# INTEREST RATE DERIVATIVES **EXPLAINED**

VOLUME 2: TERM STRUCTURE AND VOLATILITY MODELLING

Jörg Kienitz
Peter Caspers

FINANCIAL ENGINEERING **EXPLAINED**

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# Interest Rate Derivatives Explained: Volume 2

## Term Structure and Volatility Modelling

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*To Amberley, Beatrice and Benoît*
Jörg Kienitz

*To Kirsten, David, Fiona, Kendra and Noel*
Peter Caspers

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# Goals of this Book and Global Overview

## Introduction and Management Summary

The first volume of Interest Rate Derivatives Explained, Kienitz (2014b), is dedicated to introduce basic interest rate products and give an overview of the corresponding markets. There, we outlined day count conventions, defined different rates and considered products that can be priced using the current yield curves and volatility surfaces, respectively cubes. This included Interest Rate Swaps but also more involved products such as swaptions, caps and floors or constant maturity swaps and the corresponding options referencing to constant maturity swap rates such as CMS spread options.

In the current volume, we wish to extend the scope to modelling volatility and the term structure of interest rates. Such methods are important for the daily work of financial institutions since exposures need to be determined, path-dependent contracts even with early exercise features but also products including negative rates, deep in or out of the money options and alike need to be valued, processed and risk managed. Take a constant maturity swap for instance. In Interest Rate Derivatives Explained 1, Kienitz (2014b), we have assumed the entire volatility smile given. Then, a static replication argument was applied for pricing and risk management of such trades. In this volume, we wish to show how the volatility smile is build and we wish to propose methods that can be applied to a wide range of market scenarios and do not stuck as some standard models that cannot safely be applied or even do not work at all. For instance, take the SABR model, here either standard methods generate too high volatilities for ITM or OTM options, lead to arbitrage or simply the current observed rates do not fit into the models scope. We show

how to adjust models and suggest other numerical approaches that are applicable in challenges market scenarios.

Other trades and products need to address different issues. Consider path-dependence for instance. For a sound risk management, a financial institution has to be able to handle such features often embedded in interest rate trades. This can materialize as an exercise right in a bond or swap contract. Even standard products such as mortgages in Germany have built in callability features. A mortgage loan can be called off after ten years at any day with a notice period of half a year. Another aspect is accounting. With regard to applying IFRS rules, the instruments held by a financial institution may need to be accounted for by assigning a fair value. This of course can include exotic rate products.

We have structured the book in three parts. The first part of the book deals with interest rate products. We give important examples for products that cannot be priced only taking into account the current yield curves and a swaption volatility surface. The future evolution of the term structure is necessary to determine the price and to apply an efficient hedge and risk management. Products which we consider include path dependencies in many ways. One important feature we consider is callability. Bermudan swaptions are the most prominent representatives of this product class. Then, we describe how volatility is modelled. Even for European options, it is necessary to think about a sound volatility model since quotes are only available for some maturities and strikes. If the maturity or the strike of an option is not quoted, methods for inter- and extrapolation have to be considered. All strikes and maturities need to be available to apply the before-mentioned replication technique to price CMS Caps, CMS Floors, CMS Swaps or CMS Spread options. After summarizing the task of volatility modelling, we consider two popular models in detail, namely the Heston model and the SABR model. Furthermore, the models can also be used to enhance term structure models with a stochastic volatility component. This additional component helps to model observed market features and improves the quality of fitting observed option prices.

The third part is concerned with term structure models. Such models are used to evolve the current yield curve into the future. There exist several methodologies for achieving this goal. We give an overview of term structure models ranging from one factor short rates over infinite dimensional models for the instantaneous forward rate to high dimensional market models. Often the modeller has to achieve a trade off between model complexity, accuracy and numerical tractability. In fact the latter might soon become a bottle neck when we consider the current regulated markets where all kind of value adjustments, see Kienitz (2014b), have to be calculated for large and diverse portfolios. The basis of such adjustments is the generation of the future exposure. This is done by simulating risk factors for given future times called view points. Therefore, the simulation of many thousands of scenarios is the market standard method. This together with the valuation of complex options including path dependencies is very challenging. It is not hard to guess that the computational workload for fulfilling this task is immense. But once the data are available, the adjustments and exposure measures are easy to determine.

The current volume has three parts and an appendix with all together 12 chapters. Each part is dedicated to a single topic starting with products, then considering volatility modelling and finally covering term structure models. The appendix gives information on the numerical techniques that need to be applied for implementing the models and methods considered in this book. Summarizing we have:

- Part I
  - Vanilla Bonds and Asset Swaps
  - Callability Features
  - Structured Finance
  - More Exotic Features and Basis Risk Hedging
  - Exposures
- Part II
  - The Heston Model
  - The SABR Model
- Part III
  - Term Structure Models
  - Short Rate Models
  - A Gaussian Rates-Credit Pricing Framework
  - Instantaneous Forward Rate Models and the Heath–Jarrow–Morton Framework
  - The Libor Market Model
- Appendix
  - Numerical Techniques for Pricing and Exposure Modelling

Now, let us summarize some hot topics that are considered in the main body of the book. Some of those even appear the first time in book format since they were recently published and are part for ongoing research.

