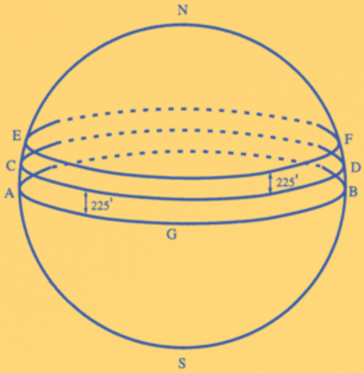
Culture and History of
Mathematics 5

Studies in the History of Indian Mathematics

C. S. Seshadri
Editor



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	१	२	३	४	७	१२	२१	३४	५४
१	१	१	२	३	५	७	१०	१४	२१
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CULTURE AND HISTORY OF MATHEMATICS 5

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CULTURE AND HISTORY OF MATHEMATICS

Editor

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Contents

Editorial Note

It gives me immense satisfaction that we could assemble a group of distinguished scholars at this Institute to conduct a seminar on the history of mathematics in India, leading to the publication of this volume. This whole endeavour owes very much to the initiative and commitment of David Mumford. I am very glad that this provided also an occasion for having amidst us the renowned indologist Frits Staal.

It is fitting that this volume is dedicated to the memory of the two outstanding scholars David Pingree and K.V. Sarma who have made pioneering contributions to this field.

It is my pleasant duty to thank all those who have contributed to this volume, as well as to the one invisible contributor Jayant Shah (Northeastern University, Boston) for his critical reading of all the material in this volume.

C.S. Seshadri
Director
Chennai Mathematical Institute

Preface

It was a great privilege to participate in the Seminar *Topics in the History of Indian and Western Mathematics* which took place at the Chennai Mathematical Institute in January and February 2008. I am a newcomer in this area and cannot read Sanskrit, but I had the opportunity to listen and learn from Sanskrit scholars, mostly Indian but one Western (Staal) and absorb their various perspectives. It has been very exciting to me over the last five years to learn something of the distinctive Indian approach to mathematics, from its beginnings in Vedic times to its wonderful achievements in algebra and the Calculus, just prior to the waves of Western invasions starting in the 16th century.

Today, there is a major resurgence of scholarship in Indian mathematics and astronomy both in India and the West, led, on the one hand, by a widespread wave of renewed interest in India in Sanskrit studies and on the other hand, by the school created by my late colleague David Pingree. Until recently, nearly all articles and books on the History of Indian Mathematics and Astronomy were nearly impossible to find in the West. But now, new translations and critical editions with commentary of many of the extant manuscripts are being published and widely distributed in the West as well as in India.

This volume contains analyses of many of the most important topics in Indian Mathematics and Astronomy, taken from talks at this seminar. Let me sketch some of the ideas from each chapter, highlighting topics which seemed to me especially significant. I will follow a roughly chronological order.

Dani's chapter deals with the oldest extant Indian mathematical works, the *Śulbasūtras*, or "Rules of the Cord", which date from as early as c.800 BCE. These are manuals on the geometry needed for erecting the fire altars central to Vedic ritual. Dani describes in detail their techniques of geometric algebra, their construction for a square whose area is the sum of those of two given squares or whose area is the same as that of a given rectangle. These anticipate many of the constructions which appear later in Book II of Euclid's *Elements*, with knotted cords replacing the Greek's use of straight edge and compass. In particular, "Pythagoras's" theorem is described here well before Pythagoras. (But note that it also occurs as early as c.1800 BCE on Babylonian tablets.) Dani then goes on to describe their approximate constructions of circles whose area is that of a given square and vice versa. His last section deals with the rational approximation given for the square root of two given in three of the four *Śulbasūtras*. Like the Babylonian approximation, it is accurate to roughly one part in a million. Dani describes a striking geometric method, first proposed by Datta, by which this approximation might have been found. But he also proposes that, since they were using very long ropes to actually lay out fire altars, it is not impossible that they could have carried out a continued fraction-like algorithm using two ropes and repeatedly taking multiples of the shorter away from the longer. The idea that such algorithms have a very long history connects to Dutta's article.

Staal—the other Westerner besides myself in this seminar—considers the origins of the number zero and proposes a link of the mathematical zero with the linguistic zero markers which were invented by the great grammarian Pāṇini. The foundations of modern computer science as well as linguistics go back to Pāṇini (c.400 BCE). Pāṇini invented formal grammar with abstract variables for various parts of an utterance and recursive rewrite rules. Sanskrit grammar was well-known to all the mathematician–astronomers and is a plausible source for many ideas which were later developed in more mathematical ways. Staal focuses on the Sanskrit word *lopa*, "something that does not appear", as a precursor to the idea of zero. As he has written in other works, Sanskrit grammar appears to grow out of the precisely formalized Vedic rituals. Astonishingly, the Vedas dealt with possible enactments of a ritual in which one participant fails to utter some required sentence. The verbal root *lup-* of *lopa* is used to describe this failure: a zero in the ritual. Thus the original zero could be the priest's lapse.

Raja Sridharan, R. Sridharan and M. D. Srinivas's chapter concerns another area of Indian mathematics: the combinatorics which was inspired first by the study of Sanskrit prosody and later by the study of musical patterns, both tonal and rhythmic. Pāṇini was followed by Piṅgaḷa (c.300 BCE) who studied Sanskrit prosody. Sanskrit was traditionally written in verse and memorized. Each line had a characteristic pattern of short and long syllables. Piṅgaḷa devised a way to order all patterns of short and long syllables in a line with n syllables, and, using this, (i) to compute the number describing each pattern, (ii) to reconstruct the pattern from its number and finally (iii) to compute the number of patterns with a fixed number of short, respectively, long syllables. The first and second use the binary number system and the third involves calculating the binomial coefficients. This beautiful foundation led to much further combinatorial work which the authors survey. For instance the Fibonacci numbers arose when they asked how many sequences have a given total length if long syllables are given length 2 and short length 1. Virahāṅka discovered the Fibonacci numbers and their recursion relation in the 7th century (well before Fibonacci!). Recursion seems to be a general theme which runs through much of Indian mathematics. Most of their chapter concerns the generalization to musical phrases and to musical rhythms, where the combinatorics gets more complex and more interesting. This story is beautifully described by the authors.

My own chapter concerns the introduction of negative numbers both in India, in China, in Greece and in modern Europe. The full arithmetic of negative numbers appears in Brahmagupta's *Brāhma-sphuṭa-siddhānta* in the 6th century and presumably arose much earlier, maybe even in the accounting practices described in Kauṭilya's *Arthaśāstra*. In contrast, the first place where this is correctly described without hesitation[1] in modern Europe is in Wallis's *Treatise on Algebra* in 1685.

[1]Other Westerners wondered whether $(-1)\cdot(-1)$ might be taken to equal -1.

This seems to me a stunning example of how far two mathematical traditions can diverge—though converging in the end.

