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B.S. Kushvah
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Applications of Fluid Dynamics

Proceedings of ICAFD 2016

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Preface

International conference on Applications of Fluid Dynamics (ICAFD 2016) was organized by Department of Applied Mathematics, Indian Institute of Technology (Indian School of Mines), Dhanbad, Jharkhand, India, in association with Fluid Mechanics Group, Department of Mathematics, Faculty of Science, University of Botswana, Gaborone, Botswana. ICAFD 2016 at IIT(ISM) Dhanbad was third in continuation following the first ICAFD 2012 in University of Botswana and second ICAFD 2014 in S.V. University, Tirupati.

The aim of the conference was to provide a platform for academicians, experts, and researchers in various disciplines of mathematics and its allied areas exclusively in applications of fluid dynamics. This international platform provided opportunities for eminent personnel to interact and suggest various corrective measures for advancement of the subject and its applications in industry, science, engineering and technology, research and development, etc. The deliberations made by an eminent mathematicians, engineers, scientists, and young researchers published in the form of conference proceeding entitled "Applications of Fluid Dynamics" by Springer in Lecture Notes in Mechanical Engineering. This international conference received overwhelming response from academicians, scientists, engineers, researchers, and most importantly research scholars of various institutions of our country such as IITs, IISc Bangalore, R&D Center, IOL, HPCL Mumbai, NITs, Central and State Universities, and foreign countries as well such as USA, New Zealand, West Indies, Botswana, Ostrava, South Africa, Russia. The financial support for organizing this international conference was extended by Jharkhand Council of Science and Technology (JCST), Ranchi, Jharkhand; Department of Science and Technology (DST-SERB), New Delhi; Defense Research Development Organization (DRDO), New Delhi; and Society of Applied Mathematics (SAM) and IIT(ISM), Dhanbad.

We are thankful, rather grateful, to the Key Speakers like Sri C.K. Asnani, Chairman and Managing Director, UCIL Jaduguda and Chief Guest of inaugural function; Prof. Dhanush Dhari Misra, Chairman, Board of Governors, Indian Institute of Technology (Indian School of Mines), Dhanbad; Prof. D.C. Panigrahi, Director, Indian Institute of Technology (Indian School of Mines), Dhanbad; Prof.

Vijay P. Singh, Guest of Honor of inaugural function and University Distinguished Professor and Caroline and William N. Lehrer Distinguished Chair in Water Engineering at Texas A&M University, USA; Prof. R. Panneer Selvam, James T. Womble Professor of Computational Mechanics and Nanotechnology Modeling, University Professor of Civil Engineering and Director of Computational Mechanics Laboratory, University of Arkansas, USA; Prof. Venkatesh Uddameri, Director, Water Resources Center, Texas Tech University, USA; Prof. Lakshmi Narasimhan, University of Botswana, Botswana; Prof. Elma Annette Hernandez Uddameri, Department of Civil, Environmental, and Construction Engineering, Texas Tech University, USA; Dr. Raj Das, Group Leader, 'Simulation and Modeling', Mechanical Engineering Department, University of Auckland, New Zealand; Dr. Sreedhara Rao Gunakala, Lecturer (Tenure track), Department of Mathematics, The University of the West Indies, St. Augustine, West Indies; Prof. J. Prakash, Head, Department of Mathematics, Faculty of Science, University of Botswana, Botswana; Prof. Sergej Hloch, Technical University of Košice in Slovakia; Prof. Alok K. Gupta, Emeritus Professor, National Center of Experimental Mineralogy and Petrology, University of Allahabad; Prof. B.V. Rathish Kumar, Department of Mathematics and Statistics at Indian Institute of Technology, Kanpur; Prof. Mridula Kanoria, Department of Applied Mathematics, University of Calcutta, Kolkata; Prof. Naveen Kumar, Department of Mathematics, Institute of Science, BHU, Varanasi; Dr. Shivendra Nath Rai, Chief Scientist, (CSIR-National Geophysical Research Institute (NGRI)), Hyderabad; Prof. Ashok Mishra, Director, "Center for Fluid Dynamics Research," CUTM, Paralakhemundi, Odisha; Prof. S.V.K. Varma, S.V. University, Tirupati; Dr. Sachin Saw, School of Mathematics and Statistical Science, Botswana International University of Science and Technology, Botswana.

