features, the German universities did not form a centralized system even after the nation's unification in 1871. They were administered instead by the individual states just as today. Moreover, since a variety of different cultural, historical, and regional factors have shaped their character and traditions, it is difficult to make sweeping generalizations about how they functioned in practice. For this reason, even though these institutions certainly shared a number of important features, they can only really be understood through an examination of their individual histories. Göttingen's *Georgia Augusta* initially served as Hanover's answer to an institutional dilemma posed by the state of Prussia ruled by the Hohenzollern monarchy. Prior to its founding, prospective candidates for the Hanoverian civil service who wished to pursue a higher education were forced to take up studies outside their home state, most notably at the Prussian University in Halle. Soon after its founding, however, young men, often from aristocratic families, thronged to the new university in the town of Göttingen, located in the southern part of Hanover. By mid-century, the Georgia Augusta not only surpassed Halle in scholarly reputation but also in popularity among German students, many of whom spent more time in beer halls than in lecture rooms (Fig. 1.1).

Historically, European universities had been dominated by the sectarian interests vested in their theological faculties. Göttingen marked an abrupt break with this longstanding tradition by *inverting* the status of its theological and philosophical faculties. Its rapid rise during the middle of the eighteenth century owed much to von Münchhausen's acumen and foresight. Up until his death in 1770, he exercised virtually complete control over the university's affairs. Moreover, his liberal policies contrasted sharply with those of Prussia's Frederick William I. The latter banished Germany's leading Leibnizian philosopher, Christian von Wolff, from Halle in 1723 for espousing doctrines inimical to the monarch's strict Pietism. Although Wolff was allowed to return (and even

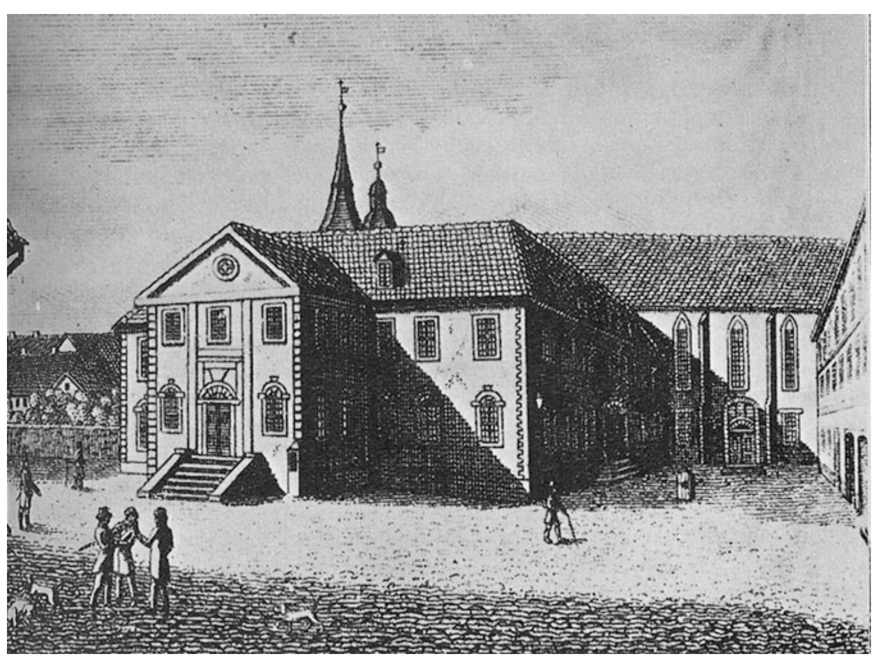

Fig. 1.1 Göttingen University and library, ca. 1815.

served as Chancellor of Halle's university after Frederick II's ascension to the throne in 1740), Halle never regained its scientific reputation. In the meantime, under von Münchhausen's beneficent leadership, Göttingen shook off the yoke of sectarianism that had restrained free thought at European universities for centuries.

Göttingen's philosophical faculty – which offered the traditional elementary training in the "liberal arts" to students who hoped to study in one of the three "higher faculties'" (theology, medicine, or law) – rapidly developed a reputation for serious scholarship based on a fertile combination of teaching and research. Initially, the vaunted ideals of *Lehr- und Lernfreiheit*, flourished to a remarkable degree in the philology seminars of Johann Matthias Gesner and Christian Gottlob Heyne. Spreading from the humanities, the reform spirit soon took hold in the natural sciences after Göttingen acquired the services of such prominent scholars as the anatomist and botanist, Albrecht von Haller; the physicist, Georg Christoph Lichtenberg; the astronomer, Tobias Mayer; and the mathematician, Abraham Kaestner. Münchhausen's innovative policies thus created the preconditions for a university dedicated to serious scholarly pursuits undertaken with a modicum of academic freedom. Within this setting, several of those who studied or taught in Göttingen went on to make lasting contributions in a number of fields. In mathematics, however, their influence would become legendary.

