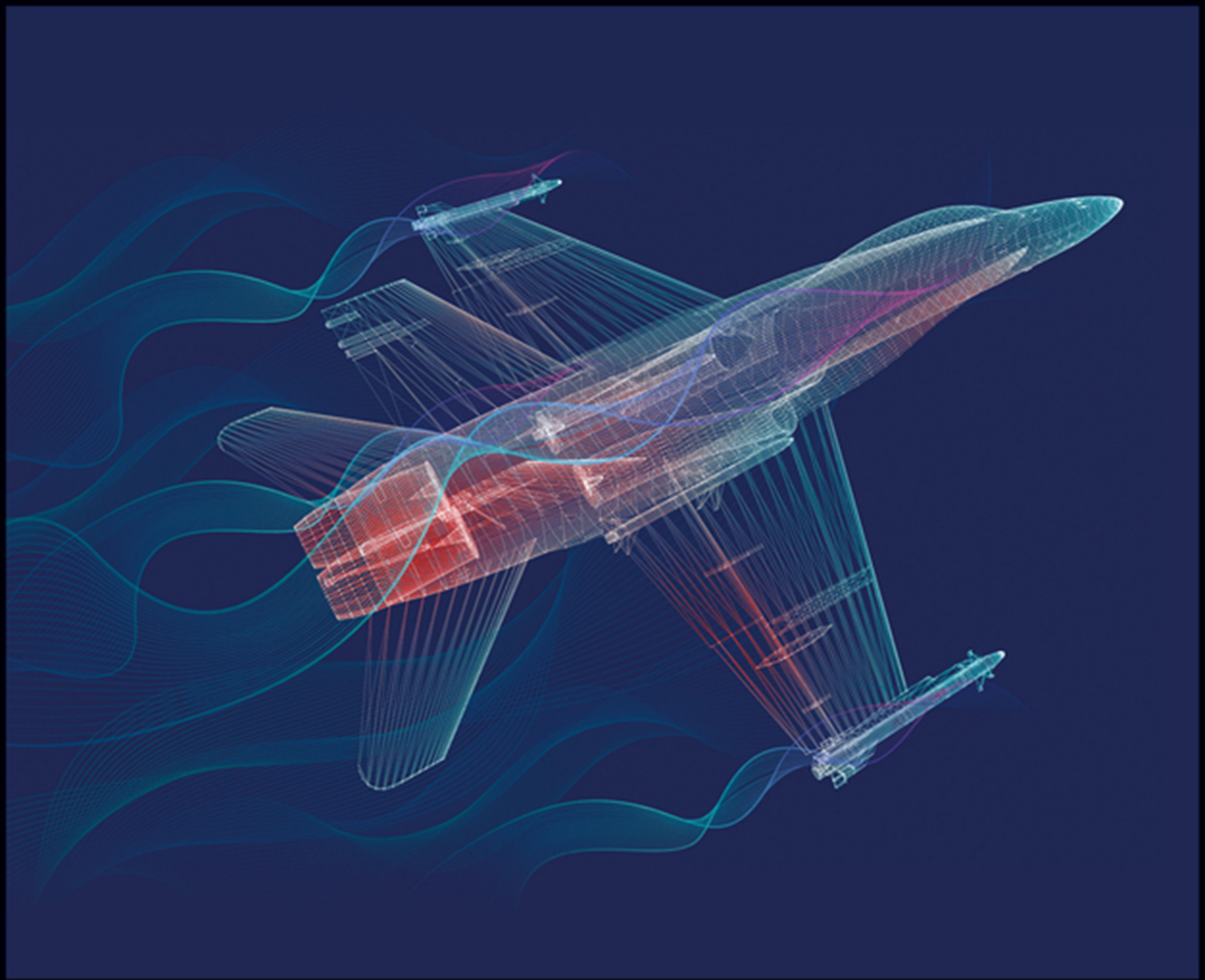

Compact Heat Exchangers

Analysis, Design and Optimization using FEM and CFD Approach

C. Ranganayakulu and K.N. Seetharamu

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Compact Heat Exchangers – Analysis, Design and Optimization using FEM and CFD Approach

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Preface

The importance of compact heat exchangers (CHEs) has been recognized in aerospace, automobile, process plants and other industries for 60 years or more. This importance is further demanded in the aerospace sector, with its requirements such as weight optimization, high compactness and high performance leading to the demand for cost-effective design and manufacturing techniques. While several books dealing mainly with heat exchangers have been published worldwide in English, no complete source of design data can be found on the many important aspects of CHE design, which an engineer can use as a comprehensive source of generalized design data for the most widely used fin surfaces. One of the first comprehensive books on the design of CHEs using primarily air or gases as working fluid was first published by Kays and London in 1967 with the SI unit edition appearing in 1984. This book is still widely used worldwide, with most design data referenced from 1967, for fin data experimentally generated. Because manufacturing technology has progressed significantly since the beginning of 21st century, many new and sophisticated forms of heat transfer surfaces have been used in CHEs. The design data for these surfaces is limited in the open literature and most of the aerospace industry CHE manufacturers keep their design data proprietary.