Dutta's contribution concerns Indian arithmetic and algebra. More specifically, it concerns three algorithms. The first, called *kuṭṭaka* (the pulverizer) constructs, for all positive integers a, b, positive integers x, y such that $ax - by$ equals plus or minus the greatest common divisor c of a and b. The first half of the algorithm is the same as the Euclidean algorithm, successive subtraction of the less from the greater, and the second half works backwards to find x and y—just as we do today. This second step was not taken by the Greeks. Kuṭṭaka appears first in rather cryptic form in the *Āryabhaṭīya* (499 CE) and soon after more explicitly in the works of Bhāskara I and Brahmagupta. They use it to construct all positive integral solutions of linear equations $ax - by = d$, a, b, d of positive integers. This had long been a concern because of the ancient Vedic method for making sense of the relative periods of the day, the lunar month, the year and the periods in planetary motion: that at the beginning of the present Kaliyuga, all the planets, the sun and moon were all lined up in one spectacular conjunction.

The second and third algorithms are concerned with "Pell's" equation $x^2 - Dy^2 = 1$ which quite plausibly arose in the search for good rational approximations x/y to $\sqrt{D}$. The second algorithm, due to Brahmagupta and called the *Bhāvāna*, is equivalent, in modern terms, to the law for multiplication of the algebraic integers $x + y\sqrt{D}$. As they put it, if $x^2 - Dy^2 = m$, $u^2 - Dv^2 = n$, then $s = xu + Dyv$, $t = xv + yu$ solves $s^2 - Dt^2 = mn$. They now played with solutions of the equations $x^2 - Dy^2 = m$ for various m and several centuries later, they did indeed find an algorithm which always finds solutions with $m = 1$, the *Cakravāḷa*. Thus they had a complete theory of Pell's equation, modulo one point—a *proof* that this worked. It is interesting that the standard proofs in modern texts are non-constructive whereas the Indian mathematicians focused instead on seeking on constructive methods and never studied non-constructive arguments. Their idea of mathematics was closer to that of applied mathematicians and computer scientists than that of pure mathematicians. Indeed the *siddhāntas* (treatises) where these algorithms were written down were manuals for actually calculating astronomical events.

Ramasubramanian and Srinivas take up the story of Indian work on Calculus. I personally feel this is a story which deserves to be much more widely known in the West. Their chapter outlines the millennium long history of these discoveries, culminating in the complete analysis of the basic calculus of polynomial and trigonometric functions, their integrals and derivatives and power series for sine, cosine and arctangent—and, of course, applications of these to astronomy. The path they took to this is quite disjoint from the path that was taken, first by Archimedes and later by Newton and Leibniz. It is extremely unlikely that any of Archimedes' work on, e.g. the Riemann sum for the integral of sine, made its way to India. Instead the Indian work seems to have taken off with their discovery of the second order finite difference equation for sine in the 5th century CE or earlier (the discrete analog of the result that sine solves the harmonic equation $y'' + y = 0$.) This appears cryptically in the *Aryabhaṭīya*, and clearly soon after, as is well described in Section 5 of Ramasubramanian and Srinivas's chapter. Also, early was an interest in summing powers of natural numbers which is described in Section 4. This early work revolves around finite differences and the corresponding sums, much like Leibniz's starting point for calculus. But the application to astronomy made it clear by the 10th century that one needed to calculate the "instantaneous velocity" as well as its finite difference approximation (see Section 6). In the 12th century, Bhāskara II used these ideas to rediscover Archimedes' derivation of the formula for the area and volume of a sphere. This derivation reduces the problem to computing the integral of sine. But, interestingly, as an applied mathematician, he completes the proof by numerically summing his sine table—even though he knows quite well that cosine differences are sines and could have done it this way. The numerical method was apparently more convincing!

The crowning achievements of this work are due to a major genius who is nearly unknown in the West: *Mādhava*, who lived in a village in Kerala in the 14th century. Only a small fragment of his work survives, but, fortunately, his and his school's work was written up in an unusual expository form in the local language, Malayalam, by Jyeṣṭhadeva in the 16th century—still over a century before Newton and Leibniz did their work. This book, the *Yukti-bhāṣā* has only now been translated into English by K. V. Śarma with commentary by the authors of this chapter and M. S. Sriram. The first volume was released with some ceremony during our seminar. The work of the Kerala school is described in Part 2 of the present chapter. Let me only mention that in addition to deriving the power series for sine and cosine and the 'Gregory' series

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

their numerical bent appears again in finding a series of ways to estimate the remainder so that this becomes a practical tool for calculating π. This led them to much more rapidly converging series such as:

$$\frac{\pi}{4} = \frac{3}{4} + \frac{1}{3^3 - 3} - \frac{1}{5^3 - 5} + \frac{1}{7^3 - 7} - \cdots$$

$$\frac{\pi}{16} = \frac{1}{1^5 + 4 \cdot 1} - \frac{1}{3^5 + 4 \cdot 3} + \frac{1}{5^5 + 4 \cdot 5} - \cdots$$

Divakaran's article proposes the compelling thesis that recursion is the central theme and technique that runs through all the Indian work in mathematics. He traces this to Pāṇini's grammatical rules and even earlier to Vedic ritual. He points out that the whole idea of decimal place-value notation can be seen as a way to describe integers recursively. Recursive generation of larger and larger numbers leads to a clear conception of infinitely large and its inverse, to a conception of the infinitely small and of the limiting process. The bulk of his article, however, focuses on the extensive use of recursion in the Kerala work on Calculus. Like Newton, he sees the introduction of power series as an algebraic analog of decimal expansions (x^n being the analog 10^n) and he discusses at length the power series expansion

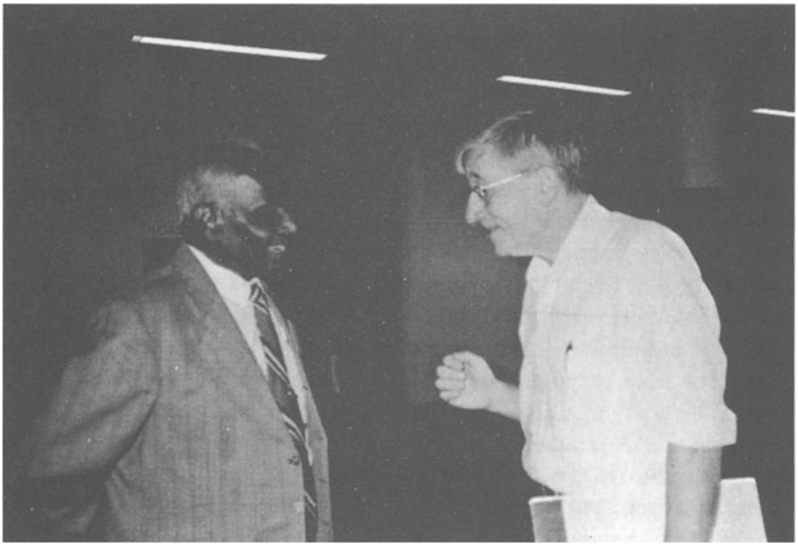
$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$$

found in the *Yukti-bhāṣā*. He then describes the huge step which one finds in the *Yukti-bhāṣā*: recursive proofs based on induction: they derive the integral of x^n using induction on n. Finally, he discusses the derivation of the power series for sine and cosine in the *Yukti-bhāṣā* and notes that it is derived by first converting the known difference equations for sine in terms of cosine and cosine in terms of sine into a summation form. This is the exact analog of converting differential equations into integral equations. Jyeṣṭhadeva, starting with a crude approximation, then recursively back-substitutes each approximation into the summation form to get a better one. Passing to the limit, he gets the power series. This back substitution is exactly what we do today to solve Fredholm integral equations.