Göttingen's famous mathematical tradition commenced with the career of Carl Friedrich Gauss, who studied, worked, and taught there for some 50 years. No book about mathematics in Göttingen can afford to neglect Gauss, the famous "Prince of Mathematicians," about whom so much has been written. That being the case, Chap. 2 deals with some mythic aspects of his life in the form of an *Intelligencer* quiz. Much of what we know about Gauss comes from a later time and a different mathematical culture that had largely broken earlier ties with astronomy and physics. Gauss embodied all three disciplines, which made him the perfect symbol for what Felix Klein hoped to achieve in Göttingen: the re-integration of pure and applied mathematics.

Gauss grew up in the city Brunswick (Braunschweig), where he was raised in a modest home as the only child of Gebhard Dietrich and Dorothea Gauss. His parents sent him to the local Volksschule, where his intellectual talent was discovered early. This brought him to the attention of Duke Karl Wilhelm Ferdinand, who financed his further education at the Collegium Carolinum in his native city. There, already as a teenager, Gauss was reading works like Newton's *Principia*, which he was able to purchase in 1794. Thanks to a royal stipend, he then had the opportunity to spend three years pursuing university studies. Years later, when he recounted why he chose to study in Göttingen rather than at the local university in Helmstedt, Gauss gave one simple reason: books (Küssner 1979, 48). Few universities, in fact, could rival the holdings in Göttingen's library. Travelling the 90 km by foot, he arrived there in October 1795 and found what he was looking for. Shortly thereafter, he reported to his former teacher Eberhard A. W. Zimmermann about how he was pouring over volumes of the Proceedings of the Petersburg Academy and other such works.

It was in this manner that the young Gauss first entered the still quiet, intellectually isolated world of the German universities. Soon thereafter, Napoleon would arrive upon the scene, but until then life went on as usual. At first undecided as to whether he should take up mathematics or philology, Gauss chose mathematics after he succeeded in proving that the 17-gon can be constructed with straightedge and compass. On 30 March 1796, he recorded the first entry in his mathematical diary: "The principles upon which the division of the circle depends, and geometrical divisibility of the same into seventeen parts, etc." (Gray 1984). In other words, he had cracked the problem of determining which regular polygons can be constructed by straightedge and compass alone, including the first non-classical case, the 17-gon. These principles led him to explore the theory of cyclotomy, thereby entering the portals that led to the theory of algebraic number fields. Symbolically, this *Tagebuch* entry may be regarded as marking at once the birth of higher mathematics in Germany along with the Göttingen mathematical tradition. By the end of the year, though still not yet 20 years of age, Gauss had already filled his scientific diary with 49 entries for results he had obtained during the preceding nine months! Several would remain secrets he took with him to his grave. In fact, he seems to have told no one about the existence of this famous little booklet documenting his early mathematical interests and findings, so its survival was something akin to a small miracle. Not until 1899 did Paul Stäckel find it among Gauss's "personal papers"; it was then in the possession of a grandson who was living in Hameln.

In 1807, six years after he had correctly predicted the location of the asteroid, Ceres, and published his monumental *Disquisitiones Arithmeticae*, Gauss was appointed professor of astronomy in Göttingen. Another nine years passed before he and his family moved into the wing of the newly built astronomical observatory, the *Sternwarte*, which he would occupy for the remainder of his life. Gauss's own star shone brightly in the mathematical firmament, but there were few others nearby. Moreover, unlike his contemporary, Augustin Cauchy, he felt no compulsion to rush into print, remaining true to his motto: "*pauca sed matura*" (few, but ripe). Even more characteristic of his conservatism, Gauss showed no interest in imparting his research results (or those of other mathematicians) in the courses he taught. Instead, he preferred to confide these only to a handful of friends and peers with whom he carried on an extensive scientific correspondence. Among the larger memoirs that

Fig. 1.2 C. F. Gauss, the Prince of Mathematicians.

he did choose to publish, several were written in Latin. In these respects, Gauss's scholarly orientation was entirely traditional (Fig. 1.2).

Throughout his career, Gauss was mainly known for his accomplishments as an applied mathematician. As a professional astronomer, he corresponded regularly with other leading practitioners, including Wilhelm Olbers (Bremen), Friedrich Wilhelm Bessel (Königsberg), and Christian Schumacher (Altona). Moreover, like several other astronomers, he worked on geodetic surveys. Beyond these standard activities, he also took part in Alexander von Humboldt's project to study global fluctuations in the earth's magnetic field. Humboldt had earlier taken note of large-scale magnetic storms, a phenomenon he hoped to study by coordinating data from a worldwide network of magnetic observatories. In 1828, he managed to win over Gauss for this endeavor when the latter attended a scientific conference in Berlin (the setting for Daniel Kehlmann's novel, *Measuring the World*, my main source for recent mythologizing about Gauss).