This book is an attempt to bring new concepts of design data generation numerically, which is cost-effective and more generic design data, which can be used more effectively by design and practising engineers. It is hoped that researchers and designers will find it of value, as well as academics and graduate students. The specialty of this book is numerical design data based on FEM and CFD. Numerical methods and techniques are introduced for estimation of performance deterioration such as flow non-uniformity, temperature non-uniformity and longitudinal heat conduction effects using FEM for CHE unit level and Colburn *j* factor and Fanning friction *f* factor data generation methods for various types of CHE fins using CFD at fin level. In addition, worked examples for single- and two-phase flow CHEs are provided, and the complete qualification tests are given for CHEs used in aerospace applications, typically, which are unavailable in open literature, as these are provided only in some standards. In order to keep the book to a reasonable size, some topics of relevance to CHE applications, which are available in other accessible books, such as *Compact Heat Exchangers* by Kays and London, 3rd edition, McGraw–Hill Book Company, New York [1984], *Compact Heat Exchangers* by J.E Hesselgreaves, Pergamon [2001] and *Plate-fin Heat Exchangers Guide to their Specification and Use* by M.A. Taylor, HTFS, UK, [1987] have been omitted, in particular those of transients (for regenerators), the effect of temperature dependent fluid properties and analytical solutions for flow in tubes.

The first three chapters of the book deal with the fundamentals of heat transfer, heat exchangers and numerical techniques of FEM and CFD. The application of FEM to compact heat exchangers is presented in Chapter 4, where heat conduction effects using FEM on evaporator tubes in pool boiling is also provided. Chapter 5 gives the complete information about CFD analysis of various fins of CHEs for generation of Colburn *j* and Fanning friction *f* factor correlations. In addition, the endurance life estimation of a typical CHE is also provided, based on FEM and CFD techniques. The basic concepts of sizing and design methodologies are provided for CHEs in Chapter 6 along with several worked examples for direct transfer type heat exchangers, boiling and condenser heat exchangers. Finally, in Chapter 7, the construction details and several qualifications tests are presented for qualification of CHEs especially for aerospace applications.

Many people have helped either directly or indirectly throughout the preparation of this book. Primarily, the authors are greatly indebted to their family members Lakshmi and Puji Krimmel (Dr C. Ranganayakulu) and Uma, Anil, Vani and Samartha Shastry (Prof. K.N. Seetharamu) for their unfailing support and patience during the development of this book. In particular, the authors wish to thank Aeronautical Development Agency and PES University, Bangalore for allowing us to work on this book. Dr C Ranganayakulu acknowledges his colleagues Mr R. Swaminathan, Mr A. Panigrahi and Mr Shahbaz Ulum for their support; in particular, Prof. Stephan Kabelac, Prof. Dieter Gorenflo, Prof. P. Nithiarasu and Prof. V. Vasudeva Rao for their guidance and encouragement; as well as Dr Saik L. Ismail, Dr A.C. Bhaskar, Dr R. Balasundar Rao, Dr M. Amaranatha Raju and Mr K.V. Ramana Murthy for their PhD works and Mr M.H. Prasad, Mr B. Mahesh and Mr C. Deepak Varma for book documentary works. Prof. Seetharamu also acknowledges Dr H.W. Lee, Dr Anvar Mydin, Dr G.A. Quadir, Dr Z.A. Zainal and in particular Dr T.R. Seetharam, Dr V. Krishna and Dr K.N.B. Murthy for their encouragement to write this book. Finally, the authors would like to thank the staff of John Wiley & Sons Ltd, UK, in particular Ms Anita Yadav and Mr Paul Beverley, for their constant support and encouragement during the preparation of the book.

C. Ranganayakulu
K.N. Seetharamu

Series Preface

The Wiley-ASME Press Series in Mechanical Engineering brings together two established leaders in mechanical engineering publishing to deliver high-quality, peer-reviewed books covering topics of current interest to engineers and researchers worldwide.