Finally, M. S. Sriram's Chapter addresses the planetary models in Indian Astronomy. As mentioned above, mathematics was usually studied together with astronomy and the two subjects advanced hand in hand. His article concentrates on their last model which is due to Nīlakaṇṭha c.1500 CE, who worked at roughly the same time as Copernicus in the West. His model is explained at length in the *Gaṇita-yukti-bhāṣā* and is described with modern formulas in Sriram's chapter. The most interesting issue is here is the historical movement from geocentric models to heliocentric models. In fact, Nīlakaṇṭha's model is 'essentially' heliocentric. In his model, one starts with the mean sun moving in the ecliptic circle. Then for each planet, one takes a plane centered at the mean sun but inclined to it to different amounts for each planet and intersecting it at the appropriate nodes. And in each of these planes one places the corresponding planet on the Ptolemaic approximation to the ellipse: an eccentric circle. Once one overcomes the dense tangle of compound Sanskrit words, a very modern model shines through.

These articles together cover a substantial portion of the exciting History of Indian Mathematics and Astronomy. Only a fraction of this has become generally known to mathematicians in the West. Too many people still think that mathematics was born in Greece and more or less slumbered until the Renaissance. Therefore, I hope that this book may serve as a way of bringing to the international mathematical community a deeper knowledge of the riches in Indian Mathematics. To scholars, however, there is another message: there is much work still to be done to study, edit and translate the many ancient manuscripts still surviving in libraries all over India. One hopes for a deeper and broader picture of the more than two millennium long history when every one of these has been looked at and analyzed.

David Mumford



David Pingree, 1933–2005

Kim Plofker

David Edwin Pingree (2 January 1933–11 November 2005) employed the fifty years of his scholarly career investigating the development of mathematics, astronomy and the related exact sciences from ancient Mesopotamia to early modern Europe and India. He published editions, translations and studies of source texts in Akkadian cuneiform, Greek, Latin, Sanskrit, Arabic and Persian, on subjects ranging from infinite series and interpolation techniques to astral magic and iconography in astrological texts. He was professionally affiliated with Harvard University as an undergraduate (B.A. in Classics and Sanskrit, 1954), graduate student (Ph.D. in Sanskrit and Indian Studies, 1960), and Junior Fellow (to 1963); the University of Chicago as a faculty member in the Oriental Institute and Departments of History, South Asian Languages, and Near Eastern Languages (1963–1971); and Brown University as a professor in the Departments of Classics and the History of Mathematics (1971–2005). Over the course of his immensely productive career he received many honors, including a Fulbright Scholarship, a Guggenheim Fellowship, and membership in several learned societies, among them the American Academy of Arts and Sciences, the American Philosophical Society, and the Institute for Advanced Study at Princeton University. Pingree was awarded the title of "Abhinavavarāhamihira" by the government of Uttar Pradesh in 1979, and in 1981 was one of the first recipients of the MacArthur Fellowship (popularly nicknamed the "Genius Grant"), together with co-honorees including the philosopher Richard Rorty, the paleontologist Stephen Jay Gould, and the computer scientist Stephen Wolfram. His total scholarly output comprised several dozen monographs and several hundred research articles, reviews, encyclopedia entries, and other works.

In his research on the history of science, David Pingree was first and foremost what might be termed a "transmissionist": he was primarily interested in what he called the "kinematics" of scientific development, the ways that scientific ideas were passed from one culture to another, and how they were transformed in the process. The first awakening of his interest in the history of science as a field of research was due, as he described it, to just such a question of transmission from the medieval Indian exact sciences to their Greek counterpart. As a Fulbright scholar in 1955, reading a Byzantine Greek manuscript on astrology in the Vatican Library, he noticed marginal notes by a commentator that included technical terms transliterated from Sanskrit. This discovery eventually inspired his important studies of the influence of Indian astrology on Arabic and Persian texts and consequently on the Greek astral science tradition of Byzantium. Pingree also explored the mirror image of this transmission in an earlier era, tracing the assimilation of Hellenistic horoscopic astrology into the Sanskrit discipline of *yavana-jātaka* via the pre-Gupta Indo-Greek kingdoms in western India. This work, which became his doctoral dissertation and his first monograph, led directly to his prolonged collaboration with the great Assyriologist historian of science Otto Neugebauer, and to his study of Akkadian, Arabic and Persian in order to track similar exchanges of scientific knowledge through the entire four-thousand-year history of the great Eurasian development of the exact sciences. (Even erudition as deep as Pingree's was forced to submit to some limitations in this quest, however: to the end of his life he remained somewhat regretful, and slightly apologetic, that he had never found time to undertake learning Chinese.)

Pingree's hypotheses about cross-cultural transmission of scientific ideas were frequently groundbreaking and sometimes controversial. A few of them were tentative speculations that never attained wide acceptance, such as his suggestion that the structure of Bhāskara II's table of easy Sines (*laghu-jyā*) might reflect influence from Islamic trigonometry. Some were plausible inferences unproven by conclusive textual evidence that were accepted by most mainstream historians but strongly resisted by some others, such as his attribution of parts of the Sanskrit *Jyotiṣa-vedāṅga*/*Vedāṅga-jyotiṣa* to Babylonian mathematical astronomy techniques in the Achaemenid period or his derivation of Āryabhaṭa's planetary mean motions around 500 CE from a hypothesized Greek source. Some were preliminary conclusions based on little-known textual sources that remain to be explored more fully, such as his deductions about the relationship of Mesopotamian and ancient Indian astral omen literature or about the recognition of Islamic optics in seventeenth-century Sanskrit astronomy. But most of his discoveries concerning scientific transmission, including his study of the Indian adoption of Hellenistic astrology, his exposition of the dependence of medieval Spanish astronomy on Indian sources, and his research on the reactions to Latin heliocentrism in eighteenth-century Rajasthan, were solidly established by masterly textual scholarship and have substantially transformed the standard narrative of scientific development.