This then led to Gauss's famous, though brief collaboration with Wilhelm Weber, who was appointed professor of physics in Göttingen in 1831 at the age of 27. Gauss had previously used spherical harmonic analysis in celestial mechanics, but he now adapted these techniques to geomagnetism. He and Weber could thereby show how to represent the global magnetic field of the Earth by combining observations at many locations. In 1834, Gauss and Weber founded the Göttingen Magnetic Union to coordinate research for a network of European observatories. Humboldt later made this into a truly international undertaking by linking with similar efforts in Britain and Russia. The British set up stations in Greenwich, Dublin, Toronto, St. Helena, Cape of Good Hope, and Tasmania, with the British East India Company adding four more in India and Singapore. Humboldt persuaded the Russian Czar to build observatories across his vast territory, making it possible to draw up worldwide magnetic charts. Gauss and Weber also demonstrated the feasibility of telegraphy by building an instrument that linked Weber's physics laboratory with Gauss's *Sternwarte*. Their collaboration briefly linked mathematics and physics within the Göttingen mathematical tradition, a bond that Klein would later seek to revive during the 1890s. For Gauss and Weber, however, their alliance of interests ended abruptly in the wake of an event that had disastrous repercussions for the whole university.

With the death of William IV in 1837 and Queen Victoria's ascension to the throne, the Hanoverian line in England ended. Ernst August, the Duke of Cumberland and a younger son of George III, thereby became King of Hanover. One of his first acts as monarch was to annul the constitution, replacing it with an older version that preserved the former privileges of the aristocracy. This evoked considerable protest among Göttingen's student body, and seven professors, Weber and the Grimm brothers among them, submitted a formal protest. All seven were removed from their positions, and three were even forced to leave the Kingdom of Hanover altogether. Passions ran high, leading the king to send in troops to maintain order; his actions left a deep scar in the university's collective psyche. Politically, the "Göttingen Seven" incident marked a serious setback for the forces of democracy and reform in Hanover. It also sent a chilling message to Göttingen scholars, who thenceforth realized that their cherished academic freedoms had strictly proscribed limits. Throughout this whole drama, Gauss stood by on the sidelines. After the blow had fallen, he hoped to see Weber (one of the least vocal among the seven) restored to his chair, but to no avail. Another member of this "Göttingen Seven" was the Orientalist Heinrich Ewald, who was married to Gauss's daughter Wilhemine. Ewald and Weber were the only two to be reappointed years later, but by then Gauss was too old to continue his collaboration with Weber. In the meantime, the results of their earlier work had been published by Gauss and his assistant, Carl Wolfgang

Benjamin Goldschmidt, in *Atlas des Erdmagnetismus: nach den Elementen der Theorie entworfen*.

Gauss's reticence and failure to speak out during this critical escapade reflected not only his own deep political conservatism but also his academic roots as well. For even though his ideas broke fresh ground and gave a decisive impulse to several new branches of pure mathematics, outwardly Gauss's career and personality bore many of the typical traits of an eighteenth-century scholar. Like Goethe and von Humboldt, he aspired to the ideal of universal knowledge. He ignored hard and fast distinctions between fields such as mathematics, astronomy, and mechanics, and his research ran the gamut from number theory and algebra to geodesy and electromagnetism. Felix Klein, whose reverence for Gauss bordered on idolatry, regarded him as the culmination of an earlier age, likening him to the crowning peak in a gradually ascending chain of mountains that drops off precipitously, leading to a broad expanse of smaller hills nourished by a steady stream flowing down from on high (Klein 1926, 62).

Looking back to the first decades of the nineteenth century, the German universities still appear very much like a backwater in the world of mathematics. However, that would change by the mid-1820s, thanks in large part to the efforts of two scientists who worked outside the mathematical field. One of these was the building commissioner August Leopold Crelle (1780–1855), a well-known figure in Berlin with broad scientific interests. His name is still remembered today as the founder of the first research journal for mathematics in Germany, *Das Journal für die reine und angewandte Mathematik*, long known simply as Crelle. Founded in 1826, its reputation for publishing outstanding original work was almost immediate, though certainly not in the direction Crelle had originally intended.

Nominally dedicated to both pure and applied mathematics, it quickly became a stronghold for pure research alone, so much so that Crelle's creation was jokingly called the "Journal für die reine unangewandte Mathematik" (pure unapplied mathematics). Its very first volume contained several articles by two brilliant young foreigners, the Norwegian Niels Henrik Abel und the Swiss geometer Jakob Steiner. One of Abel's papers presented his famous result that the general quintic equation cannot be solved by means of an algebraic formula. This landmark result stands at the threshold of Galois theory and thus represents one of the great advances in the history of algebra. Altogether, Abel published six articles and notes in this first volume, Steiner five, and the 22-year-old C. G. J. Jacobi contributed a paper in analysis. Crelle clearly had a sharp eye for young talent; when he was seen in the company of Abel and Steiner strolling around Berlin, local humorists made up the idea of calling the two young men Crelle's sons, Cain und Abel.