First, we have a wide coverage of products including Bermudan style derivatives with the Bermudan swaption as the most important one. Other more exotic interest rate derivatives are also still in the trading and banking books of financial institutions. We cover TaRN, floating rate notes or range accruals. You find valuable information on these types of trades including examples on how they work and the coupon mechanism works.

Then, we consider the very important topic of exposures. As already outlined in Kienitz (2014b), this is a very hot topic at the moment. The exposure profile of trades and portfolios is the key to measure counterparty credit risk either for regulatory or for accounting purposes. Exposure measures and examples for many common interest rate derivatives are considered. This includes multi-callable swaps where a Bermudan swaption is embedded into a swap-type contract. All is illustrated with pictures and graphs.

We give a broad overview of volatility modelling which is a very important topic and there has been a great body of research. We point to the corresponding literature and cover two of the models commonly applied in interest rate markets, namely the Heston and the SABR models. The latter models appeared most prominent in the quantitative finance literature. Foremost we have to mention Hagan et al. (2015) and Antonov et al. (2015). We think the reader will appreciate that we included the new developments here with new approximation formulas, numerical schemes for achieving a no-arbitrage representation of the probability density and even methods to use the newly proposed Free Boundary SABR model. We do not know of any other book covering this together with the new market paradigms of negative rates and Bachelier volatilities.

When it comes to term structure modelling financial institutions can choose from a variety of different models. To this end, we outline the main approaches to term structure modelling including ways to account for a stochastic basis. After laying out the different approaches we consider some representatives of each model class, namely

- Short Rate models with a focus on the Gaussian Short Rate model class
- Cheyette models with unspanned stochastic volatility
- Libor Market models with many different correlation structures

The reader will appreciate that many of the concepts are illustrated using spreadsheets that can be downloaded, see Section "Code".

At the end of the book you find a round up of numerical methods that are necessary to apply the models in practice. This extents the exposition from volume 1 where we considered bootstrapping, yield curve calibration and interpolation techniques. In Appendix A, we outline the application of transformation techniques which can be applied but are not only restricted to the Heston model. In fact many jump models and stochastic volatility models can be tackled with the described techniques. Then, we cover the PDE approach using finite difference approximations of continuous quantities in some detail. The method is often applied in financial mathematics and we use it in our exposition to implement the SABR model. Finally, one of the most important numerical methods called Monte Carlo simulation is described. This technique is more important than ever. This is due to the fact that exposure for large portfolios that depend on a large number of risk factors has to be considered. Monte Carlo methods are the only tractable way of achieving this.

## Code

Some of the methods and models are illustrated using spreadsheets. For instance, the different parsimonious approaches to model volatility and correlation in the context of Libor Market models or some flavours of using approximation formulae for the SABR model are illustrated in this way. All the examples are for pedagogical use only. The sheets cannot be used for sound modelling the interest rate markets but can serve as the basis for creating proprietary implementations and generating ideas.

The material for this book and for the first volume are available via www.jkienitz.de. There you also find additional material and further illustrations on quantitative finance, mathematical modelling and related topics.

Many publicly available software libraries have term structure models already implemented. We especially mention QuantLib (www.quantlib.org) and ORE (www.opensourcerisk.org).

## Further Reading

This book can of course not give all the nitty gritty details and cannot provide a full account of all products, models and numerical techniques. To this end, we put together a list of relevant literature the reader might consult after reading this book. We decided to give hints on further reading with respect to the three parts of the book. For the appendix on numerical methods, we place the references for further reading directly below the last section of that chapter.

We also suggest further to the given references to do a search on the well-known preprint services including SSRN, ResearchGate or arXiv.

**Part I**

For the first part, we refer to Andersen and Piterbarg (2010a) and Andersen and Piterbarg (2010c). Many of the products discussed in this first part are also analysed and further explained in these books. Another standard reference is the compendium Brigo and Mercurio (2006). Also many products are considered there and many closed form solutions for derivatives products as well as intuition, hedging issues and further ideas are covered in a well written and clear manner. If you are looking for an account on recent advances in exposure modelling for rates consider to read Lichters et al. (2015). Many approaches such as the CSA Floor at 0 or tackling derivatives in the multi curve framework are reviewed and described in detail.

Some further books on valuation which are relevant for the first part are Henrad (2014) and Kenyon and Stamm (2012). They cover the changes the interest rate markets have undergone after August 2007 and how derivatives and options are tackled in this new era. Finally, we mention Kienitz (2014a). This book is the first part to the current one and has all the definitions for the underlying quantities of the derivatives considered in the first part. Furthermore, some basic derivatives have already been considered there.

