Outstanding Contributions to Logic 12

Hans van Ditmarsch
Gabriel Sandu *Editors*

Jaakko Hintikka on Knowledge and Game-Theoretical Semantics

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Gabriel Sandu is professor of philosophy at the University of Helsinki. In the past he was also Director of Research at CNRS and professor of philosophy at Paris 1, Panthéon-Sorbonne. His research is on theories of truth; dependence and independence between quantifiers, and the application of game-theoretical methods to the study formal languages which extend ordinary first-order languages (IF languages). This work, done in collaboration with Jaakko Hintikka, challenges the universalist conception of logic and language according to which one cannot express semantic relations in one and the same language. The most recent focus has been on importing concepts from classical game-theory (Nash equilibria) into logic. The resulting notions of truth and logical consequence have led to Nash equilibrium semantics.

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Chapter 1
Short Overview of the Development of Hintikka's Work in Logic

Gabriel Sandu

Abstract I will present a short overview of Hintikka's main ideas in logic, starting with his early work on constituents and model sets, continuing with his contributions to epistemic logic, up to his later work in game-theoretical semantics and the Interrogative Model of Inquiry.

1.1 Introduction

Throughout his career Hintikka developed and applied two fundamental logical tools that he created during his young years: *constituents* [6] and *model set*s [7]. They share a common feature: both are partial descriptions of a (possible) world in an underlying first-order language. Hintikka's teacher G.H. von Wright was a great source of inspiration and so was the small community of Finnish philosophers at that time, which included Eino Kaila and Erik Stenius. Constituents and distributive normal work were the methodological basis of Hintikka's work in inductive logic. Model sets or *Hintikka set*s as they are now called led to new proofs of completeness for first-oder logic and were integrated later on into the tree (analytic tableaux) method [24, 34]. They became the methodological pillar of Hintikka's later work in *Knowledge and Belief* and Hintikka's own version of the *Picture theory of language*.

1.2 Constituents and Distributive Normal Forms

Hintikka learned about constituents and distributive normal forms from the lectures of his teacher, G.H. von Wright. The lectures took place at the University of Helsinki during 1947–1948. (Some of the details of my presentation are from [30]). We fix a monadic first-order language. From the primitive predicate symbols of the language,

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one can generate mutually exclusive predicates (Q-predicates) in an obvious way. Thus if we assume that the language possesses only two monadic predicates, M_1 and M_2, we get 4 Q-predicates

$$\begin{aligned}Q_1(x) &= M_1(x) \wedge M_2(x)\\ Q_2(x) &= M_1(x) \wedge \neg M_2(x)\\ Q_3(x) &= \neg M_1(x) \wedge M_2(x)\\ Q_4(x) &= \neg M_1(x) \wedge \neg M_2(x).\end{aligned}$$

A *constituent* tells us which Q-predicates are instantiated and which ones are empty in an underlying universe of individuals. Thus the logical form of a constituent (with quantifier depth 1) is:

$$C = \pm \exists x Q_1(x) \wedge \ldots \wedge \pm \exists x Q_4(x).$$

Constituents are mutually exclusive and jointly exhaustive, and each constituent specifies a “possible world”. The disjunction of all constituents is called by von Wright a tautology, which, when presented in this way, is said to be in distributive normal form.

Hintikka, 21 years old, set himself the task to extend distributive normal forms to the entire first-order logic with relation symbols. The project resulted in his doctoral dissertation, *Distributive Normal Forms in the Calculus of Predicates*, [6], where Hintikka showed, among other things, that each formula in first-order logic is equivalent to a disjunction of (canonical) constituents. In the particular case in which the sentence is a consistent generalization (quantificational sentence without individual constants), Hintikka showed that it can be expressed as a finite disjunction of constituents (each generalization has a finite quantificational depth.) Hintikka’s results are better known to the community from [6].

Constituents and distributive normal forms became the methodological pillar of what later on came to be known as Hintikka’s school in inductive logic and philosophy of science, which involved, in addition to Hintikka himself, his students R. Tuomela, R. Hilpinen and I. Niiniluoto. One of the main applications of distributive normal forms was to Carnap’s program in inductive logic. By dividing probabilities among constituents, Hintikka was able to show that universal generalizations have non-zero probabilities in an infinite universe, a result that Hintikka presented at the LMPS Congress in Jerusalem in 1964, and later on in print in [22].

Another application of constituents was to information theory. Hintikka took his probability measures on constituents as the basis for *measures of information*, an idea he explores in [23]. Risto Hilpinen [4, 5] applied constituents to the problem of developing a plausible rule for accepting statements on the basis of their probabilities. Yet another application of constituents was to the problem of theoretical terms [31, 37]. Work in these areas led also to fruitful interaction with P. Suppes, which resulted into two edited books, [22, 23].

1.3 Model Sets

In [7] the author introduced models sets as a new tool in logical semantics, and constructed a new proof of the completeness of first-order logic. A model set is a set of sentences in the relevant logical language which constitutes a partial description of a possible state of affairs.

One starts with a first-order language L and assumes it has an infinite number of individual constants. A model set μ is any set of sentences of L which satisfies some very intuitive closure conditions:

(i) For any atomic sentence A, not both A and $\neg A$ belong to μ
(ii) If $A \wedge B$ belongs to μ, then both A and B belong to μ
(iii) If $A \vee B$ belongs to μ, then either A or B belongs to μ
(iv) If $\neg\neg A$ belongs to μ, then A belongs to μ.

The clauses for negated disjunctions and conjunctions are the duals of the clauses for disjunctions and conjunctions, respectively. More interestingly, model sets rely on a substitutional interpretation of quantifiers:

(v) If μ contains $\forall x A$, it contains $A(x/c)$ for each constant of L which *occurs in* μ.
(vi) If $\exists x A$ belongs to μ, then $A(x/c)$ belongs to μ for at least one constant of L.