The other key talent scout was Wilhelm von Humboldt's brother, Alexander, who discovered Peter Gustav Lejeune-Dirichlet (1805–1859) when both were living in Paris. Dirichlet grew up as the youngest child in a large family from Düren, a town situated between Aachen and Cologne on the left bank of the Rhine. At the time of his birth, this region belonged to the French Empire, but with the 1815 Congress of Vienna, it was granted to Prussia. Already at age twelve, Gustav Dirichlet showed a strong interest in and aptitude for mathematics, so his parents sent him to Bonn, where he attended the Gymnasium for two years. Afterward he went to the Cologne Gymnasium, where he was a pupil of the physicist Georg Simon Ohm of Ohm's Law fame, but left without a graduation certificate. By then, Dirichlet was intent on pursuing a career in mathematics, a decision that led him to Paris in May 1822. There he was hired as a private tutor by General Foy, a lucky turn of events that enabled him to remain in the French capital.

Over the next five years, he studied under or met with many of the era's most illustrious figures, including Fourier, Hachette, Laplace, Lacroix, Legendre, and Poisson. During these years, Dirichlet also spent a good bit of time struggling with Gauss's *Disquisitiones Arithmeticae,* a book that would remain important to him throughout his entire life. His hopes for a career in mathematics took a turn for the better after Alexander von Humboldt came to hear about him through Fourier and Poisson (Fig. 1.3). This was surely no accident. Having spent nearly 18 years in Paris, Humboldt knew nearly every scientist there. In later years, he recalled his sense of duty in keeping a watchful eye out for budding German talents in every field, whether astronomers, physicists, chemists, or mathematicians. His efforts clearly helped set the stage for the flowering of scientific excellence at the Prussian universities. Soon after leaving Paris in 1826, he arranged for Dirichlet's first academic appointment in Breslau by asking Gauss to write a letter of recommendation on his behalf.

This brings us to Chap. 3, which focuses on events from this time and the sharply contrasting personalities of Gauss and Dirichlet. The famous Gaussian motto *pauca sed matura* was often taken as an admonition declaring that one should strive for perfection in mathematical publications, voluntarily holding back new results until they can be presented elegantly. In practice, however, Gauss often took advantage of this austere attitude toward publication by referring to his "works in progress." Thus, when corresponding with potential competitors he made it a point to inform them that he was already familiar with their results: he had just not found time yet to polish them up for publication. Chapter 3 takes up this theme by way of Gauss's early efforts to uncover a number-theoretic law of biquadratic reciprocity together with Dirichlet's parallel discoveries along the same lines.

The latter found dramatic expression in a letter Dirichlet wrote to his mother in late October 1827. I found a typescript

Fig. 1.3 Statue of Alexander von Humboldt at Humboldt University located on Unter den Linden in Berlin.

of a portion of this letter among Klein's papers, as this document was presented in a seminar on the psychology of mathematics (Klein Nachlass 21A). One of the participants was the philosopher Leonard Nelson, whose mother was a granddaughter of Gustav Dirichlet. Gert Schubring had, in fact, tracked down the three parts of Dirichlet's Nachlass, which he described in (Schubring 1984) and (Schubring 1986). The latter article contains interesting information about Nelson's political activities, indicating why he was a likely target for Nazi persecution, even though he had died some years before Hitler assumed power. For this reason Nelson's papers, including the family documents related to Dirichlet, were placed in the Murhard library in Kassel, now part of the Kassel University Library. By far the most important documents for Dirichlet's biography can be found in this portion of the Nachlass, which contains the many letters he wrote to his mother. This includes the original of the letter I translated using the typescript in Klein's papers, which contains some minor inaccuracies. Schubring also pointed out a glaring mistake soon after my essay appeared in print (see his letter to the editor in *Mathematical Intelligencer* 12(1)(1990): 5–6.): the photograph that appears on the cover of *MI* and in my original paper is not a picture of the young Dirichlet, although it was identified as such in the Göttingen collection. I should also add that the story surrounding this dramatic letter and how Dirichlet eventually unlocked the secrets of biquadratic reciprocity – though with no help from Gauss – has recently been retold by Urs Stammbach in (Stammbach 2013).

Little more than a year after he came to Breslau, Dirichlet was on his way to Berlin, again thanks to the influence of Alexander von Humboldt. He would remain in the Prussian capital until 1855, at which time he succeeded Gauss in Göttingen. It was also through Humboldt that Dirichlet met and later married Rebecka Mendelssohn-Bartholdy, the younger sister of the famous composer (Fig. 1.4). Although she was overshadowed as a musician by her brother Felix and older sister Fanny, Rebecka nevertheless maintained many friendships with celebrities and artists of the Romantic period. She continued to cultivate these cultural ties when the couple moved to Göttingen. During his years in Berlin, Dirichlet exerted a strong influence on several talented mathematicians who went on to brilliant careers: Kummer, Eisenstein, Kronecker, Riemann, and Dedekind.