In light of the above-mentioned controversies, Pingree has sometimes been rashly relegated (by those acquainted with only a few isolated fragments of his work) to the company of reactionary Orientalists like the nineteenth-century John Bentley and G. R. Kaye who took it for granted that original scientific discoveries were by default, Greek, and that treatises in, say, Arabic or Sanskrit or Chinese must represent mere borrowings and imitations. Pingree himself, however, was far from sharing this ill-informed view, which he scornfully dismissed as the vice of "Hellenophilia" unworthy of a serious historian:

> Hellenophiles, it might be observed, are overwhelmingly Westerners, displaying the cultural myopia common in all cultures of the world but, as well, the arrogance

> that characterized the medieval Christian's recognition of his own infallibility and that has now been inherited by our modern priests of science... If it is evident that for a historian the proposition that the Greeks invented science must be rejected, it necessarily follows that they did not discover a unique scientific method... Babylonian and Indian mathematics are frequently criticized for relying not on proofs but on demonstrations. But without axioms and without proofs Indian mathematicians solved indeterminate equations of the second degree and discovered the infinite power series for trigonometrical functions centuries before European mathematicians independently reached similar results... Those who deny the validity of alternative scientific methods must somehow explain how equivalent scientific "truths" can be arrived at without Greek methods. And in their denial they clearly deprive themselves of an opportunity to understand science more deeply. (Pingree, "Hellenophilia versus the history of science", *Isis* 83.4 (1992), 554–563)

It should be noted that the "infinite power series" referred to above were familiar to Pingree largely through the Sanskrit editions of the Kerala-school treatises published by his brilliant and indefatigable colleague K. V. Sarma. The historiographic approach that they, and the studies in the present work, espouse–namely, the careful exploration of the intellectual content of a scientific tradition within its own cultural context and in its encounters with other cultures–is what David Pingree recognized as the true vocation of a historian of science.

Department of Mathematics, Union College, Schenectady NY, USA.

K. V. Sarma (1919–2005)[1]

M. S. Sriram

Born at Chengannur in Kerala on 27th December 1919, Krishna Venkateswara Sarma had his school education in Attingal near Thiruvananthapuram. He completed his B.Sc. degree with Physics as the major subject in 1940, from Maharaja's College of Science, Thiruvanathapuram. His family tradition of Sanskrit scholarship influenced Sarma to join the M.A. course in Sanskrit at Maharaja's College of Arts, Thiruvanathapuram, which he completed with distinction in 1942. During 1943–51, he was in charge of the Manuscripts Section of the Kerala University Oriental Research Institute and Manuscripts Library. It is here that he acquired expertise in deciphering and critically editing palm-leaf and paper manuscripts of Sanskrit and Malayalam texts. During this period, he prepared an analytical catalogue of nearly 50,000 manuscripts of the library.

From 1951 to 1962, Prof. Sarma was in the Department of Sanskrit, University of Madras, where he was associated with the project of compiling the *New Catalogus Catalogorum of Sanskrit Works and Authors*, under the direction of the great Sanskritist V. Raghavan. It was also the time when his life-long pre-occupation with the Kerala school of Astronomy and Mathematics began to take shape and he started painstakingly collecting manuscripts on Astronomy, Astrology and Mathematics, critically editing and translating many of them. Some of his early publications in this genre were *Grahacāranibandhana* of Haridatta, *Siddhāntadarpaṇa* of Nīlakaṇṭha, *Veṇvāroha* of Mādhava, *Goladīpikā* and *Grahanāṣṭaka* of Parameśwara. During this period, Prof. Sarma also came under the influence of the renowned scholar T. S. Kuppanna Sastri, in collaboration with whom he edited the main text of the Vākya system, *Vākyakaraṇa*, with the commentary of Sundararāja.

At the invitation of Acharya Viswa Bandhu, Prof. Sarma moved in 1962 to the Visvesvaranand Institute of Sanskrit and Indological Studies of the Panjab University at Hoshiarpur. He served as the Director of the Institute during 1975–80 and stayed on at the Institute till 1983. This period of his stay at Hoshiarpur was indeed very productive and he published more than 50 books, mostly on the

[1] This is based on the obituary which appeared in Ind. Jour. Hist. of Sci. 41(2006), 231-246, which also includes a bibliography of publications of K. V. Sarma.

Kerala School of Astronomy. These include very important seminal works such as *Dṛggaṇita* of Parameśvara, *Golasāra* of Nīlakaṇṭha, *A History of the Kerala School of Hindu Astronomy*, *Līlāvatī* of Bhāskarācārya with *Kriyākramakarī* of Śaṅkara and Nārāyaṇa, *Tantrasaṅgraha* of Nīlakaṇṭha with the commentaries *Yuktidīpikā* and *Laghuvivṛti* of Śaṅkara, *Jyotirmīmāṃsā* of Nīlakaṇṭha, and *Gaṇitayuktayaḥ*.

In 1983, Prof. Sarma returned to South India to settle down in Chennai. His important publications during this period include: *Indian Astronomy: A Source Book* jointly with the renowned historian of science B.V.Subbarayappa, *Vedāṅga Jyotiṣa* of Lagadha and *Pañcasiddhāntikā* of Varāhamihira, on which he had worked in collaboration with T. S. Kuppanna Sastri. From 1990 onwards, Prof. Sarma had been working on a critical edition and English translation of the celebrated Malayalam work *Gaṇita-yukti-bhāṣā* of Jyeṣṭhadeva (c.1530 AD). He requested K. Ramasubramanian, M.D.Srinivas and M.S.Sriram to prepare detailed explanatory notes in English. Prof. Sarma also edited a Sanskrit version of *Gaṇita-yukti-bhāṣā* which appeared in 2004, and compiled an important catalogue, *Science Texts in Sanskrit in the Manuscripts Repositories of Kerala and Tamil Nadu*, which includes a list of nearly 3,500 works related to science and technology. In fact, he continued to be relentlessly active till his very death (on January 13, 2005).

Prof. Sarma has to his credit several publications also on diverse aspects of Sanskrit learning such as Vedas, Itihāsas and Purāṇas, Dharmaśāstras, etc. In fact, he has authored more than 100 books and 500 articles. His outstanding contribution consists in searching for and bringing to light many of the seminal works of Kerala School of Astronomy, which show that the tradition of Mathematics and Astronomy continued to flourish till late middle ages at least in the South of India. They also present a detailed view of the methodology of these sciences, on issues such as justification of mathematical and astronomical results and procedures, and the importance of continuous examination and revision of planetary theories. It is mainly due to Prof. Sarma's painstaking work on primary sources that the work of the Kerala School has been brought to the attention of historians of Mathematics, and opened a new perspective on Indian contributions during the late medieval period.

Just as in the case of his illustrious predecessors such as Bibhutibhushan Datta, Avadhesh Narayan Singh and others, and his own contemporary and collaborator K.S.Shukla, Prof. Sarma did not receive even in his own country, the recognition and accolade, which he richly deserved. He was of course awarded the D.Litt degree of Panjab University in 1977 and, in 1992, was bestowed the Certificate of Honour by the President of India. He was also conferred the honorary degree of Vācaspati by Kendriya Samskrita Vidyapeetham, Tirupati, in 2003.

Prof. Sarma, with the wish that his legacy should continue, founded the Sree Sarada Education Society & Research Centre during the 90s, donating his lifetime savings and invaluable collection of books and manuscripts; since his demise, the Centre has published critical editions of some of the texts that he had been working on during his last days. One hopes that the Centre will receive support and encouragement from scholars and funding agencies so as to sustain the torch that Prof. Sarma lit.