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1

Basic Heat Transfer

1.1 Importance of Heat Transfer

The subject of heat transfer is of fundamental importance in many branches of engineering. A *mechanical engineer* may be interested in knowing the mechanisms of heat transfer involved in the operation of equipment, such as boilers, condensers, air preheaters and economizers, and in thermal power plants, in order to improve performance. Refrigeration and air-conditioning systems also involve heat-exchanging devices, which need careful design. *Electrical engineers* are keen to avoid material damage due to hot spots, developed by improper heat transfer design in electric motors, generators and transformers. An *electronic engineer* is interested in knowing the most efficient methods of heat dissipation from chips and other semiconductor devices so that they can operate within safe operating temperatures. A *computer hardware engineer* wants to know the cooling requirements of circuit boards, as the miniaturization of computing devices is advancing rapidly. *Chemical engineers* are interested in heat transfer processes in various chemical reactions. A *metallurgical engineer* may need to know the rate of heat transfer required for a particular heat treatment process, such as the rate of cooling in a casting process, as this has a profound influence on the quality of the final product. *Aeronautical engineers* are interested in knowing the heat transfer rate in electronic equipment that uses compact heat exchangers for minimizing weight, in rocket nozzles and in heat shields used in re-entry vehicles. An *agricultural engineer* would be interested in the drying of food grains, food processing and preservation. *Civil engineers* need to be aware of the thermal stresses developed in quick-setting concrete, and the effect of heat and mass transfer on buildings and building materials. Finally, an *environmental engineer* is concerned with the effect of heat on the dispersion of pollutants in air, diffusion of pollutants in soils, thermal pollution in lakes and seas and their impact on life (Incropera et al. [1]).

The study of heat transfer can offer economical and efficient solutions to critical problems encountered in many branches of engineering. For example, we could consider the development of heat pipes that can transport heat at a much greater rate than copper or even silver rods of the same dimensions, even at almost isothermal conditions. The development of modern gas turbine blades, in which the gas temperature exceeds the melting point of the material of the blade, is possible by providing efficient cooling systems, and this is another example of the success of heat transfer design methods. The design of computer chips, which encounter heat fluxes of the same order those occurring in re-entry vehicles, especially when the surface temperature of the chips is limited to less than 100 °C, is another success story for heat transfer analysis.

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Although there are many successful heat transfer designs, further developments are still necessary in order to increase the lifespan and efficiency of the many devices discussed above, which can lead to many more inventions. Also, if we are to protect our environment, it is essential to understand the many heat transfer processes involved and to take appropriate action, where necessary.

1.2 Heat Transfer Modes

Heat transfer is the exchange of thermal energy between physical systems. The rate of heat transfer is dependent on the temperatures of the systems and the properties of the intervening medium through which the heat is transferred. The three fundamental modes of heat transfer are conduction, convection and radiation. Heat transfer, the flow of energy in the form of heat, is a process by which a system changes its internal energy, hence it is of vital use in applications of the *first law of thermodynamics*. Conduction is also known as diffusion, not to be confused with diffusion related to the mixing of constituents of a fluid.

The direction of heat transfer is from a region of high temperature to another region of lower temperature, and is governed by the *second law of thermodynamics*. Heat transfer changes the internal energy of the systems from which and to which the energy is transferred. Heat transfer will occur in a direction that increases the entropy of the collection of systems. Heat transfer is that section of engineering science that studies the energy transport between material bodies due to a temperature difference (Bejan [2], Holman [3], Incropera and Dewitt [4], Sukhatme [5]). The three modes of heat transfer are

- Conduction
- Convection
- Radiation.

The conduction mode of heat transport occurs either because of an exchange of energy from one molecule to another, without the actual motion of the molecules, or because of the motion of any free electrons that are present. Therefore, this form of heat transport depends heavily on the properties of the medium and takes place in solids, liquids and gases if a difference in temperature exists.

Molecules present in liquids and gases have freedom of motion, and by moving from a hot to a cold region, they carry energy with them. The transfer of heat from one region to another, due to such macroscopic motion in a liquid or gas, added to the energy transfer by conduction within the fluid, is called *heat transfer by convection*. Convection may be free, forced or mixed. When fluid motion occurs because of a density variation caused by temperature differences, the situation is said to be a free or natural convection. When the fluid motion is caused by an external force, such as pumping or blowing, the state is defined as being one of forced convection. A mixed convection state is one in which both natural and forced convections are present. Convection heat transfer also occurs in boiling and condensation processes.

All bodies emit thermal radiation at all temperatures. This is the only mode that does not require a material medium for heat transfer to occur. The nature of thermal radiation is such that a propagation of energy, carried by *electromagnetic waves*, is emitted from the surface of the body. When these electromagnetic waves strike other body surfaces, a part is reflected, a part is transmitted and the remaining part is absorbed. All modes of heat transfer are generally present in varying degrees in a real physical problem. The important aspects in solving heat transfer problems are identifying the significant modes and deciding whether the heat transferred by other modes can be neglected.