**Part II**

Since the concept of volatility is of course not only relevant for modelling interest rates, there are many papers either devoted to volatility or covering volatility modelling for other asset classes. For volatility modelling a standard reference is Gatheral (2006). That book gives a great overview of many techniques and gives hints to further reading. The ground breaking articles for local volatility are Dupire (1994) and Derman and Kani (1994). These references are cited in many papers and books covering volatility modelling. Another good source of information and worth for building your intuition is Rebonato (2004). Here, different approaches with many illustrations and outlining their practical relevance are covered.

A very recent book that is written by a market practitioner is Bergomi (2016). This book covers the instruments that have volatility as well as the dynamics of the volatility as risk factors. For instance, forward starting options are covered, volatility index futures and options, the dynamics of local volatility, uncertain volatility and its usage and many more facts are considered in great detail.

Now, for the special case of the Heston Stochastic Volatility model which was introduced in Heston (1993) many papers and even a book dedicated to the model do exist. The book is Rouah (2015) and covers the standard but also many variants of the Heston model and numerical methods. Since then many researchers and practitioners used this model and contributed in terms of applications or numerical methods. Relevant literature for implementing the Heston model and applications to calibration are Lord and Kahl (2005), Albrecher et al. (2006), Forde et al. (2012), Antonov et al. (2008), for simulation Andersen (2008), Staunton (2007) and Chan and Joshi (2010), for long stepping schemes (Bin 2007). The Heston model is also applied for enhancing existing models with a stochastic volatility component such as market models or for considering hybrid models. For enhancing market models, see Piterbarg (2003), Kiesel and Lutz (2010) and for an example of a hybrid model we recommend Kammeyer and Kienitz (2012a, b, c).

The other popular model is the SABR model. There is also a large body of literature available. The main reference for the SABR model is Hagan et al. (2002) but there have been research papers and books thereafter. For instance Hagan et al. (2005) propose an expression for the density which is often necessary when dealing with CMS derivatives.

After the events in 2007 and 2008 we saw that the standard method of applying SABR namely using the approximation technique was not valid anymore and methods had to be considered to remove arbitrage and cover with low rates and high volatility. We refer to Doust (2012), Hagan et al. (2015), Hagan et al. (2016), Antonov and Spector (2012), Antonov and Spector (2013), Kienitz (2015), Kienitz et al. (2017). Other methods to account for the negative rates include Antonov et al. (2015). The authors introduce a new local volatility function to the standard SABR model that changes the model behaviour and leading to a new way of modelling rates.

For considering the SABR model together with term structure models, we refer to Mercurio and Morini (2009) or Rebonato et al. (2009). The latter covers all the aspects necessary to use a market model with SABR-type stochastic volatility.

If you are interested in implementing the models we suggest to consult Kienitz and Wetterau (2012). This reference covers most of the techniques described in the above references. It also provides working Matlab source code and the reader can see how the models work and can play around with parameters, run simulations and calibration.

**Part III**

Term structure models are covered in a three volume compendium Andersen and Piterbarg (2010a, b, c). Another standard reference has already been mentioned for the products covered in Part I of this book. It is Brigo and Mercurio (2006). Both books have a wealth of information, tips and tricks from practitioners and well-known researchers.

If you are interested in short rate modelling you can work through several research papers but the above-cited references have all the stuff you need to successfully apply short rate models. From pricing basic instruments as well as numerical methods to tackle exotic products are covered in the cited books. Furthermore, the references there point to the original papers if you wish to consider reading the original articles.

For the instantaneous forward rate models, we are not aware of a book that has a broad coverage of the models and the numerical techniques which are applied here. We suggest to take Andreasen (2005), Cheyette (1994) or Trolle and Schwartz (2009) as a starting point. A nice summary was written by a student of J. Kienitz from UCT, see Schumann (2016).

The case of the LGM model is considered in many papers by P. Hagan. For a detailed description, you can also consult Lichters et al. (2015). They treat the case of multi-currency LGM and combining it with different markets by using for instance foreign exchange extensions.

If you consider to work with a Libor market model we suggest to take Rebonato (2002) as a reference. Modern extensions with stochastic volatility are covered in Piterbarg (2003), Kiesel and Lutz (2010), Antonov et al. (2008) and Rebonato et al. (2009).

Another very useful reference is the homepage of John Schoenmakers at WIAS Berlin (http://www.wias-berlin.de/people/schoenma/). There you find many papers on different aspects of Libor Modelling. Furthermore, his book Schoenmakers (2005) covers the basics as well as advanced and very technical aspects of modelling Libor rates.

For the modern aspects of multi curve models and stochastic basis approaches, we suggest to consider Grbac et al. (2015), Mercurio and Xie (2012) and Mercurio (2010).

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# Part I

## Products