The clauses for $\neg\forall x$ and $\neg\exists x$ should be obvious. Identity introduces some complications that will not be our concern here.

Hintikka's model sets share some basic features with other similar semantic systems of that time, Carnap's *state-descriptions* and Quine's *truth-sets*: none of them relies on the (model-theoretic) notions of model and reference, and all of them treat quantifiers substitutionally. We define them shortly following [25] who makes the comparison with Hintikka's model sets straightforward.

A Carnapian state-description for L [1] is any set S of atomic or negated atomic sentences of L which satisfies a stronger version of clause (i):

(i') For any atomic sentence A, either A or $\neg A$ belongs to S (but not both).

One can then define the notion '*A holds in the stated description S* for L':

(a) For an atomic sentence A, A holds in S if A belongs to S;
(b) $\neg B$ holds in S if B does not hold in S;
(c) $B \wedge C$ holds in S if both B and C hold in S; and finally
(d) $\forall x B$ holds in S if $B(c/x)$ holds in S for every individual cinstant c in L.

We obtain the clause for the existential quantifier through the usual definition. A is *logically true* if A holds in every state-description S. A is *logically entailed by a set of sentences Γ* if for every state-description S: if every sentence of Γ holds in S, also A holds in S. [10] analyzes the connection between model sets and Carnapian state-descriptions. Still I find that models-sets are closer in spirit to truth-sets.

A truth-set for L [32], is any set S of sentences of L that satisfies the same conditions for propositional connectives as Hintikka's model sets, except that they are now formulated in the 'if and only if' style:

(a') For an atomic sentence A: $\neg A$ belongs to S iff A does not;
(b') $B \wedge C$ belongs to S iff both B and C belong, etc.
(c') $\forall x B$ belongs to S iff $B(c/x)$ belongs to S for every constant c in L.

The notions 'A is *logically true'* and 'A is *logically entailed by S*' are defined in an obvious way.

It can be easily seen that every truth-set for L is a model set for L and every model set for L is a subset of at least one truth-set for L. Also from a model set one can extract at least one model (in the model-theoretic sense) by a Henkin style technique: the universe of the model consists of the constants occurring in the model set (we assume, for simplicity that L has no identity and no function symbols); the interpretation of a n-place predicate symbol R which occurs in the model set consists of those sequences $(c_1, ..., c_n)$ such that $R(c_1, ..., c_n)$ belongs to the model set; and for those relation symbols P which do not occur in the model set, their interpretation is arbitrary. A straightforward result which is known nowadays as Hintikka's lemma says that every sentence in the model set is true in the corresponding model.

State descriptions and truth-sets are complete and infinite descriptions of a "possible world". Complete, given that for any atomic sentence A, either A or $\neg A$ belongs to the model set; and infinite given that L contains an infinite number of atomic sentences. On the other side, there are model sets μ which are *partial descriptions* of a "world": for some atomic sentence A, neither A nor $\neg A$ belong to μ. And given the clause for the universal quantifier that restricts witnessing instances only to the constants of L which occur in the model set, there are model sets which contain $\forall x A$ and which are finite. Hintikka often emphasizes these two features of model sets.

Although model sets may be finite and partial, they permit the definition of logical truth and logical consequence in a somehow roundabout way [25]. A is *logically true in the model set sense* if $\neg A$ does not belong to any model set for L. And A *is logically entailed by a set of sentences* S if $\neg A$ does not belong to any model set for L of which S is a subset.

[7] proves the following result:

(*) Let S be an arbitrary set of sentences of L. Then S is consistent if and only if S extends to a model set (in an eventually richer language), i.e., there is a model set of which S is a subset.

In view of a result due to Henkin which shows that a set S of sentences is consistent if and only if S has a model (in the model-theoretical sense), Hintikka's result (*) reads: S has a model if and only if S may be extend to a model set. [25].

1.4 Von Wright: An Essay in Modal Logic

With model sets in place, one of the major challenges Hintikka took was to see how this notion and (*) could be generalized to alethic (*it is necessary*, *it is possible*), deontic (*it is obligatory*, *it is permitted*) and epistemic (*the agent knows*, *believes*) modalities. The context of Hintikka's work was provided by C.I. Lewis' and von Wright's work on modal logic.

[26] considered alethic principles like

(a) If necessarily p, and p entails q, then necessarily q

$$\frac{\Box p \quad \Box(p \to q)}{\Box q}$$

(b) Whatever is a logical law is necessary
(c) If it is necessary that p, then it is necessary that it is necessary that p

$$\Box p \to \Box\Box p$$

and investigated various modal systems to deal with them.

Von Wright [38] investigates four groups of modalities:

- alethic modalities (*necessary*, *possible*, *contingent*, *impossible*)
- epistemic modalities (*verified* or *known to be true*, *undecided*, *falsified* or *known to be false*)
- deontic modalities (*obligatory*, *permitted*, *forbidden*, *indifferent*)
- existential modalities (*universal*, *existing*, *empty*).