On behalf of organizing committee, we express our sincere gratitude to Prof. D. D. Misra, Chairman, BoG, IIT(ISM) and Chief Patron, ICAFD-2016 for their encouragement. We express our sincere thanks to Prof. D.C. Panigrahi, Director, IIT(ISM), Prof. Thabo T. Fako, Vice-Chancellor, University of Botswana, Botswana and Patrons, ICAFD 2016 for their guidance and source of inspiration. We also express our thanks to IIT(ISM) and University of Botswana administrations for providing infrastructural support to organize an international event at such a large scale smoothly and successfully. We acknowledge the support and motivation received from each one of them who have directly or indirectly helped us to organize this international event in a grand way. We also put on record about the contribution of Research Associate and Scholars namely Dr. Pintu Das, Sultana Begam, Ayan Chatterjee, Chandan Kumar Thakur, Affreen Akhter, Rakesh Kumar Singh, Rohit Kumar, Animesh Samanta, and Manish Chaudhary for helping us to organize an international event apart from preparing and formatting the manuscript. Finally, the execution of international event is possible with support of one and all.

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About the Editors

Dr. M.K. Singh is an Associate Professor in the Department of Applied Mathematics at the IIT(ISM), Dhanbad, Jharkhand, India. Dr. Singh has published more than 50 research papers in National and International journals and conferences of repute. He has edited three books and published three chapters in his area of expertise. Dr. Singh has guided more than five M.Phil. and Ph.D. students. Dr. Singh has more than fifteen years of teaching and research experience in Applied Mathematics. His research interest is in the field of Solute Transport Modeling. He has executed three Minor and Major Research Projects successfully. Dr. Singh is the life member/member of various professional bodies. Dr. Singh has organized various national and international events, i.e., workshops, training programmes, conferences, etc. Dr. Singh is the panel examiner of Ph.D. in Central Universities/Institutes of our country and abroad.

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Dr. J. Prakash is working as a Professor in the Department of Mathematics, Faculty of Science, University of Botswana, Botswana. He is having more than forty years of teaching experience in various countries such as India, Iraq, Zambia, New Zealand, and Botswana at university levels. He is having research interest in bio-fluid dynamics, fluid mechanics, computational fluid dynamics (CFD), mathematical modeling, mathematical methods, etc., with more than 175 research publications in various International journals. He has organized international conferences at India and abroad. Dr. Prakash has received C.V. Raman International Fellowship Award for African Researchers under Visiting Fellowship Program from FICCI Science and Technology, International Cooperation Division, Government of India. He has also received Global Visionary Award on January 13, 2014, by Vision Foundation, Ahmedabad, India.

Applications of Fluid Dynamics: An Introduction

Mritunjay Kumar Singh

Locke has said "Mathematics is a way to settle in the mind a habit of reasoning." Basically, two types of reasoning exist in mathematics such as inductive and deductive. Inductive reasoning in which statement or propositions are made on general observation and experience while deductive reasoning based on self-evident truth in which statements are the product of mind. Mathematics in widest sense is the development of all types of deductive reasoning. A true lover of MATHEMATICS should be Methodical, Analytical, Technical, Humble, Ethical, Meritorious, Assertive and Attentive, Thoughtful and Tenacious, Introspective and Indigenous, Cautious and Curious, Studious. Subsequently, mathematics enhances mind power, apprehension, talent, honesty, enthusiasm, memory, ambition, thirst of knowledge, intelligence and intricacy, confidence and clarity, and self-realization. A genuine learner of mathematics is certainly interested in finding out the answers of some of the questions which are as follows: Why should everybody learn mathematics? What is the place of mathematics in any scheme of education? What are the aims and objectives of teaching mathematics to everybody? How does it make contribution in the development of an individual? Mathematics must contribute toward the acquirement of the values like knowledge and skill, intellectual habits and power, desirable attitude, and ideals. Moreover, it also possesses practical value, disciplinary value and cultural value, etc., as well. As per Hamilton, the study of mathematics cures the vice of mental distraction and cultivates the habit of continuous attention. As we all know, mathematics demands hard work from the learner and hardworking people are very much needed in any developing society. We can establish the very good relation of mathematics with the other subject such as Physics, Chemistry, Biology, Agriculture, Engineering, Social Science, Economics, Logic. Some of the views of mathematics regarding relationship with