1.3 Laws of Heat Transfer

It is important to quantify the amount of energy being transferred per unit time, and this requires the use of rate equations. For heat conduction, the rate equation is known as *Fourier's law*, which is expressed for one dimension as

$$q_x = -k\frac{dT}{dx} \tag{1.1}$$

where q_x is the heat flux in the x direction (W/m^2); k is the thermal conductivity (W/m · K), a property of material, and dT/dx is the temperature gradient (K/m).

For convective heat transfer, the rate equation is given by *Newton's law of cooling* as

$$q = h\left(T_w - T_a\right) \tag{1.2}$$

where q is the convective heat flux; (W/m^2); $(T_w - T_a)$ is the temperature difference between the wall and the fluid and h is the convection heat transfer coefficient, (W/m^{2}K).

The convection heat transfer coefficient frequently appears as a boundary condition in the solution of heat conduction through solids. We assume h to be known in many such problems. In the analysis of thermal systems, we can again assume an appropriate h if not available (e.g. heat exchangers, combustion chambers). However, if required, h can be determined via suitable experiments, although this is a difficult option.

The maximum flux that can be emitted by radiation from a black surface is given by the *Stefan–Boltzmann law*:

$$q = \sigma T_w^4 \tag{1.3}$$

where q is the radiative heat flux, (W/m^2); σ is the Stefan–Boltzmann constant (5.669×10^{-8}) in W/m^{2}K^4; and T_w is the surface temperature (K).

The heat flux emitted by a real surface is less than that of a black surface and is given by

$$q = \varepsilon \sigma T_w^4 \tag{1.4}$$

where ε is the radiative property of the surface and is referred to as the *emissivity*. The net radiant energy exchange between any two surfaces, 1 and 2, is given by

$$Q = F_\epsilon F_G \sigma A_1 \left(T_1^4 - T_2^4\right) \tag{1.5}$$

where F_ϵ is a factor that takes into account the nature of the two radiating surfaces; F_G is a factor that takes into account the geometric orientation of the two radiating surfaces and A_1 is the area of surface 1. When a heat transfer surface, at temperature T_1, is completely enclosed by a much larger surface at temperature T_2, the net radiant exchange can be calculated by

$$Q = qA_1 = \in_1 \sigma A_1 \left(T_1^4 - T_2^4\right) \tag{1.6}$$

With respect to the laws of thermodynamics, only the first law is of interest in heat transfer problems. The increase of energy in a system is equal to the difference between the energy transfer by heat to the system and the energy transfer by work done on the surroundings by the system, that is,

$$dE = dQ - dW \tag{1.7}$$

where Q is the total heat entering the system and $\boldsymbol{W}$ is the work done on the surroundings. Since we are interested in the rate of energy transfer in heat transfer processes, we can restate the first law of thermodynamics as follows:

The rate of increase of the energy of the system is equal to the difference between the rate at which energy enters the system and the rate at which the system does work on the surroundings, that is,

$$\frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt} \tag{1.8}$$

where t is the time.

The important fluid properties associated with conduction phenomena are presented in Appendixes A2–A8.

1.4 Steady-State Heat Conduction

Heat conduction is the transfer of heat (internal energy) by microscopic collisions of particles and movement of electrons within a body. The microscopically colliding objects, which include molecules, atoms and electrons, transfer disorganized microscopic kinetic and potential energy, jointly known as internal energy. Conduction takes place in all phases of matter: solids, liquids, gases and plasmas. The rate at which energy is conducted as heat between two bodies is a function of the temperature difference (temperature gradient) between the two bodies and the properties of the conductive medium through which the heat is transferred. Thermal conduction is originally called diffusion.

Steady-state conduction is the form of conduction that happens when the temperature difference(s) driving the conduction are constant, so that (after an equilibration time), the spatial distribution of temperatures (temperature field) in the conducting object does not change any further. In steady-state conduction, the amount of heat entering any region of an object is equal to the amount of heat coming out (if this are not so, the temperature would be rising or falling, as thermal energy is tapped from or trapped in a region).

For example, a bar may be cold at one end and hot at the other, but after a state of steady-state conduction is reached, the spatial gradient of temperatures along the bar does not change any further, as time proceeds. Instead, the temperature at any given section of the rod remains constant, and this temperature varies linearly in space, along the direction of heat transfer.