The starting point of von Wright's investigations was the observation that the formal relations between concepts in one group are analogous to those of the concepts in the other groups. For instance, in the class of deontic modalities, if a proposition is obligatory, then its negation is forbidden. Its counterpart in alethic modalities is 'if a proposition is necessary, then its negation is impossible', which also holds. Von Wright develops his former technique on constituents into a method which decides, together with the truth-tables, whether a modal sentence expresses a "truth of logic" or not. By the latter von Wright means a sentence whose truth depends "upon the specific logical nature of modal concepts" (p. 10), e.g.

$$\Diamond A \wedge \Box(A \to B) \to \Diamond B.$$

Here is an illustration of von Wright's technique for the modal system he calls M_1 which studies M_1—sentences, that is, truth-functional compounds of atomic M_1—sentences and/or atomic N_1—sentences, where:

- Atomic M_1—sentences, are atomic sentences prefixed with $\Diamond$ or truth-functional compounds of atomic sentences, where the compound is prefixed with $\Diamond$

- Atomic N_1—sentences, are atomic sentences prefixed with $\Box$ or truth-functional compounds of atomic sentences, prefixed with $\Box$.

Von Wright shows how the modal principles

(I) If $\Diamond(A \vee B) \leftrightarrow (\Diamond A \vee \Diamond B)$.
(II) If A and B are logically equivalent, then $\Diamond A$ and $\Diamond B$ are logically equivalent (i.e. they have the same truth-values).

provide, in combination with the truth-table method, a decision procedure for each M_1—sentence. It goes like this. Each propositional formula A has a disjunctive normal form, that is, it can be expressed as a disjunction of conjunctions of atomic sentences or their negations. By principle (II), $\Diamond A$ is equivalent to $\Diamond B$ where B is the disjunction normal form of A; and by principle (I), $\Diamond A$ is equivalent to the disjunction of, say, m conjunctions, each prefixed with $\Diamond$. The latter are (modal) constituents. So it seems that the truth-value of each atomic M_1—sentence could be determined from the truth-values of its constituents by the truth-table method, provided that the constituents can appear in the truth-tables in any combination of truth-values (i.e. are independent).

To understand this later requirement, consider the propositional formula $\Diamond(A \vee \neg A)$. The disjunctive normal form of $(A \vee \neg A)$ (when the list of atomic formulas consists only of A) is $(A \vee \neg A)$. Its constituents are $\Diamond A$ and $\Diamond\neg A$. But given that $(A \vee \neg A)$ is a tautology, then it cannot be so, according to von Wright, that both $\Diamond A$ and $\Diamond\neg A$ are false. Thus the following principle is still needed in addition to (I) and (II):

(III) Any propositional formula A is itself possible or its negation is possible.

The impact of (III) should be clear. By (I), $\Diamond(A \vee \neg A)$ is equivalent with $(\Diamond A \vee \Diamond\neg A)$ and by (III) the row in its truth table in which both $\Diamond A$ and $\Diamond\neg A$ are false, is deleted. Then $\Diamond(A \vee \neg A)$ comes out as "logically true in the system M_1".

What about $\Diamond(A \wedge \neg A)$? The disjunctive form of $(A \wedge \neg A)$ is empty, i.e. it is a 0-term disjunctive-sentence. We would like the truth table for $\Diamond(A \wedge \neg A)$ to be always F but we can't get this result from the principles listed so far and the truth-tables. Von Wright adds another principle to his list:

(IV) If a proposition is a tautology, then the proposition that it is necessary is a tautology too.

(IV) ensures that $\Box\neg(A \wedge \neg A)$ is a tautology. But $\Box\neg(A \wedge \neg A)$ is an abbreviation of $\neg\Diamond(A \wedge \neg A)$. By the truth-table method, $\Diamond(A \wedge \neg A)$ is logically false in the system M_1.

A similar method applies to atomic N_1 sentences and then to any M_1 sentence. Finally von Wright shows that these principles combined with the truth-table method shows that $\Diamond A \wedge \Box(A \rightarrow B) \rightarrow \Diamond B$ is a logical truth in the system M_1.

Von Wright (Chap. 4) also constructs a system of epistemic modalities by using epistemic counterparts of the principles (I)–(IV). They are obtained by replacing "possible" by "not falsified" and then by defining the other epistemic modalities in terms of "falsified". Thus A is falsifed, FA, expresses the same proposition as the proposition that the negation of A is verified, $V\neg A$. And A is undecided can be expressed by $\neg VA \wedge \neg V\neg A$ or equivalently by $\neg FA \wedge \neg F\neg A$. Thus from the point of view of "formal behaviour" "the verified corresponds to the necessary, the undecided to the contingent, and the falsified to the impossible."

Von Wright notices the analogy between the alethic "it is true that p but not neccessary that p" which expresses the contingency of p and the epistemic "it is true that p but not known (verified) that p" which expresses the epistemical contingency of p. But he also notices a difference between them:

> Now certainly a proposition may be true without being known to be true. And certainly someone may intelligibly say "it is true that p, though nobody knows it". But if he said "It is true that p, though nobody knows it, not even I" we should feel there was something linguistically wrong. (p. 32)

In his review of [38], Strawson [36] takes this difference to throw doubts on the whole enterprise of epistemic logic: "Facts of this kind may lead us to wonder how far a system of epistemic modalities can contribute to the philosophical elucidation of words like "know"." Later on in *Knowledge and Belief*, [11] offers a solution to this "Moorean" paradox (cf. below.)

Von Wright also deals with combinations of epistemic and existential modalities, that is, quantified epistemic logic. Of these combinations he is interested in epistemic-existential sentences (*de dicto*), e.g. "It is known that something is red", existential-epistemic sentences (*de re*), e.g. "Something is known to be red" and the system which combines both. The first two requires no new governing principles, but the third one requires two new principles (p. 49):

(V) If it is known that everything possesses a certain property, then everything is known to possess that property.
(VI) If there is a thing which is known to possess a certain property, then it is known that something possesses this property.

Von Wright points out that none of these principles is convertible. Later on in *Knowledge and Belief* Hintikka will show, using model sets, that both (V) and (VI) are valid ("sustainable").