The year 1844 marked a turning point for mathematics in Berlin with the arrival of Carl Gustav Jacob Jacobi (1804–1851) as a salaried member of the Prussian Academy of Sciences (Fig. 1.5). Jacobi, a native of Potsdam, had been teaching in Königsberg alongside the physicist Franz Neumann and the astronomer Friedrich Wilhelm Bessel. Jacobi and Neumann founded the famed mathematical-physical seminar that would later serve as a model for several seminars established at other German universities. These seminars were the principle vehicle for educating future researchers, an innovation that made Germany a magnet for so many young foreign scholars during the latter half of the nineteenth century. Dirichlet and Jacobi got along splendidly, and the latter was soon a regular guest at the musical soirees hosted by Rebecka Dirichlet.

During this period, Berlin stood in the forefront of Jewish emancipation, a major theme in (Elon 2002). Rebecka's grandfather, the philosopher Moses Mendelssohn, had paved the way for the *Haskalah* (Jewish enlightenment) of the eighteenth and nineteenth centuries. In 1763, he was granted the status of *Schutzjude* (Protected Jew) by the King of Prussia, which assured him the right to live in Berlin without being disturbed by local authorities. In keeping with this

Fig. 1.4 Drawings by Wilhelm Hensel of Dirichlet and young Rebecka Mendelssohn-Bartholdy.

Fig. 1.5 Carl Gustav Jacob Jacobi.

family tradition, the Dirichlets were on close terms with other prominent liberal families in Berlin, including Rahel and Karl Varnhagen von Ense. Both supported the political uprisings in 1848 that called for democratic reforms in the German states. After this failed, Rebecka helped two prominent revolutionaries escape after the Prussian army had crushed the last remaining resistance in July 1849. One of those arrested, Gottfried Kinkel, was sentenced to life imprisonment in Spandau, just west of Berlin. His friend Carl Schurz escaped, and a little more than a year later, he managed, partly with the help of Rebecka Dirichlet, to free Kinkel from the Spandau prison. Both became prominent figures in exile, Kinkel as a writer in London, and Schurz as a general in the army of Abraham Lincoln during the Civil War (Lackmann 2007, 244–245).

The third figure in this circle of important Berlin mathematicians contrasted sharply with the other two, who belonged to an intellectual elite and its surrounding salon culture. In fact, Jakob Steiner, one of the most celebrated mathematicians of his time, stood poles apart from German high culture in general (Fig. 1.6). Whereas younger contemporaries, like Jacobi and Abel, produced brilliant work in their early twenties, Steiner was nearly forty before he gained an appointment as extraordinary professor in Berlin.

Fig. 1.6 Jakob Steiner, before he grew a beard.

Afterward he enjoyed a legendary career, attracting throngs of students to his courses on synthetic geometry. Yet he was never promoted to a full professorship (*Ordinariat*), probably because his Swiss-German dialect, gruff personality, and outspoken liberal views on political matters led many of his colleagues to avoid him whenever they could. Not only was Steiner regarded as unpleasant, he was also thought to be uncultivated and far too uncouth to be welcomed into Berlin's elite society.

Born in 1796 near the small Swiss village of Utzenstorf, 25 km north of Bern, Steiner grew up as the youngest of eight children in a farming family. The local school he attended offered its charges only the most rudimentary skills, so that at age 14 Steiner could barely write. Afterward some deep inner urge must have taken hold of him, leading him to pursue a more advanced education. In the spring of 1814, at age 18, he left home to take up studies at Johann Heinrich Pestalozzi's school in Yverdon. There Steiner's genius for geometry was quickly discovered, and soon he was even allowed to teach classes. Four years later, he left this stimulating and supportive atmosphere to take up studies in Heidelberg, the beginning of a long, difficult road to academic success. All his life, Steiner saw himself as a crusader for the teaching methods he learned and practiced at Pestalozzi's school. He gave an early testimonial to this effect in a document that he submitted to the Prussian Ministry of Education on 16 December 1826:

> The method used in Pestalozzi's school, treating the truths of mathematics as objects of independent reflection, led me, as a student there, to seek other grounds for the theorems presented in the courses than those provided by my teachers. Where possible I looked for deeper bases, and I succeeded so often that my teachers preferred my proofs to their own. As a result, after I had been there for a year and a half, it was thought that I could give instruction in mathematics (Lange 1899, 19).