1.4.1 One-Dimensional Heat Conduction

A one-dimensional approximation of the heat conduction equation is feasible for many physical problems, such as plane walls and fins (Bejan [2], Holman [3], Incropera and Dewitt [4], Ozisik [6]). In these problems, any major temperature variation is in one direction only and the variation in all other directions can be ignored. Other examples of one-dimensional heat transfer occur in cylindrical and spherical solids in which the temperature variation occurs only in the radial direction. In this section, such one-dimensional problems are considered for steady-state conditions, in which the temperature does not depend on time. Time-dependent and multidimensional problems will be discussed in later sections.

The steady-state heat conduction equation for a plane wall, shown in Figure 1.1, is

$$kA\frac{d^2T}{dx^2} = 0 \tag{1.9}$$

where $\boldsymbol{k}$ is the thermal conductivity and $\boldsymbol{A}$ is the cross-sectional area perpendicular to the direction of heat flow. The problem is complete with the following description of the boundary conditions.

At $x = 0$, $T = T_1$; and at $x = L$, $T = T_2$

The exact solution to Equation 1.9 is

$$kAT = C_1x + C_2 \tag{1.10}$$

On applying the appropriate boundary conditions to Equation 1.10, we obtain

$$C_2 = kAT_1 \tag{1.11}$$

And

$$C_1 = -\frac{kA(T_1 - T_2)}{L} \tag{1.12}$$

Therefore, substituting constants C_1 and C_2 into Equation 1.10 results in

$$T = -\frac{(T_1 - T_2)x}{L} + T_1 \tag{1.13}$$

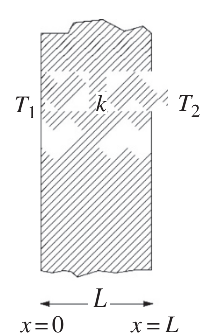

Figure 1.1 Heat conduction through a homogeneous wall.

The above equation indicates that the temperature distribution within the wall is linear. The heat flow, Q, can be written as

$$Q = -kA\frac{dT}{dx} = \frac{kA(T_1 - T_2)}{L} \tag{1.14}$$

Example 1.1 The wall of an industrial furnace is constructed from 0.3 m thick fireclay brick having a thermal conductivity 1.7 W/m · K. Measurements made during steady-state operation reveal temperatures of 1400 and 1150 K at the inner and outer surfaces, respectively, as shown in Figure 1.2. What is the rate of heat loss through a wall that is 0.5 m by 1.2 m on a side?

Solution

Known: Steady-state conditions with prescribed wall thickness, area, thermal conductivity and surface temperatures.

Find: Wall heat loss.

Schematic: Figure 1.2

Assumptions:

Steady-state conditions
One-dimensional conduction through the wall
Constant thermal conductivity

Analysis: Since heat transfer through the wall is by conduction, the heat flux be determined from Fourier's law. Using Equation 1.1 gives

$$q_x = k\frac{\Delta T}{L} = 1.7\frac{\text{W}}{\text{m}}\cdot\text{K} \times \frac{250\,\text{K}}{0.15\,\text{m}} = 2833\,\text{W/m}^2$$

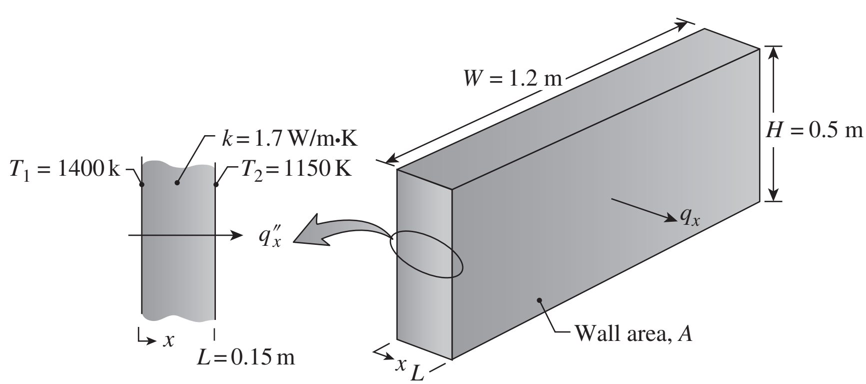

Figure 1.2 One-dimensional heat conduction slab.

The heat flux represents the rate of heat transfer through a section of unit area, and it is uniform (invariant) across the surface of the wall. The heat loss through the wall of area, $A = H \times W$ is then

$$(HW)\, q_x = (0.5\,\text{m} \times 1.2\,\text{m})2833\,\text{W/m}^2 = 1700\,\text{W}$$

Comments: Note the direction of heat flow and the distinction between heat flux and heat rate.