The decision method in this case is completely similar to the previous one, i.e. we reduce the original VE-sentence to a truth-function of atomic constituents, the only difference being that the atomic constituents have now the form FC where C is a constituent in a monadic predicate language (see Sect. 1.1), that is, a specification of a possible world built up from disjoint unary predicates of the underlying language and the existential quantifiers or their negations. Skipping over many details, the normal form of the VE-sentence $VEA \vee \neg FUA$ (here EA is an abbreviation of $\exists x A$ and UA of $\forall x A(x)$) turns out to be

$$\neg(\neg F(\neg EA \wedge E\neg A) \vee \neg F(\neg EA \wedge \neg E\neg A)) \vee (\neg F(\neg E\neg A \wedge EA) \vee \neg F(\neg E\neg A \wedge \neg EA))$$

which is a truth-function of the atomic VE-constituents $F(\neg EA \wedge E\neg A)$, $F(\neg EA \wedge \neg E\neg A)$ and $F(\neg E\neg A \wedge EA)$. Thus we can check, by the truth-table method whether this formula is a logical truth or not. The only restriction on the distribution of truth-values (which does not apply to this case), is that if a sentence has a maximal number of VE constituents (the disjunction of the corresponding E-constituents is a tautology), then not all of them can be falsified.

Finally von Wright deals also with "higher-order" modalities (e.g. "it is possible that it is necessary that p") for which he needs a new principle of reduction:

(VII) If it is possible that a certain property is possible, then the property is possible.

Von Wright shows that, if this principle is adopted, then higher-order modal sentences can be shown to be equivalent to truth-functional complexes of of first-order modal properties.

In Appendix II, von Wright investigates various axiomatic systems and compares them to C.I. Lewis's systems.

One interesting point. Von Wright points out that if 'verified' or 'known to be true' refer to the actual knowledge of some particular person, then the counterparts of Lewis' principles may fail.

1.5 Knowledge and Belief

Hintikka did not follow von Wright's methodology but applied his earlier notion of model sets to the investigation of the satisfiability of quantified deontic sentences [9] and that of quantified alethic sentences [22]. For reason of space we cannot deal with these matters here.

In [11] Hintikka's purpose is to extend the notion of model set in order to show that sentences involving knowledge and belief are consistent ("defensible"):

> … we are led to ask how the properties of model sets are affected by the presence of the notions of knowledge and belief; how, in other words, the notion of model set can be generalized in such a way that the consistency (defensibility) of a set of statements remains tantamount to its capacity of being embedded in a model set. What additional conditions are needed when the notions of knowledge and belief are present? (idem, p. 34)

One of Hintikka's main insights was that "In order to show that a given set of sentences is defensible, we have to consider a set of model sets" (idem, p. 35). In other words, model sets must be combined into a modal system so that a model set may have other model sets (state of affairs, possible worlds) in the system that are alternatives to it. Model systems appear for the first time in [8, 9]. In [9] Hintikka tells us that he is following the notes of a manuscript. Apparently these notes were the basis of his seminars at Harvard during 1958–1959 where he uses the model sets technique to obtain completeness proofs for the quantified modal systems M, S4 and S5. The manuscript was never published.

In [11], the author's target are epistemic notions like "the agent a knows that p", $K_a p$, or "it is possible, for all the agent knows that p", $P_a p$, which add new closure conditions on model systems of the form:

(C.K) If $K_a p$ belongs to a model set μ (in a model system Φ), and if μ^* is an alternative to μ (with respect to the agent a) in Φ, then p belongs to μ^*.
(C.¬K) If $\neg K_a p$ belongs to a model set μ, then $P_a \neg p$ belongs to μ.
(C.P) If $P_a p$ belongs to a model set μ, then there is at least one alternative μ^* to μ in Φ such that p belongs to μ etc.

Various constraints are imposed on the alternative relation in order to obtain the desired properties of knowledge and belief. For knowledge, it is required that the alternative relation be at least reflexive and transitive. They lead to further closure conditions like

(C.K*) If $K_a p$ belongs to a model set μ, then p also belongs to μ.
(C.KK*) If $K_a p$ belongs to a model set μ in some model system Φ, and if μ^* is an alternative to μ (with respect to the agent a) in Φ, then $K_a p$ belongs to μ^*.

Let me point out, right from the start, that Hintikka is concerned with *virtual knowledge*, that is, knowledge of cognitively perfect agents who are sufficiently clever to be able to carry out the implications of what they know. Thus e.g. the constraint (C.K*) means that whenever you say "a knows that p", it would be indefensible (inconsistent, irrational) for you to deny on the same occasion, that p. *Knowledge and Belief* contains many indefensibility arguments. The proof of the indefensibility of a statement p is interpreted, in the spirit of the model set technique, as an aborted attempt to describe a state of affairs in which p would be true; and in the same spirit "every proof of the fact that a statement p implies epistemically another statement q is, intuitively speaking, an aborted attempt to describe consistently a state of affairs (with alternatives) in which p would be true but q false." (idem. p. 45). Here is one Hintikka's examples.

We show that "$K_a p \wedge K_a q$" virtually implies "$K_a(p \wedge q)$" by trying to build up a model set in which the former is true and the latter is false:

$K_a p \wedge K_a q \in \mu$ (first assumption)
$\neg K_a(p \wedge q) \in \mu$ (second assumption)
$P_a \neg(p \wedge q) \in \mu$ (second assumption and (C.¬K)
$\neg(p \wedge q) \in \mu^*$ for some alternative μ^* to μ (from second assumption and (C.P))

Skipping over a couple of steps, which lead to $K_a p \in \mu$ and $K_a q \in \mu$, we infer by (C.K):

$$p \in \mu^*$$
$$q \in \mu^*$$

which together with $\neg(p \wedge q) \in \mu^*$ entails a contradiction.