Department of Theoretical Physics, University of Madras.

Geometry in the Śulvasūtras

S. G. Dani

Śulvasūtras are compositions pertaining to the fire rituals performed by the Vedic Indians. The rituals involved constructions of altars and fireplaces in a variety of shapes, involving geometric theory. Some of the theory is explicitly enunciated, while some other aspects of the knowledge at that time can be inferred from the constructions. We present here an overview of the geometric ideas contained in the Śulvasūtras.

*Yajna*s, or fire rituals, formed an integral part of life in the Vedic culture, going back to 1500 BCE or earlier, and extending until about the sixth century BCE. Some of these concerned sacrifices to be performed regularly by a householder (*gṛhastha*), while performance of certain others was prescribed for bringing about fulfilment of specific aims or desires, which included both material (acquiring cows, vanquishing an enemy etc.) and transcendental (securing place in heaven) aspects.

In view of the great significance attached to the *yajna*s meticulous attention was paid to a variety of details in their planning and execution. The rituals involved construction of altars (*vedi*) and fireplaces (*agni*) in a variety of intricate shapes. Over a period the procedures for construction of the shapes appear to have got more formalised and acquired a degree of sophistication in geometrical terms. The Śulvasūtras mark the peak in the geometrisation of the altar building activity of the Vedic era.

The Śulvasūtras are often referred to as 'manuals' for construction of the altars and fireplaces. While there is a certain valid analogy here, it should be borne in mind however that the contents are not limited to prescribing steps or procedures for the construction of the altars and fireplaces. They also describe various geometric principles involved, and set up a body of geometric ideas. In Baudhāyana and Āpastamba Śulvasūtra there are separate sections devoted to geometric theory.

The Vedic people were a heterogeneous community, with many *śākhās* (branches), having nevertheless a common cultural identity. The different *śākhās* had their versions of Śulvasūtras, transmitted orally from generation to generation within the community (branch). There are nine extant Śulvasūtras of which four, Baudhāyana, Āpastamba, Mānava and Kātyāyana Śulvasūtras are of significance from a mathematical point of view. The dates of the Śulvasūtras are uncertain, but it is generally believed that they were composed sometime during the period 800 – 200 BCE; for the individual Śulvasūtras the ranges would be, Baudhāyana (800 – 500 BCE), Āpastamba and Mānava (650 – 300 BCE) and Kātyāyana (300 BCE – 400 CE), according to Kashikar, as quoted in [11].

The root *śulv* means 'to measure', and the name Śulvasūtra would correspond to "theory of mensuration" (see [4]). The word *śulva* also means 'rope' in Sanskrit, which indeed was a major equipment employed in measurements. In the body of the Śulvasūtras the word *śulva* does not appear; instead the word *rajju* is used for rope. On the other hand, while most measurements involved in the constructions were indeed carried out with ropes, there are instances where a *bamboo* rod was used instead. This suggests that the name Śulvasūtras indeed was meant to convey mensuration in the conceptual sense, and not only as operations with the rope. The meaning of *śulva* as rope is presumably a later development.

Like other *Vedāṅgas* (appendages of the *Veda*s) the Śulvasūtras are composed in the *sūtra* (aphoristic) style, characterised by short sentences with nouns often compounded at great length and verbs avoided as much as possible, rather than running prose, presumably for reasons of convenience in reciting them. The text, which is in prose form in other respects, has been divided by later commentators into convenient segments, treated as individual *sūtras*, and grouped into Chapters. As presented in [19] (the scheme which we shall follow in the sequel for reference), Baudhāyana has 21 Chapters adding to 285 sūtras, Āpastamba has 21 Chapters adding to 202 sūtras, Mānava has 16 Chapters adding to 228 sūtras, and Kātyāyana has 6 Chapters adding to 67 sūtras.

There is a considerable overlap in the contents of the different Śulvasūtras indicating that the works are expositions from a common stream of knowledge. There are also significant differences, which may be attributed to the different branches they come from and the difference in their period. For general reference in this respect the reader is referred to [19]; (see also [1], [6], [13], [14] and [16]).

The contents of the Śulvasūtras can be broadly categorised into two groups, one consisting of Geometric theory, and another dealing with various details about the constructions of various *vedi*s and *agni*s; in Baudhāyana and Āpastamba the sections dealing Geometric theory are arranged in the beginning. From the other part also one can draw some inferences about the geometric knowledge at that time. The preponderant aspect in this part however is the description of the "Architecture" of tilings involved in the construction of complex figures needed for the *vedis* and *agnis*; (Kātyāyana Śulvasūtra however consists mostly of theory part - this also explains its being shorter than the others). The designs of some of the special fireplaces involve elaborate figures resembling falcons and other birds, tortoise, chariot wheels, circular trough (with a handle), pyre, etc., whose constructions are described, quite elaborately, in the form of tiling by bricks in certain primary shapes. Apart from the issue of achieving likeness with the desired figure, in terms of rectilinear constructions, there are also other stipulations involved, such as the number of bricks to be used etc.. On account of these there are also some arithmetical and combinatorial features involved, in a rather scattered form, in the architectural description of the layout of the fire altars. A study of this aspect would be of interest. We shall however not concern ourselves with it here, and will confine to geometry in the Śulvasūtras, including the principles and constructions described explicitly, as well as those which can be seen to be involved implicitly.

The geometric contents from the Śulvasūtras will be discussed in the following sections taking up various themes. Before going over to the main contents, a few words would be in order regarding the units of measurement involved. They had various units for measures of lengths. Measures of many of the altars are given in terms of *puruṣa* (meaning man), which was about $7\frac{1}{2}$ feet, stipulated as the height attained by the performer of the sacrifice, *yajamāna*, with uplifted arms. A commonly occurring small unit is *aṅgula* (meaning finger, in width); *puruṣa* comprised of 120 *aṅgula*s, so an *aṅgula* was about $\frac{3}{4}$ th of an inch. *pada* (meaning foot, given as 15 *aṅgula*s in Baudhāyana and 12 *aṅgulas* in Kātyāyana), *prādeśa* (12 *aṅgula*), *aratni* (24 *aṅgula*) are some of the other length measures that occur frequently.

The Baudhāyana Śulvasūtra gives in the beginning (sūtra 1.3) names of 18 different units of length measure. The smallest among them, *tila* (sesame seed) is $\frac{1}{34}$ of an *aṅgula*. It was postulated by Thibaut (see [20], page 15) that the unit owes its origin to the fact that they had a formula for $\sqrt{2}$ involving the fraction $\frac{1}{34}$; (the formula will be discussed later). Many of the intermediate units do not bear a simple fractional relation with *purusa* however; e.g. a *bāhu* is 36 *aṅgula*s, a *yuga* is 86 *aṅgula*s, etc.. The units must have arisen from the context of performance of specific *vedis* and many of them occur infrequently.