1.4.2 Three-Dimensional Heat Conduction Equation

The determination of temperature distribution in a medium (solid, liquid, gas or a combination of phases) is the main objective of a conduction analysis, that is, to know the temperature in the medium as a function of space at steady state and as a function of time during the transient state. Once this temperature distribution is known, the heat flux at any point within the medium, or on its surface, may be computed from Fourier's law, Equation 1.1. The knowledge of the temperature distribution within a solid can be used to determine the structural integrity via a determination of the thermal stresses and distortion. The optimization of the thickness of an insulating material and the compatibility of any special coatings or adhesives used on the material can be studied by knowing the temperature distribution and the appropriate heat transfer characteristics.

We shall now derive the conduction equation in Cartesian coordinates, as per Carslaw and Jaeger [7], by applying the energy conservation law to a differential control volume, as shown in Figure 1.3. The solution of the resulting differential equation, with prescribed boundary conditions, gives the temperature distribution in the medium.

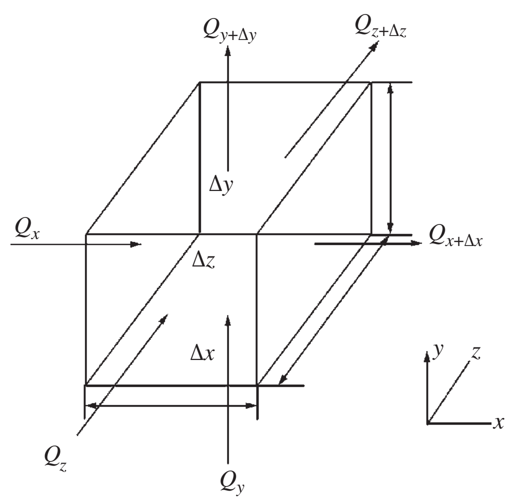

Figure 1.3 A differential control volume for heat conduction analysis.

A Taylor series expansion results in

$$Q_{x+dx} = Q_x + \frac{\partial Q_x}{\partial x}\Delta x$$
$$Q_{y+dy} = Q_y + \frac{\partial Q_y}{\partial y}\Delta y \qquad (1.15)$$
$$Q_{z+dz} = Q_z + \frac{\partial Q_z}{\partial z}\Delta z$$

Note that the second- and higher-order terms are neglected in the above equation. The heat generated in the control volume is $G\ \Delta x\ \Delta y\ \Delta z$ and the rate of change in energy storage is given as

$$\rho\, \Delta x\, \Delta y\, \Delta z\, c_p \frac{\partial T}{\partial t} \qquad (1.16)$$

Now, with reference to Figure 1.3, we can write the energy balance as inlet energy + energy generated = energy stored + exit energy that is,

$$G\,\Delta x\,\Delta y\,\Delta z + Q_x + Q_y + Q_z = \rho \Delta x \Delta y \Delta z \frac{\partial T}{\partial x} + Q_{x+dx} + Q_{y+dy} + Q_{z+dz} \qquad (1.17)$$

Substituting Equation 1.15 into the above equation, and rearranging, results in

$$-\frac{\partial Q_x}{\partial x}\Delta x - \frac{\partial Q_y}{\partial y}\Delta y - \frac{\partial Q_z}{\partial z}\Delta z + G\,\Delta x\,\Delta y\,\Delta z = \rho\,\Delta x\,\Delta y\,\Delta z\, c_p \frac{\partial T}{\partial t} \qquad (1.18)$$

The total heat transfer Q in each direction can be expressed as

$$Q_x = \Delta y\,\Delta z\, q_x = -k_x \Delta y\,\Delta z \frac{\partial T}{\partial x}$$
$$Q_y = \Delta x\,\Delta z\, q_y = -k_y \Delta x\,\Delta z \frac{\partial T}{\partial x} \qquad (1.19)$$
$$Q_z = \Delta x\,\Delta y\, q_z = -k_z \Delta x\,\Delta y \frac{\partial T}{\partial z}$$

Substituting Equation 1.19 into Equation 1.18 and dividing by the volume, $\Delta x\ \Delta y\ \Delta z$, we get

$$\frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k_z \frac{\partial T}{\partial z}\right] + G = \rho c_p \frac{\partial T}{\partial t} \qquad (1.20)$$

Equation 1.20 is the transient heat conduction equation for a stationary system expressed in Cartesian coordinates. The thermal conductivity, k, in the above equation is a vector. In its most general form, the thermal conductivity can be expressed as a tensor, that is,

$$k = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix} \qquad (1.21)$$

Equations 1.20 and 1.21 are valid for solving heat conduction problems in anisotropic materials with a directional variation in the thermal conductivities. In many situations, however, the thermal conductivity can be taken as a non-directional property, that is, it is isotropic. In such materials, the heat conduction equation is written as (constant thermal conductivity)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{G}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1.22}$$

where $\alpha = k/\rho c_p$ is the *thermal diffusivity*, which is an important parameter in transient heat conduction analysis.