An interesting testimonial concerning Steiner's unusual teaching style, but also his cantankerous personality, comes from the pen of the English geometer Thomas Archer Hirst (1830–1892). After taking his doctorate in Marburg in 1852, Hirst spent the academic year 1852–53 in Berlin, where he attended the lectures of Steiner and Dirichlet. He later became a fixture in the British community as a member of the Royal Society, the British Association for the Advancement of Science, and the London Mathematical Society. In his diary, he recorded this impression of Steiner:

> He is a middle-aged man, of pretty stout proportions, has a long, intellectual face, with beard and moustache, and a fine prominent forehead, hair dark and rather inclining to turn grey. The first thing that strikes you on his face is a dash of care and anxiety almost pain, as if arising from physical suffering. . . . He has rheumatism. All these point to physical nervous weakness. His Geometry is famed for its ingenuity and simplicity. He is an immediate pupil of Pestalozzi: in his youth he was a poor shepherd boy, and now a professor.
>
> His argument is that the simplest way is the best; he tries ever to find out the way Nature herself adopts (not always, however, to be relied upon). Mathematics he defines to be the "science of what is self- evident." . . .
>
> I listened with great interest to [Prof. Riess] talk about Dirichlet, Jacobi and Steiner. He told me fully the relations on which the latter stands with them all, and truly it is unexplainable. Riess says his vulgarity has by them all been slightly borne in consideration of his undoubted genius. But that some time ago without provocation Steiner cut them all. The probable reason is that Steiner, naturally of a testy disposition, which has been increased, too, by bodily illness, feels slighted that he has been 33 years "Ausserordentliche" [Extraordinary] Professor. The reason is clear: firstly he does not know Latin, and that among German professors is held as a necessity: 2nd he is so terribly one-sided on the question of Synthetical Geometry that as an examiner he would not be liked. The more I hear, the more I am determined to see him and study him for myself (Gardner and Wilson 1993, 622–624).

In earlier years, Steiner spoke often with Jacobi about common geometrical interests, and later he did the same with Weierstrass. After his colleague's death in 1863, Weierstrass continued to uphold and honor the Steinerian tradition by teaching his standard course on synthetic geometry (Biermann 1988). It should be noted that Jacobi was no longer living at the time Hirst visited Berlin. Thus, the period when Dirichlet, Jacobi, and Steiner were together in Berlin lasted only seven years, and even these were clearly troubled ones as far as Steiner's relations with his colleagues were concerned. This era in Berlin came to an end in 1855 with the death of Crelle and the departure of Dirichlet for Göttingen, where he assumed the chair in mathematics formerly occupied by Gauss (his position as director of the astronomical observatory, however, remained for many years unfilled).

There followed a brief, but brilliant era for mathematics in Göttingen, now represented by Dirichlet and two talented young researchers – Bernhard Riemann and Richard Dedekind – both of whom profited greatly from nearly daily contacts with their older friend, who was still only fifty. Dirichlet's decision to leave Berlin for a more provincial city was made easier by the fact that he no longer had to put up with teaching at the Military school in the Prussian capital. Rebecka surely would have preferred staying, but saw that her husband had no desire to negotiate better conditions with Prussian ministerial officials. She corresponded regularly with her friends, playing for sympathy by calling the little university town she now lived in "Kuhschnappel," the name of a rural village in Saxony made famous in the novel *Siebenkäs* by Jean Paul (Lackmann 2007, 246). It took her little time, though, to make their new home into a local musical center. After briefly renting an apartment, Dirichlet bought a large house located near the town's center from a colleague (today it serves as student housing (see Fig. 1.7)). Here they lived for the next few years with their two youngest children and Dirichlet's mother. Rebecka opened their new home to upwards of seventy guests at a time and with plenty of gaiety for all. Dedekind, a gifted pianist and cellist, played waltzes here for the dancers. The famed violinist Joseph Joachim performed in the Dirichlet home, and it was during a stay in Göttingen that he was invited by Rebecka to hear a young sopranist named Agathe von Siebold. One year later, she met Joachim's close friend, Johannes Brahms, to whom she was briefly engaged.

Fig. 1.7 The house at Mühlenstrasse 1 in Göttingen where the Dirichlets lived from 1856–1859.

Not long after his arrival, Dirichlet was contacted by Johann Karl Kappeler. As President of the Swiss School Board, Kappeler was seeking advice about a suitable candidate for the chair in mathematics at the newly founded Federal Polytechnical School in Zürich (Frei u. Stammbach 1994, 37). Not surprisingly, Dirichlet named both Riemann and Dedekind, which prompted Kappeler to travel to Göttingen so that he could hear lectures by both men! He chose Dedekind, who in 1858 became the first in a long line of eminent German mathematicians to spend part of their careers teaching in Zürich. Incidentally, it was while teaching a course in analysis there that Dedekind realized his inability to prove a standard property of the real numbers. That insight was what led him to invent so-called Dedekind cuts, his famous method for constructing the real numbers; however, he only published this much later in 1872 (Fig. 1.8).