The other Śulvasūtras use many of the units described by Baudhāyana but there is no systematic listing or a comprehensive statement on their interrelations as in Baudhāyana Śulvasūtra. Some other measures, *vitasti*, *ūrvasthi*, *aṇūka* are also mentioned in Āpastamba Śulvasūtra.

For the area of rectilinear figures they had the notion as we have today. They were aware that for similar figures the ratio of the areas equals the square of the ratio of the lengths of the corresponding sides, as is clear from usage of the idea at various places; in Āpastamba Śulvasūtra there is also an elucidation of this with some examples, including with fractional sides $1\frac{1}{2}$ and $2\frac{1}{2}$ (sutras 3.6 to 3.9). The square units and the corresponding linear unit were known by the same name, the meaning being understood from the context; e.g. *puruṣa* could mean the unit of length as well as the area of the square with that length ("square *puruṣa*", so to speak), depending on whether it referred to length or area; we shall also adopt this as a convention in the sequel when referring to the Śulvasūtras units, rather than prefixing the term "square".

1. Construction of Rectilinear Figures

Though inevitably there are some 3-dimensional features to the fireplaces described in the Śulvasūtras, the geometric ideas of significance chiefly concern planar geometry. These involve the concepts and construction of rectilinear figures such as squares, rectangles, symmetric (isosceles) trapezia and triangles, rhombuses, as well as circles, as primary figures.

The rectilinear figures sought to be drawn had a bilateral symmetry; viz. isosceles triangles, symmetric trapezia, rectangles. The east-west line served as the line of symmetry. Towards construction of these figures with prescribed sizes for the sides, the sutras principally describe steps to draw perpendiculars to the line of symmetry; these are however packaged into complete procedures for drawing the desired figures, as may be seen in some examples discussed below. The issue of drawing a perpendicular to a given line at a given point on it is addressed in the Śulvasūtras in two essentially different ways, involving the following principles (described here in modern formulation):

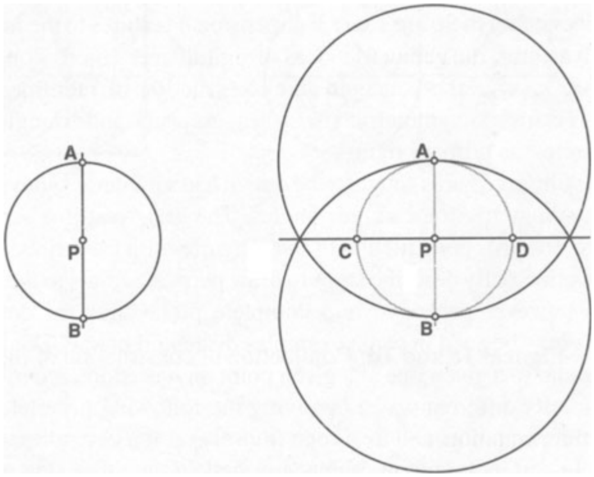
- given two circles with equal radii intersecting each other, the line joining their (two) points of intersection is perpendicular to the line joining their centres, at the midpoint of the line segment joining the centres.
- (converse of Pythagoras theorem) in a triangle with sides with lengths a, b, c if $c^2 = a^2 + b^2$ then the sides with lengths a and b are perpendicular to each other.

These principles are *not* enunciated in the Śulvasūtras, though they are implicitly at work in their constructions (see however the discussion at the end of §2). A large number of sūtras describe constructions of squares and trapezia via application of one of the above, for various specific given sizes, as we shall see in some detail below.

The first statement as above is of course what we commonly use for drawing perpendiculars, to a given line at a given point, in Euclidean geometry: take two points equidistant from the given point and draw two intersecting circles with these as centres and join the points of intersection. This procedure is involved in various constructions in Śulvasūtras for drawing perpendiculars in the same way as we now do. It is however not isolated as a procedure for drawing perpendiculars, but forms a part of the package prescribed for construction of various figures (in the framework as indicated above): thus a construction of a square in Baudhāyana (sutras I.22 to I.28 [19] consists of the following steps, described as an aggregate (see Figure 1):

i) take a rope of the desired side of the square and mark the midpoint;

ii) place a pole at the desired midpoint (say at P as in Figure 1a) and tying the ends of the rope to it draw the circle around it by the mark (at the midpoint) and place poles at the points where the circle meets the east-west line, (A and B as in Figure 1);



Figures 1a and 1b: Beginning of Baudhāyana construction of a square

iii) draw circles with centres at points A and B by the length of the rope, and mark the points where the line joining their points of intersection meets the original circle, to obtain the north-south line and mark the points where the line meets the circle as in (ii) (C and D as in Figure 1b);

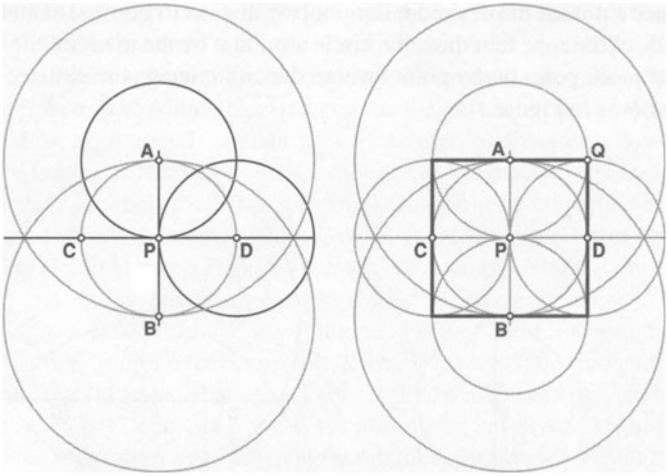
(iv) draw circles by the mid-point on the rope, tieing both its ends at A and then at D and mark the point where they meet (Q as in Figure 1d); this is one of the vertices of the square, and the others can be obtained similarly.

Notice that the first three steps are designed to draw the perpendicular bisector to the line of symmetry and marking the midpoints of the sides of the desired square, and in (iv) these are used to produce the vertices of the square.

Other constructions are also described in a similar vein, as a package for producing the intended figure, without reference to steps involved in each other.

The procedure for drawing the circles involved in the above construction is by tying one end of a rope to a pole placed at the point chosen to be the centre and tracking the point at a distance equal to the selected radius; this plays the role of the compass as used in school geometry now.

The same principle as above was also used with a rope in another way (see Figure 2): given a line and a point on it, say P, to draw the perpendicular to the line at P, take two points on the line equidistant from P, on either side of the line, affix



Figures 1c and 1d: Completion of construction of the square

poles at the two points and tie a rope (loosely) at the two poles; then stretching the rope holding at its midpoint, points are marked on either side where it lies on the ground; the line joining these two points is the desired perpendicular to the original line at P.

It may be seen that though the procedure is different the same principle underlies this construction. This variation for producing perpendiculars also appears as part of construction of some rectilinear figures; in particular Baudhāyana construction of a rectangle given in sutras I.36 to I.41 adopts this procedure.