If the analysis is restricted only to steady-state heat conduction with no heat generation, the equation is reduced to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \tag{1.23}$$

For a one-dimensional case, the steady-state heat conduction equation is further reduced to

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0 \tag{1.24}$$

The heat conduction equation for a cylindrical coordinate system is given by

$$\frac{1}{r}\frac{\partial}{\partial r}\left[k_r r\frac{\partial T}{\partial r}\right] + \frac{1}{r^2}\frac{\partial}{\partial \varnothing}\left[k_\varnothing\frac{\partial T}{\partial \varnothing}\right] + \frac{\partial}{\partial z}\left[k_z\frac{\partial T}{\partial z}\right] + G = \rho c_p\frac{\partial T}{\partial t} \tag{1.25}$$

where the heat fluxes can be expressed as

$$\begin{aligned} q_r &= -k_r\frac{\partial T}{\partial r} \\ q_\varnothing &= -\frac{k_\varnothing}{r}\frac{\partial T}{\partial \varnothing} \\ q_z &= -k_z\frac{\partial T}{\partial z} \end{aligned} \tag{1.26}$$

The heat conduction equation for a spherical coordinate system is given by

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[k_r r^2\frac{\partial T}{\partial r}\right] + \frac{1}{r^2 sin^2\theta}\frac{\partial}{\partial \varnothing}\left[k_\varnothing\frac{\partial T}{\partial \varnothing}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[k_\theta\sin\theta\frac{\partial T}{\partial \theta}\right] + G = \rho c_p\frac{\partial T}{\partial t} \tag{1.27}$$

where the heat fluxes can be expressed as

$$\begin{aligned} q_r &= -k_r\frac{\partial T}{\partial r} \\ q_\varnothing &= -\frac{k_\varnothing}{r\sin\theta}\frac{\partial T}{\partial \varnothing} \\ q_\theta &= -\frac{k_\theta}{r}\frac{\partial T}{\partial \theta} \end{aligned} \tag{1.28}$$

It should be noted that for both cylindrical and spherical coordinate systems, Equations 1.25 and 1.27 can be derived in a similar fashion as for Cartesian coordinates by considering the appropriate differential control volumes.

1.4.3 Boundary and Initial Conditions

The heat conduction equations discussed above will be complete for any problem only if the appropriate boundary and initial conditions are stated. With the necessary boundary and initial conditions, a solution to the heat conduction equations is possible. The boundary conditions for the conduction equation can be of two types or a combination of these two – the *Dirichlet* condition, in which the temperature on the boundaries is known and/or the *Neumann* condition, in which the heat flux is imposed (see Figure 1.4) as per Lewis et al. [8]:

Dirichlet condition

$$T = T_0 \text{ on } \Gamma_T \tag{1.29}$$

Neumann condition

$$q = -k\frac{\partial T}{\partial n} = C \text{ on } \Gamma_{qf} \tag{1.30}$$

In Equations 1.29 and 1.30, T_0 is the prescribed temperature; Γ is the boundary surface; n is the outward direction normal to the surface; and $\boldsymbol{C}$ is the constant flux given. The insulated, or adiabatic, condition can be obtained by substituting $C = 0$. The convective heat transfer boundary condition also falls into the *Neumann* category, and can be expressed as

$$-k\frac{\partial T}{\partial n} = h(T_w - T_a) \text{ on } \Gamma_{qc} \tag{1.31}$$

It should be observed that the heat conduction equation has second-order terms and hence requires two boundary conditions. Since time appears as a first-order term, only one initial value (i.e. at some instant of time all temperatures must be known) needs to be specified for the entire body, that is,

$$T = T_0 \text{ all over the domain } \Omega \text{ at } t = t_0 \tag{1.32}$$

where t_0 is a reference time.

The constant or variable temperature, conditions are generally easy to implement as temperature is a scalar. However, the implementation of surface fluxes is not as straightforward.

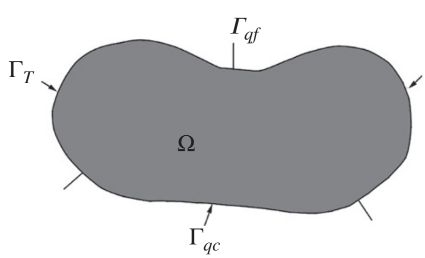

Figure 1.4 Boundary condition.