Riemann enjoyed the strong support of Dirichlet, with whom he had studied in Berlin. On the other hand, his relationship with Gauss, whose career was already nearing its end when Riemann first came to Göttingen in 1846, had been distant by comparison. Gauss, however, did play a role in drawing out some of Riemann's boldest ideas, which he expressed in a manuscript written in 1854 for an auspicious occasion: Riemann's final qualification to join the faculty as a *Privatdozent.* This text would later exert a strong influence on developments in differential geometry, followed by a wave of new interest after Einstein showed its relevance for interpreting gravitational effects as variations of curvature in a space-time manifold. In short, Riemann's lecture came to be regarded as a central document presaging the "relativity revolution," the theme of Part IV. The third essay "Geometry and Physical Space" (Chap. 20) in Part IV touches on some of the novel ideas in Riemann's lecture in connection with Einstein's theory of gravitation. A few brief remarks about the events of 1854 would therefore seem appropriate here.

Fig. 1.8 Richard Dedekind.

In that year, Riemann wrote his younger brother Wilhelm that he had given "hypotheses on the foundations of geometry" as the third topic for his habilitation lecture. Hoping that the faculty would choose one of his first two topics, he learned that Gauss wanted to hear the third. Probably Gauss knew nothing about the direction of Riemann's thoughts just then, but he may have wondered whether the young candidate had something new to say about recent speculations concerning non-Euclidean geometry. Originally, Gauss wanted to postpone the colloquium until August owing to his poor health. Riemann had to pester him so that the final act for his qualification as a *Dozent* could take place on June 10, 1854. The ordeal began at 10:30 and ended at one o'clock, but no protocol survives, which means that nothing is known about what Gauss might have asked during the examination. Writing some 20 years after the event, Dedekind mentions that immediately after the colloquium Gauss spoke to Wilhelm Weber with great excitement about the depth of Riemann's ideas. Whether Riemann himself ever learned of this no one can say, and we can only speculate as to why he never published this monumental essay.

During his lifetime, Riemann's ideas exerted a strong influence on research in real and complex analysis, though his use of topological tools such as Riemann surfaces also met with strong resistance (Bottazzini and Gray 2013, 259–341). This part of his legacy, however, stands in sharp contrast with the speculations on foundations of geometry as set forth in his habilitation lecture; in fact, these ideas exerted no impact at all during his lifetime. His famous lecture only appeared in print after his death when Dedekind managed to locate it among his friend's posthumous papers (Riemann 1868). In all likelihood, Dirichlet would have heard about Riemann's lecture after his arrival in Göttingen, but if so, no trace of such discussions seems to have survived. In fact, personal misfortunes soon intervened that would cut short the careers of both men. Consequently, the university's promising potential as a research center never actually took root. In the summer of 1858, Dirichlet suffered a heart attack from which he at first recovered; but then his wife's sudden death led to his own in May 1859. Riemann was then appointed as his successor, but by this time his own health was already in a precarious state. Suffering from tuberculosis, he spent most of his time convalescing in Italy. When he died there in 1866, the mathematical school in Berlin was in full ascendency.

The year 1866 was a fateful one, not only for Göttingen mathematics but for the course of German political history as well. In early July, the Austrian army was crushed by the Prussians at Königgrätz, the latter fighting under the command of Helmuth von Moltke. His forces were outnumbered, but they had a decisive technical advantage over the Austrians, who had to stand while reloading their traditional rifles. This made them easy targets for the Prussians, since they had fast-firing, breech-loading guns, which they could reload while lying on their stomachs. This battle proved decisive for Bismarck's larger strategy, which aimed to nullify the Austrians before uniting the German states under the dominion of the Prussian monarchy. The big loser in this Seven-Weeks War between Austria and Prussia, though, was the Kingdom of Hanover, which had allied itself with the Austrian side. As a consequence, it disappeared from the map altogether, taken, so to speak, as war booty by the Prussians. Riemann was residing in Göttingen until June, when he left for Italy just as the Hanoverian army was about to be defeated near Langensalza. When he died a month later, the Georgia Augusta was already a Prussian university.

Since 1855, Berlin had been led by the triumvirate of Ernst Eduard Kummer, Karl Weierstrass, and Leopold Kronecker. All three exemplified ideals fully consistent with the neo-humanist tradition that had long animated Berlin's philosophical faculty, which included both the humanities as well as the natural sciences. Thus, their appeal to purism and systematic rigor in mathematics was thoroughly in accordance with the research orientation in other disciplines. This ethos served not only to instill a spirit of high purpose, it also gave clear definition to the special type of training imparted in Berlin, thereby providing graduates of the program with a sense of collective identity. In many cases, the Berlin tradition also gave its members a feeling of self-assuredness that outsiders might envy or even despise, but which surely had to be acknowledged.