Using this technique Hintikka is able to transform all the modal theorems of C.I Lewis S4 of strict implication into valid principles of epistemic logic. He also gives a solution to some traditional puzzles, like Moore's puzzle of saying and disbelieving.

Finally, he defends his program in epistemic logic against Quine's criticisms of modal logic by showing that substitutivity of identity and existential generalization make sense in modal contexts, provided certain assumptions are fulfilled. Let me shortly say few words about each of these matters.

1.6 Moore's Paradox

In [11] Hintikka discusses Moore's paradox on "saying and disbelieving". He starts by noticing that there is something logically queer about someone asserting

1. p but I do not believe that p

even if it is not self-contradictory (indefensible) according to the criteria he set up. He offers the following explanation of the absurdity of (1).

It is expected from anyone (say b) who asserts the sentence

2. p but a does not believe that p
   "that it is possible for him to believe what he says, that is, it would be defensible for him to say
3. "I believe that the case is at follows: p but a does not believe that p"". (idem p. 52)
   (3) is of the form
4. $B_b(p \wedge \neg B_a p)$
   while (1) is of the form
5. $B_a(p \wedge \neg B_a p)$

Now Hintikka shows that (5), unlike (4), is indefensible in his system. To show this, he follows the usual *reductio ad absurdum* proof, and supposes (5) belongs to a model set. Then using the transitivity of belief, he derives a contradiction (p. 52). Hintikka adds that he has offered a solution to Moore's puzzle which does not invoke any additional principles to the ones he so far introduced. Perhaps a short remark should be added to this. True, Hintikka does not strengthen the *logical* principles that govern knowledge and belief. He does introduce, however, perhaps without noticing, an extra-assumption, which is, as we saw above, a norm of assertion: assert a sentence only if you believe it (i.e. it is defensible).

1.7 Hintikka and Quine's Criticism of Modal Logic

Hintikka's work in epistemic logic went against Quine's arguments to the effect that quantifier rules like existential generalization and substitutivity of identity are misguided in alethic contexts. Hintikka acknowledges that none of these rules holds uniformly in epistemic contexts. That is, one cannot always infer

1. a knows that Dr. Jekyll is a murderer (i.e., $K_a(M(j))$)

from the premises.

2. a knows that Mr. Hyde is a murderer (i.e., $K_a(M(h))$)

and

3. Dr. Jekyll is the same man as Mr. Hyde (i.e. $j = h$).

Neither can one infer

4. $(\exists x)K_a(M(x))$

from (2).

For Quine, the failure of substitutivity in the first example indicates the referential opacity of the position occupied by the term "Mr. Hyde". This feature is also responsible for the impossibility of existential generalization in the second example. Quine's solution was to restrict these rules to referentially transparent contexts.

For Hintikka [11], the failures are not *failures of referentiality*, that is, they are not due, as Quine sometimes seems to suggest, to the way in which our singular terms refer to objects. The source of the failures has to do rather with *multiple referentiality*, that is, with the fact that a has to consider several epistemic alternatives to the current one. In some of these "possible worlds" the proper names "Dr. Jekyll" and "Mr. Hyde" refer to two distinct men (p. 102). For Hintikka substitutivity of identity makes perfectly good sense in epistemic contexts, provided that a knows that Mr. Hyde is the same man as Dr. Jekyll. This, in turn, comes down to the principle that the two names refer to the same individual in all a's epistemic alternatives [1]. In an analogous way, Hintikka goes on, "quantifying in" goes smoothly whenever singular terms like "Mr. Hyde" names the same individual in every relevant epistemic alternative. Hintikka represents the last requirement by '$\exists x K_a(x = h)$' and takes it to be equivalent (in this simple case) to the principle that a *knows who* Mr. Hyde is (p. 112).

1.8 Cross-Identification and "Knowing Who"

In his review of [11], Chisholm [2] points out In his review of points out that Hintikka's proposal to restore existential generalization and substitutivity of identity pushes him towards metaphysics (essentialism). For instance, a knowing who Mr. Hyde is presupposes a method of *cross-identification* on the basis of which one would have to be able to establish when an individual in one world is the same as an individual in another world. Chisholm reviewed several criteria of cross-identifications, including essential properties, but did not find any of them fully acceptable. [3] ended up on a rather sceptical note: if we had a satisfactory answer to the question of

[1]Hintikka's solution is basically the same solution as that given by Kanger much earlier in Kanger (1957b). Kanger was with Hintikka among the first ones to develop a "possible worlds" semantics for modal logic in Kanger (1957a)

knowing who, we would also have criteria to distinguish essential from non-essential properties.

Chisholm criticisms motivated Hintikka to develop methods of cross-identification in the years to come. In [13] he introduces the distinction between *public* and *perspectival identification*. I may have heard of Barack Obama, know who he is (the President of US) but have never seen him. When I finally see him, I identify him perspectivally, that is, I place him on my visual map. Or, I may be in a situation in which I have seen him, but fail to associate him with Barack Obama, i.e. fail to identify him publicly. When this happens I know who Barack Obama is. Hintikka developed the distinction between "two modes of identification" in [16] and applied it, inspired by Kaplan's work, to the logic and semantics of demonstratives in [19].