Though these methods based on the orthogonality principle as above have been used in various constructions, on the whole the *sūtrakāra*s show greater predilection towards using the other principle, namely the converse of Pythagoras theorem. It is believed that the Egyptians also used triangles with sides 3, 4 and 5 (in some

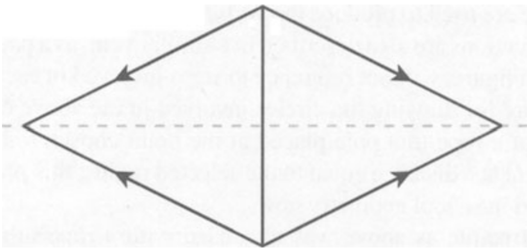
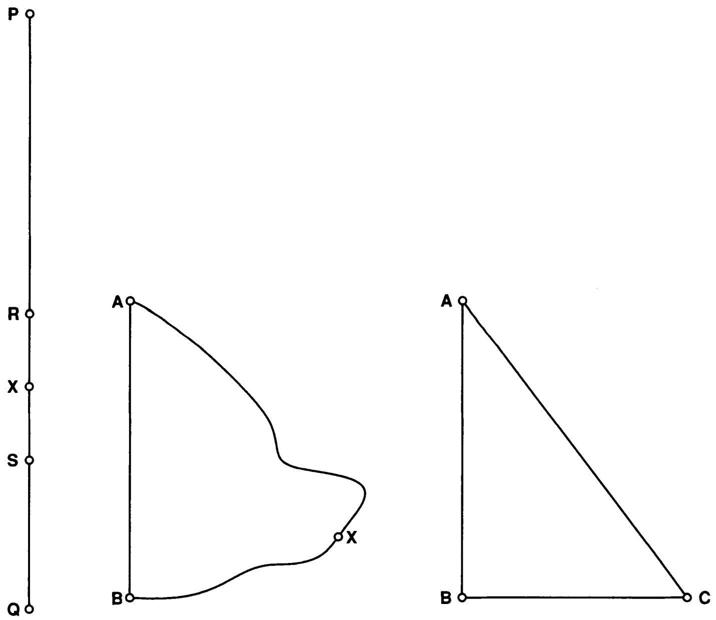


Figure 2: Another procedure for drawing a perpendicular

units) for construction of perpendiculars, but this has now been discounted (see [8], Appendix 5). It is not clear whether there is any other instance historically of using converse of Pythagoras theorem for the purpose of drawing perpendiculars.

Let me now describe in some detail how this application was made, and discuss the possible convenience for which it was preferred. Take a rope, with endpoints marked P and Q, with length a (see Figure 3). Let R be the midpoint of PQ (on the string), S the midpoint of QR and X the midpoint of RS. Now tie the two ends of the string to two poles placed a distance $a/2$ along the line to which a perpendicular is to be drawn, with the Q end at the point where the perpendicular is to be drawn. Let A and B be the points on the plane (ground) where the ends P and Q are tied. Now stretch the rope, holding it at the point X as above, on one side of the line AB, and mark the point on the plane where X lies, say C (see Figure 3(c)). Notice that B and C are at a distance $3a/8$ and A and C are at a distance $5a/8$. Thus the sides BC, AB and AC are in the proportion $3 : 4 : 5$, and since $3^2 + 4^2 = 5^2$, by the converse of the Pythagoras theorem the angle $\angle ABC$ is a right angle. We have thus constructed a perpendicular to the line AB at the point B.



Figures 3a, 3b and 3c: Construction of perpendicular by the *Nyanchana* method

A rope with markings as above, which can be preserved, can thus be used as an *instrument* to draw a perpendicular, essentially at one stroke; the distance $a/2$ to be kept between A and B is also available by a mark on the rope as the distance between P and R. In directing a *yajamāna* towards drawing a perpendicular (as a step in the construction of the *vedi*), it would be simpler for the priest to use this approach, than those with the orthogonality principle discussed above.

The above procedure depends on the fact that a triangle with sides in the proportion 3 : 4 : 5 is a right angled triangle, by the converse of the Pythagoras theorem. The Śulvasūtras describe also analogous procedures using in place of $(3, 4, 5)$ other triples (a, b, c) such that $a^2 + b^2 = c^2$; typically they are triples of integers, that we now call *Pythagorean triples*, and their multiples by a fraction, but occasionally some incommensurable triples are also involved. As in the above procedure it involves marking a point X so that when the rope is stretched holding at that point we would get a right angled triangle. Such a point is called *Nyanchana*; *Nyanchana* means "lying with face downwards" and in this context signifies that the marked point is to be plotted on the ground. A procedure involving use of the triple $(5, 12, 13)$ goes as follows: having chosen a distance a between the poles, a rope of length one and half times the measure is taken (thus extending the rope by $a/2$), and the *Nyanchana* mark is set at a distance a sixth of the extended piece, namely $a/12$ from the joining point. The *Nyanchana* mark then divides the string in the proportion 5 : 13, and steps analogous to those described above will yield a perpendicular at the pole on the side of the shorter segment. It may be noted that the Pythagorean triple is unrelated to the length a in either case.

In [3] I have discussed the theme of Pythagorean triples with regard to the Śulvasūtras. Here I will therefore introduce it only briefly to put the topic in perspective.

The main role of the Pythagorean triples in Śulvasūtras was their use in producing perpendiculars via the converse of Pythagoras theorem. The two triples $(3, 4, 5)$ and $(5, 12, 13)$ are a common occurrence in this respect in the Śulvasūtras. These are primitive triples (there is no common integer factor greater than 1). Some multiples of these triples (non-primitive) were also commonly in use; the triple $(15, 36, 39)$ seems to have been an especially familiar one, and perhaps much older than the Śulvasūtras themselves (see [3] for some observations on this). In Āpastamba two more primitive Pythagorean triples occur in the description of the the construction of the *Mahāvedi*: $(8, 15, 17)$ and $(12, 35, 37)$ (Asl. 5.3 - 5.5); this is the only place where they occur in Āpastamba Śulvasūtra.

The *Mahāvedi* was in the shape of a symmetric trapezium with a base of 30 units, height of 36 units and face (side opposite to the base) of 24 units (see Figure 4); to give an idea of the physical size (though it shall not concern us further) it may be mentioned that the unit involved is either a *pada* or a *prakrama*, the latter being $\frac{1}{4}$th of *puruṣa*, and the height then works out to be about 20 meters.

Āpastamba gives four constructions for the *Mahāvedi* all based on the *Nyanchana* method as discussed above. The first one known as the *ekarajjuvidhi* ("one-rope process") involves a rope of length 54 units, with markings at 36 and 12 units from the two ends respectively and a mark at the midpoint of the remaining middle

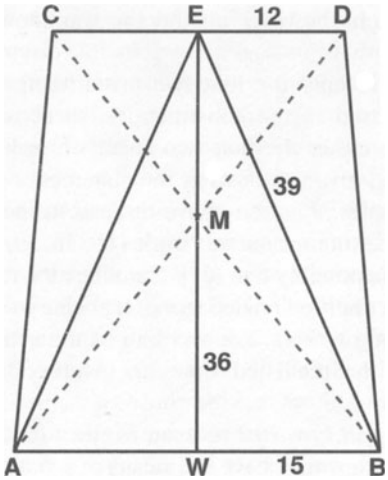


Figure 4: The *Mahāvedi*

portion of 6 units as the *Nyanchana* mark. Thus we have a subdivision into 15 and 39 units, and the desired height being 36 units this enables fixing the two vertices A and B at the base by the *Nyanchana* method (through triangles EWA and EWB. The vertices at the face CD are also obtained in the same way, by plotting the mark at 12 from the end rather than the original *Nyanchana* mark. The other constructions involve two cords.