Kummer's career had followed in the wake of Dirichlet's: he succeeded him first in Breslau and then again in 1855, when the latter left Berlin for Göttingen. Like Dirichlet, he also married into the Mendelssohn family; Kummer's first wife, Ottilie, was the daughter of Nathan Mendelssohn and a cousin of Rebecka Mendelssohn-Bartholdy, the wife of Dirichlet. Although Berlin's leaders were, each in his own way, advocates of disciplinary purism, they came from very different backgrounds: Kummer was a Protestant Prussian, Weierstrass a Catholic Westphalian, and Kronecker a wealthy Jewish businessman from Silesia. Weierstrass eventually became the most prominent of the three. His lectures drew huge numbers of auditors, few of whom had any chance of understanding what he was saying. That seemed not to matter, though, as word got round that his cycle of lectures on analysis was the very latest word on the subject (Bottazzini and Gray 2013, 343–486). Since he never wrote these up for publication, but rather continuously revised their content during each cycle, it was very difficult to learn Weierstrassian analysis without attending his lectures. (Felix Klein managed to do so through the help of his friend Ludwig Kiepert.)

Chapter 4 sketches the rivalry that developed between Berlin and Göttingen. This conflict arose only after the era when Gauss, Dirichlet, and Riemann lived and taught at the Hanoverian university. One should also take note that none of these three luminaries succeeded in establishing a mathematical school there comparable with the one Jacobi founded in Königsberg. After the passing of Riemann, this older Göttingen tradition gave way to new trends that became common characteristics of *Wissenschaft* in general as academic specialization began to take hold. By the time young Felix Klein first set foot in the town, three years after Riemann's death, the Göttingen of Gauss and Riemann was already the stuff of legends. Klein joined the circle of students who surrounded Alfred Clebsch, a leading exponent of the Königsberg tradition and a staunch opponent of the then dominant mathematical culture in Berlin.

Riemann's demise left Richard Dedekind as the last living representative of the older Göttingen mathematical legacy. Dedekind had studied under Gauss, Dirichlet, and Riemann before becoming a *Privatdozent*. In 1862, after three years at the Zurich Polytechnique, he accepted a professorship at the Polytechnic Institute in his native Brunswick, where higher mathematics played virtually no role in the curriculum. He would remain in Brunswick all his life, shunning any opportunity to assume a more distinguished position at a leading German university. From this unlikely outpost, Dedekind produced work that would inspire others for decades to come, including several editions of Dirichlet's *Vorlesungen über Zahlentheorie*, which he supplemented with new concepts and results that were to prove fundamental for modern algebra and algebraic number theory. Like Riemann, Dedekind was intent on pursuing original ideas that were well ahead of their time. Emmy Noether later paid tribute to him by saying "es steht alles schon bei Dedekind" (everything can be found in Dedekind), an exaggeration that actually reflected her own modesty (Dedekind 1930–32). Such hero worship certainly has its endearing side, particularly when coming from such a towering figure as Noether, whose father was no doubt a kind of role model in this regard. Not only was Max Noether an eminent algebraic geometer, he also studied the works of other leading mathematicians with great care. Moreover, like him, Emmy Noether proved to be a true mathematical scholar, as attested by were work on the Cantor-Dedekind correspondence (Noether and Cavaillès 1937). Similarly, Dedekind made an important scholarly contribution when he joined with Heinrich Weber in preparing the first edition of Riemann's *Werke*, which appeared in 1876.

Yet despite his redoubtable mathematical stature, Dedekind always preferred to work in quiet isolation. His modest, almost reclusive personality made him ill-suited for the task of promoting and sustaining the Göttingen tradition of Dirichlet and Riemann. And so finding a suitable successor for Riemann proved difficult: Dedekind had no desire to leave Brunswick, nor did Leopold Kronecker wish to give up his position in Berlin, where as a member of the Prussian Academy of Sciences he could teach whenever and whatever he wanted.

Eventually the ministry settled on Königsberg-trained Alfred Clebsch, who already headed a small research group in Giessen. After moving to Göttingen in 1868, this fledgling school quickly matured, attracting several promising new talents. That same year, Clebsch joined forces with another Königsberg product, Carl Neumann, in founding a new journal, *Die Mathematische Annalen.* By doing so, they effectively threw down the gauntlet to the Berlin mathematicians, who till then had dominated the scene through Crelle's journal, which since 1855 was edited by Carl Wilhelm Borchardt, a former pupil of Jacobi and close friend of Weierstrass. Clebsch stood on quite bad terms with the Berliners, who took a critical view of his influence, including his use of mixed methods in fields like algebraic geometry and invariant theory (see Chap. 4). Unfortunately for Göttingen, Clebsch's tenure at the Georgia Augusta was brief: in November 1872, he contracted diphtheria, which quickly extinguished his young life. He was only 39 years old when he died.

Following Clebsch's sudden and premature death, Göttingen became just another outpost for the now dominant Berlin school. His professorship was briefly occupied until 1875 by Lazarus Fuchs and afterward by Hermann Amandus Schwarz; both had studied in Berlin and would later become members of its faculty. In the meantime, memories of the