1.9 Rigid Designation

In the context of alethic modalities, Hintikka's argument for the legitimacy of "quantifying in" whenever a proper name refers to the same individual in all the relevant possible worlds, led him to discuss, later on in his work, Marcus' and Kripke's work on "direct reference" and "rigid designators". Although Hintikka contemplated both the descriptive and the rigid designator accounts of proper names, he did not endorse any of them but ended up defending an intermediate position where his methods of cross-identification (both in alethic and epistemic contexts) do not constitute an abbreviation (sense) of the proper name but combine with the context to identify the referent. Hintikka is not completely clear on these matters and his later work did not bring more light on these issues. In contrast to Hintikka's position, Kripke thinks that the problem of cross-identification does not arise in the context of alethic modalities: possible worlds are postulated, and so are the individuals with whom we populate them.

These matters have been extensively debated and I will not explore them in more details here. But I think the following needs to be said. In the context of model sets and model systems which is, roughly, that of Hintikka's work before 1973, Hintikka's formulation of "a knows who Mr. Hyde is" as $\exists x K_a(x = h)$ does not guarantee, contrary to what Hintikka thinks [11, p. 111], that h refers to one and the same individual in every possible world in which h exists. The only rules governing the impact of $\exists x K_a(x = h)$ on model systems are:

(C.EK=EK=*) If $\exists x K_a(b = x) \in \mu$, and μ^* is an epistemic alternative to μ with respect to a, then $\exists x K_a(b = x) \in \mu^*$.

(C.EK=) If $\exists x K_a(b = x) \in \mu$, then $\exists x(b = x) \in \mu$.

The former condition makes "knowing who" to behave in the same way as knowing that [11, p. 116]. The second conditions tells us that if a knows who b is, then b exists (Hintikka's intrepretation). Hintikka needs these conditions to show the self-sustenance of the principle

$$\exists x K_a A \rightarrow K_a \exists x A$$

that we discussed earlier in connection with von Wright's work.

So let us why none of these rules ensures that 'b' refers to one and the same individual in every possible world in which b exists. For suppose that $\exists x K_a(b = x) \in \mu$, and μ^* is an epistemic alternative to μ. From the two conditions combined, we get that $\exists x(b = x) \in \mu$ and $\exists x(b = x) \in \mu^*$. The most we can get from these conditions, using the model sets technique, is that $b = c \in \mu$ and $b = d \in \mu^*$ for some constants c and d. The two conditions are compatible with both the "descriptive" and "rigid" interpretation of proper names. In other words, the non-referential semantics with its substitutional interpretation of quantifiers the technique of model sets relies on cannot enforce that 'b' refers to one and the same individual in every relevant possible world.

Hintikka came to realize later on that $\exists x K_a(x = h)$ and the substitutional interpretation of quantifiers it relies on cannot ensure that 'h' refers to one and the same individual in every possible world in which h exists. Or so I would like to think. For instance, in [21] Hintikka and Sandu claim that when quantifiers are interpreted objectually, then $\exists x K_a(b = x)$ and $\exists x\Box(b = x)$ express that 'b' is a "rigid designation" in epistemic and alethic contexts, respectively (p. 181; the references are to [19]). They also argue that this effect cannot be accomplished with substitutionally interpreted quantifiers and even present some arguments against the latter (p. 184) on independent grounds connected with partially ordered quantifiers and Independence-Friendly logic (But see also the next section).

1.10 Model Sets and the Picture Theory of Language

Hintikka's result (*) on model sets mentioned earlier [7] formed the basis of Hintikka's own conception on the picture theory of language that he develops in details in Chap. 2 ("Quantification and the Picture Theory of Language") of [14]. Roughly, model sets can now serve as pictures in Wittgenstein's sense of the word. Hintikka highly appreciated Stenius' interpretation of the picture theory in the *Tractatus*, explored in [35]. An important ingredient in Stenius' account of the picture theory is that of a *key of interpretation*, that is, a function which maps the individual constants and predicate symbols of a given language to possible individuals and properties of appropriate arities in the logical space. Given a key of interpretation, each atomic sentence functions as a picture of a (possible) fact or state of affairs that is isomorphic to it.

Hintikka [14] retains the notion of key of interpretation but associates the state of affairs generated by it for a given atomic sentence with its truth-conditions. Suppose the only atomic sentences occurring in a model set are, say $a_1 R b_1$ and $a_1 R b_2$. We can choose a key of interpretation which maps 'a_1' to a_1, 'b_1' to b_1, 'b_2' to b_2 and the relation symbol 'R' to the relation Q which holds between a_1 and b_1, and between a_1 and b_2. We have thus formed a model (in the model-theoretical sense) with universe a_1, b_1 and b_2 and with the interpretation function being the key of interpretation. The atomic sentence '$a_1 R b_1$' is then defined to be true in the model if and only the individuals assigned to 'a_1' and 'b_1' by the interpretation function stand in the relation assigned to 'R' if and only if a_1 stand in the relation Q to b_1; etc. This is exactly the basic ideas behind the technicalities in the so-called Hintikka's Lemma. The problem is now to extend this account to compound sentences. Hintikka thought that model sets offer him a way to accomplish this, an idea he explores in [14].

Consider a universally quantified sentence. If we manage to embed it into a model set, then we reduce it to its substitutional instances, which, at the end of the process reduce to atomic sentences, which are pictures in the former sense. As emphasized in [14, p. 47], on this account, quantified sentences are not, strictly speaking, pictures, they are *recipes* for constructing pictures. We are told that "... what most immediately corresponds to reality of which quantificational sentences speak are the outcomes of model sets construction which are often obtained only by a long and complicated process" [14, p. 51].

Gradually Hintikka came to be aware of the fact that one still needs to compare model sets and pictures with the world (see J. Acero's paper in this volume). Model sets are often very complicated and the models obtained from model sets consist, after all, of syntactic material, as illustrated in our toy example. The substitutional interpretation of quantifiers makes the underlying "language games" associated with them "indoor games" as Hintikka sometimes call them. In [14] a change of perspective takes place: he moves from model sets to "outdoor games", that is, *semantical games* and *game-theoretical semantics*: they are now the link which mediate between sentences and the world through the activities of seeking and finding individuals in the world (This point is nicely illustrated in Acero's paper.)