It may be noticed that the diagonals of the trapezium meet at a point M which together with the base and face makes isosceles triangles AMB and CMD whose symmetric half parts have lengths given by the triples (15, 20, 25) and (12, 16, 20) respectively. Āpastamba's second procedure consists of drawing the two triangles using that these are multiples of the triple (3, 4, 5).

The third construction is based on the triple (5, 12, 13) and its multiple (15, 36, 39), using that half the base and half the face, viz. 15 and 12, occur in these triples; unlike in the first two constructions one of the vertices of the right angled triangle involved (the one on the east-west line) is of no significance to the diagram itself. It is the last construction in which the triples (8, 15, 17) and (12, 35, 37) appear. Again since 15 and 12 occur in these, the vertices at the base and the face may be plotted with these triples, respectively, with poles at distances of 8 and 35 from the base and face respectively. Manifestly the construction would no longer be as elegant as the earlier ones, and Āpastamba could not have missed noticing that. From the overall context it is clear that the aim has been to quench the curiosity about the various ways the *Nyanchana* method could be used to plot the vertices, and no practical requirement is involved. This involves finding triples with 15 and 12 as one of the first two entries. Incidentally, Āpastamba exhausts such triples. There is no clue however whether this was known, and if so how it was realised.

In Baudhāyana, while only the first two primitive triples, viz. $(3, 4, 5)$ and $(5, 12, 13)$, are found used in the constructions, there is a list of 5 primitive Pythagorean triples (or rather the first two terms of each, which of course determines the third term) given, following the statement of Pythagoras theorem. Apart from the four triples as in the above discussion the triple $(7, 24, 25)$ also forms part of the list; the (non-primitive) triple $(15, 36, 39)$ is also included along with the others, which is possibly due to the familiarity with it in a wider context, and its association with tradition on account of its being involved in the Mahāvedi (see [3] for a discussion on this). The wording of the sūtra, its location and the overall context indicate that the listed triples are given as illustrative examples for the Pythagoras theorem.[1]

There are no other primitive Pythagorean triples found in the Śulvasūtras. It would seem that though they may have the means of producing more triples, if not an infinite family, they would have had no motivation for it, with the ones that occur having specific objectives that are adequately met in their context.

The *Nyanchana* procedure was also used in a construction with an incommensurable triple (not a multiple of Pythagorean triples); the construction is also interesting from another point of view (see below). To construct a square of side a Āpastamba gives the following procedure: on a rope mark three points, two endpoints and a point in between whose distance from one endpoint is $a/2$, and from the other endpoint it is equal to the length of the diagonal of the square with side $a/2$. The rope is now tied to two poles, one at the desired centre of the square and the other at the point at distance $a/2$ on the east-west line, by the endpoints as above, with the longer side (from the in between mark) being attached to the centre. The rope is then stretched, holding it at the middle marked point. Where it lays on the ground is one of the vertices of the desired square; the other vertices are plotted similarly. The procedure presupposes being able to mark a point at a distance equal to the diagonal of a square of side $a/2$, so the construction is in a way "circular", from the point of view of logical development. However, the diagonal of a square was such a common occurrence in their practice that it was I suppose treated as a "tangible" quantity. Having once constructed a square one could produce ropes with markings as required, and then employ them for later construction of squares following the above procedure. It has been suggested by some authors (see Footnote 4) that an approximate numerical expression for $\sqrt{2}$ was used to get the

[1] Indeed, a Pythagorean triple does not, strictly speaking, illustrate Pythagoras theorem, since one would need to know that the triangle corresponding to the triple as side-lengths is a right angled triangle; the nuance of the illustration here would be more like "when you draw rectangles with sides 3 and 4, 5 and 12, ..., the diagonals will be 5, 13, ..., and you see that the area produced by the diagonal is the sum of the that of the squares on the sides."

diagonal of the square with side $a/2$; this however seems unlikely, considering the common usage of ropes all around, which furthermore would give a more accurate measure, without the cumbersome subdivision that would be involved in producing the approximate value using the formula for $\sqrt{2}$ (see §6).

For the constructions of the figures the Śulvasūtras adopted meticulous procedures, involving drawing the perpendiculars, which was accomplished by the methods that we discussed above. Surprisingly, despite extensive use of these methods, conceptualisation of the perpendicular or right angle seems to have eluded them. Absence of the concept may have led to view the task of drawing each symmetric trapezium with different given dimensions individually, as is noticeable especially in Āpastamba Śulvasūtra. While there is a degree of unity in the descriptions, with the concept of the perpendicular a more uniform prescription could have been given for the constructions. Furthermore in several constructions, including in the *Mahāvedi* as seen above, special Pythagorean triples (not necessarily primitive) were sought depending on the desired sizes. While the latter may have offered an amusing diversion, in practice it would have been simpler to have a unified way of drawing perpendiculars, even with *Nyanchana* method if that was found more convenient, and marking the point on the perpendicular line at the desired distance, producing the line if necessary. This possibility does seem to have been realised at some stage. In [3] I have noted that the *vedis* described in the Asl. 6.3-6.4 (*nirūḍhapaśubandha vedi*), Asl. 6.6, Asl. 6.7 (*paitṛki vedi*), Asl. 6.8 (*uttara vedi*), and Asl. 7.1 involve varied shapes (two trapezia of different dimensions, two squares and an oblong rectangle), but the construction of each of them refers to "Having stretched (the cord) by the mark at fifteen" (*pancadaśikenaivāpāyamya*), and taking various contextual factors into account concluded that the phrase is used as a way of saying "Having drawn a perpendicular"; the desired point on the perpendicular line is meant to be marked on that line by measuring out the requisite distance. This marks a step towards conceptualisation of the perpendicular at a practical level, which however does not seem to have been abstracted further.

2. Pythagoras Theorem and its Applications

The most notable feature of the Śulvasūtras in terms of geometric theory is the statement of the so called Pythagoras theorem. This stands out especially in the context of the fact that some of them, especially Baudhāyana, predate Pythagoras. There has been a variety of speculation in this respect, including that Pythagoras may have got it from the Indians (A. Bürk quoted in [10]) or, in broader terms, that there may have been a common source for the geometry of the Greeks and the Indians (see Seidenberg [17] and [18]). Available inputs seem inadequate to have a meaningful discussion on this, and in any case we will not go into this aspect here. The main discussion below will pertain to the role of the theorem in the overall context of the Śulvasūtras themselves.