Equation 1.30 can be rewritten with the direction cosines of the outward normal as

$$k_x \frac{\partial T}{\partial x} \tilde{\imath} + k_y \frac{\partial T}{\partial y} \tilde{m} + k_z \frac{\partial T}{\partial z} \tilde{n} = C \text{ on } \Gamma_{qf} \tag{1.33}$$

Similarly, Equation 1.31 can be rewritten as

$$k_x \frac{\partial T}{\partial x} \tilde{\imath} + k_y \frac{\partial T}{\partial y} \tilde{m} + k_z \frac{\partial T}{\partial z} \tilde{n} = h(T - T_a) \text{ on } \Gamma_{qc} \tag{1.34}$$

where, $\tilde{\imath}$, $\tilde{m}$ and $\tilde{n}$ are the direction cosines of the appropriate outward surface normals.

The general energy equation for heat conduction, taking into account the spatial motion of the body is given by

$$\frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k_z \frac{\partial T}{\partial z}\right] + G = \rho c_p \left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right) \tag{1.35}$$

where u, v and w are the components of the velocity in the three directions, x, y and z respectively.

1.5 Transient Heat Conduction Analysis

In the above, we have discussed steady-state heat conduction, in which the temperature in a solid body is assumed to be invariant with respect to time. However, many practical heat transfer applications are unsteady (transient) in nature and in such problems the temperature varies with respect to time. For instance, in many industrial plant components, such as boilers or refrigeration and air-conditioning equipment, the heat transfer process is transient during the initial stages of operation, so the analysis of transient heat conduction is very important.

1.5.1 Lumped Heat Capacity System

In this section, we consider the transient analysis of a body in which the temperature is assumed to be constant at any point within and on the surface of the body at any given instant of time. It is also assumed that the temperature of the whole body changes uniformly with time. Such an analysis is called a *lumped heat capacity* method and is a simple and approximate procedure in which no spatial variation in temperature is allowed. The change in temperature in such systems varies only with respect to time. It is therefore obvious that the lumped heat capacity analysis is limited to small-sized bodies and/or high thermal conductivity materials. Consider a body at an initial temperature T_0, immersed in a liquid maintained at a constant temperature T_a, as shown in Figure 1.5. At any instant in time, the convection heat loss from the surface of the body is at the expense of the internal energy of the body. Therefore, the internal energy of the body at any time will be equal to the heat convected to the surrounding medium, that is,

$$-\rho c_p V \frac{dT}{dt} = hA(T(t) - T_a) \tag{1.36}$$

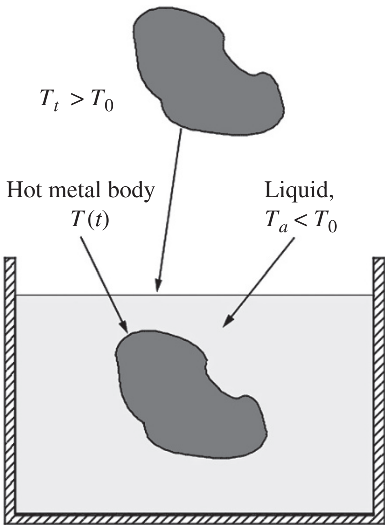

Figure 1.5 Lumped heat capacity system: A hot metal body is immersed in a liquid maintained at a constant temperature.

where ρ is the density, c_p is the specific heat and V is the volume of the hot metal body; A is the surface area of the body; h is the heat transfer coefficient between the body surface and the surrounding medium; t is the time; and $T(t)$ is the instantaneous temperature of the body.

Equation 1.36 is a first-order differential equation in time, which requires an initial condition to obtain a solution. As mentioned previously, the initial temperature of the body at time $t = 0$, is T_0. Applying the variable separation concept to Equation 1.36, we get

$$\frac{dT}{T(t) - T_a} = -\frac{hA}{\rho c_p V} dt \tag{1.37}$$

Integrating between temperatures T_0 and $T(t)$, we obtain

$$\int_{T_0}^{T(t)} \frac{dT}{T(t) - T_a} = -\int_0^t \frac{hA}{\rho c_p V} dt \tag{1.38}$$

Note that the temperature changes from T_0 to $T(t)$ as the time changes from 0 to t.

Integration of the above equation results in a transient temperature distribution as follows:

$$\ln\left(\frac{T - T_a}{T_0 - T_a}\right) = \frac{-hAt}{\rho c_p V} \tag{1.39}$$

or

$$\frac{T - T_a}{T_0 - T_a} = e^{\left[-\frac{hA}{\rho c_p V}\right] t} \tag{1.40}$$

The quantity $\rho c_p V/hA$ is referred to as the time constant of the system because it has the dimensions of time. When $t = \rho c_p V/hA$, it can be observed that the temperature difference $(T(t) - T_a)$ has a value of 36.78% of the initial temperature difference $(T_0 - T_a)$.