1.11 The Interrogative Model of Inquiry

Hintikka's work on epistemic logic turned out to be highly stimulating in logic, philosophy and AI. In the "second generation" of epistemic logic, a "social" dimension was added (multi-agent epistemic logic) which led to such notions as *distributive knowledge* and *common knowledge*; and in a "third generation" a "dynamic" aspect was added on top of the previous two, which stimulated the development of epistemic foundations of game theory, Dynamic Epistemic Logic and the work of the Amsterdam school.

Hintikka and his collaborators started to develop their own version of "dynamic logic" in the early 1980. But Hintikka's "dynamic logic" targeted different phenomena than the ones I just mentioned. Hintikka's *Interrogative Model of Inquiry (IMI)* integrated Hintikka's earlier work on epistemic logic with the semantics of questions and presuppositions in an all-embracing system of reasoning and argumentation, [19]. He often liked to present *IMI* in the form of a game played by an idealized scientist, the Inquirer, against Nature (the subject -matter under investigation). The game is played on a fixed model (the universe) which is thought to encode our actual world or some part of it. The Inquirer has some background knowledge, encoded in a theory T, and his goal is to solve a given problem C. At each stage the Inquirer has a choice to make between a logical move, that is, a deduction that he makes from what he knows so far, and an interrogative move, that is, a question he puts to Nature. "Question" is just a technical term here standing for any new observation or measurement the Inquirer might make. He adds the received "answers" to his background theory T. At the end of the day the Inquirer is supposed to establish whether C or its negation follows from the theory T and the set of received answers. Epistemic logic has a crucial role to play here given the requirement that answers must be known (or believed with a certain probability). By making certain assumptions on the ingredients of the model, Hintikka was able to analyze certain crucial concepts in philosophy of science (explanation, induction, etc.), although some of the issues remain controversial.

The significance of Hintikka's work was recognized by the Swedish Royal Academy of Sciences which in 2005 awarded Hintikka the Rolf Schock Prize in logic and philosophy "for his pioneering contributions to the logical analysis of modal concepts, in particular the concepts of knowledge and belief".

1.12 Game-Theoretical Semantics

Another direction in which Hintikka's earlier work on quantifiers took him is *Game-theoretical semantics* (GTS). I mentioned earlier that GTS was developed as a revolt against the model set view of analyzing quantifiers which is merely syntactical. In [14] he moves to *game-theoretical semantics* which illustrate for him the need to mediate the interpretation of quantifiers through activities of seeking and finding individuals in the world. Some of the main ideas appear for the first time in [12, 14]. They are fully developed in [15]. Here too, against the stream, Hintikka builds up a systematic programme for the treatment of quantifiers in natural language as an alternative to [33] view of quantification theory as the "canonical notation" of all scientific discourse, and against Montague's treatment of quantifiers in [29].

Hintikka's semantical games for first-order languages are well known. I will not review them here. A semantical game for a first-order sentence A is played by two players, Myself and Nature, on a model M which interpretes the nonlogical symbols of A. Truth (falsity) in M is defined as the existence of a winning strategy for Myself (Nature). Hintikka observes that this definition of truth is equivalent to the standard model-theoretical notion of truth, but notices its heuristic, linguistic and philosophical advantages, which include, among other things, a game-theoretical analysis of natural language quantifiers and pronouns, an illustration of Wittgenstein's ideas of language games, etc.

In [15], Hintikka, inspired by Henkin's work on branching quantifiers, and against Quine's *first-order thesis,* gives examples of natural language sentences which, in his opinion, require a greater expressive power than ordinary first-order logic. The idea behind branching quantifiers is that they can express certain patterns of dependence and independence of quantifiers which cannot be expressed in ordinary, first-order logic. One such pattern is

- For every x for every y there is a z which depends only on x and there is a w which depends only on z

rendered by Henkin as the branching prefix

$$\begin{Bmatrix} \forall x & \exists z \\ \forall y & \exists w \end{Bmatrix}$$

The branching form is intended to indicate that $\exists z$ depends only on $\forall x$ and $\exists w$ depends only on $\forall y$. The intended interpretation is taken care of nicely by the game-theoretical interpretation which is now extended to cover this new patterns of dependence and independnece in terms of games of imperfect information. In this game, when choosing a value for z Myself knows only the value of x and when choosing a value for w he knows only the value of y. Hintikka gives the following example of a natural language sentence which exemplifies this pattern:

1. Every writer likes a book of his almost as much as every critic dislikes some book he has reviewed.

The interpretation of this kind of examples has been longly debated.

Hintikka continued to develop these ideas during the late 1970s and 1980s, producing with his collaborators extensive research on the analysis of pronouns, conditionals, definite descriptions and intentional phenomena. Hintikka's work on branching quantifiers led to IF (independence-friendly) logic, a logical system introduced with Sandu in [20]. It has been the main focus of Hintikka's efforts during the last 20 years. In [17] he argues that IF logic is the right logic for the foundations of mathematics and for the logical representation of natural language. During the last years of his life he devoted much of his work to show how an extension of IF logic with probabilities constitutes "the true logic of experiments in quantum theory". Hintikka's work in this area led to several logics of dependence and independence with applications to quantum theory and social choice theory. He was often afraid of running out of time when developing this programme. He presented his last thoughts on this topic in a session on the Philosophy of Physical Sciences that he chaired on 7th of August 2015 at the Congress for Logic, Methodology and Philosophy of Science in Helsinki.

Hintikka mentioned several times that he had only one true teacher: G.H. von Wright. Once, in a meeting in Paris I heard G.H. von Wright saying that Hintikka was his only true student. Hintikka himself had many students. I think that this is due to Hintikka's generosity. He liked to share his ideas with his students in order to develop them jointly. He did the same with his colleagues, providing us constantly with "food for thought".

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Gabriel Sandu is professor of philosophy at the University of Helsinki. In the past he was also Director of Research at CNRS and professor of philosophy at Paris 1, Panthéon-Sorbonne. His research is on theories of truth; dependence and independence between quantifiers, and the application of game-theoretical methods to the study formal languages which extend ordinary first-order languages (IF languages). This work, done in collaboration with Jaakko Hintikka, challenges the universalist conception of logic and language according to which one cannot express semantic relations in one and the same language. The most recent focus has been on importing concepts from classical game-theory (Nash equilibria) into logic. The resulting notions of truth and logical consequence have led to Nash equilibrium semantics.

Chapter 2
From Pictures to Semantical Games: Hintikka's Journey Through Semantic Representationalism

Juan José Acero

Abstract This essays examines Hintikka's trajectory through Semantic Representationalism from the classical, i.e. Wittgensteinian, Picture Theory of Meaning to Game-Theoretical Semantics. It starts by asking what makes a sentence a representation of a fact and what conditions enable a sentence to represent something. It is argued that at the end of his journey Hintikka conceives of the representational function of sentences as arising from the norms that regulate their place in verification practices. As a consequence of it, the analysis of semantic representation in terms of an isomorphic relation between sentences and facts is replaced by a theory based on the concept of winning strategy in a semantical game. This maneuver allows Hintikka to emphasize that semantical representation is shaped by normative constraints that rule the specific activities involved in those games.

Semantical Representationalism takes language's main function to be that of allowing their users to say how things are, to represent them as having certain properties and holding certain relations; and to describe facts as well as states of affairs and situations. Of course, there may be other functions, but all of them depend in the last analysis on the representational role of language. In terms of philosophical semantics, Hintikka not only favors Semantical Representationalism, but he has also led it to its most pioneering positions. In this respect, the initial chapters of his book *Logic, Language-Games and Information* [8] present one of the most compelling journeys within recent philosophy of language and philosophy of logic. Hintikka sets off from a country in which Semantical Representationalism exhibits well-known features, and he plunges deeply into unexplored territory. The starting point of his journey is the Picture Theory of Meaning that Wittgenstein sketched in his *Tractatus Logic-Philosophicus* [28], and the end of Hintikka's journey is Game-Theoretical Semantics. Although both theories lie in Semantic Representationalism's orbit, they differ significantly. Ostensibly, those differences would derive from Hintikka's insight into the relevance of game theory's concepts and techniques for semantical analysis. This

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is not the whole truth, because what creates such a vast distance between the Picture Theory of Meaning and Game-Theoretical Semantics is the philosophical insights that those views respectively articulate.

Hintikka's first steps in [8] take him from conceiving representations as *pictures*—that is, as based on isomorphic relations between sentences or propositions and facts—to conceiving them as truths in a model. In a second phase, the concept of truth in a model gives way to that of belonging to a model set or, alternatively, being embeddable in a model set. Models sets are partial descriptions of possible worlds. This feature entitles them to play the role which the pictures introduced by Wittgenstein in the *Tractatus* had. Sentences are pictures in a derivative way, namely, insofar as they can be embedded in model sets. Now, there is a solid reason to prefer Hintikka's Picture Theory of Meaning over the classical version, namely its explanatory power. The range of sentences on whose semantical properties the theory casts light is broader than the range covered by the classical version. The latter suffers from two limitations, both of them worth being taken seriously. The idea of a comparison between language and reality, if not built into Semantical Representationalism, is continuous with it, because the ability to negotiate representations, i.e. to generate and to interpret them, involves the power to control their mutual adjustment, and this in turn requires the ability to compare such representations to what they stand for. However, comparisons cannot often be made straightforwardly, at a glance—a requirement Hintikka systematically recognizes. His own variety of the Picture Theory of Meaning can be made suitable for overcoming such a disadvantage by turning it into a system of principles that rule the comparison processes. On the other hand (and this is its second and fatal weakness), even after having been adapted to turn the comparison between language and reality into a step-by-step processes, Hintikka's Picture Theory of Meaning offers those steps as indoor activities. To determine whether a model set fits a possible world (or a part or aspect of it), it is necessary to follow a step-by-step process which is sensitive to the rest of the sentences in the model set. Thus conceived, nothing requires either to set up the right language-to-world relations or to certify that they work appropriately. No matter which specific form is given to Semantical Representationalism, this view gives full credit to the idea that language-to-world comparisons must take place outdoors. Therefore, Hintikka's Picture Theory of Meaning cannot be the final word in the search for a fully satisfactory alternative to the classical, Tractarian, view. In [8] Hintikka argues that Game-Theoretical Semantics is in a much better position to constitute such an alternative. According to this theory, a sentence is a picture if it can be verified in a semantical game. On the one hand, semantical games are outdoor processes that set the comparison between language and reality in a dynamic framework that calls for rule-governed activities. On the other hand, in principle no kind of sentence is beyond such a dynamic framework. Accordingly, Game-Theoretical Semantics is the culmination of the journey which started with the classical Picture Theory of Meaning and went through Hintikka's variety both in the static and the dynamic formats.

This essay analyzes Hintikka's journey in its first part (Sects. 2.1 and 2.2). Two questions guide the analysis. The first one is the What-Is question: What is a