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the other subject were quoted as follows: Mathematics gives the final shape to the rules of physics. Mathematical nature of the chemical equation can be explored. The claim of any particular branch of natural philosophy to be considered as a science can be assessed only on the basis of the amount of mathematics employed in it. The proportions of human body are strictly mathematical. A human being's beauty and attractive form depends on his/her proportionate buildup. The whole figure is six times the length of the foot which holds well for the form either is slender or plump. For example, the Greek made all their statues according to this rule. The hand, from the wrist to the middle finger, is one tenth of the whole stature. The scientific calculation about the normal weight in pounds is double the height in inches. Mathematics in engineering deals with surveying, leveling, designing, estimating constructions, computing, etc. Applied mathematics includes fluid mechanics in detail and explores their applications extensively in science, engineering and technology as well.

Fluid mechanics is defined as the science that deals with the behavior of fluids at rest (fluid statics) or in motion (fluid dynamics). Fluid mechanics is also referred to as fluid dynamics by considering fluids at rest as a special case of motion with zero velocity. Fluid dynamics has several subdisciplines itself, including aerodynamics (the study of air and other gases in motion) and hydrodynamics (the study of liquids in motion). Fluid dynamics has a wide range of applications, including calculating forces and moments on aircraft, determining the mass flow rate of petroleum through pipelines, predicting weather patterns, understanding nebulae in interstellar space, and modeling fission weapon detonation. Some of its principles are even used in traffic engineering, where traffic is treated as a continuous fluid, and crowd dynamics. Fluid mechanics is encountered in almost every area of our physical lives. Blood flows through our veins and arteries, a ship moves through water and water flows through rivers, airplanes fly in the air and air flows around wind machines, air is compressed in a compressor and steam expands around turbine blades, a dam holds backwater, air is heated and cooled in our homes, and computers require air to cool components. All engineering disciplines require some expertise in the area of fluid mechanics. Fluid mechanics, especially fluid dynamics, is an active field of research with many unsolved or partly solved problems. Fluid mechanics can be mathematically complex, and can best be solved by numerical methods, typically using computers. A modern discipline, called computational fluid dynamics (CFD), is devoted to this approach to solving fluid mechanics problems. The solution to a fluid dynamics problem typically involves calculating various properties of the fluid, such as flow velocity, pressure, density, and temperature, as functions of space and time. Before the twentieth century, hydrodynamics was synonymous with fluid dynamics. This is still reflected in names of some fluid dynamics topics, like magnetohydrodynamics and hydrodynamic stability, both of which can also be applied to gases (Munson et al. 1990; Yunus and Cimbala 2006; Potter and Wiggert 2008).

Magnetohydrodynamics is also known as MHD or magneto-fluid dynamics or hydromagnetics which is the study of the magnetic properties of electrically conducting fluids. Examples of such magneto-fluids include plasmas (electrically conducting gas), liquid metals, and salt water or electrolytes. The word MHD is derived from magneto—meaning magnetic field, hydro—meaning water, and dynamics—meaning movement. One of the most famous scholars associated with MHD was the Swedish Physicist Hannes Alfvén (1908–1995), who received the Nobel Prize in Physics (1970) for fundamental work and discoveries in magneto-hydrodynamics with fruitful applications in different parts of plasma physics. The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn polarizes the fluid and reciprocally changes the magnetic field itself. The set of equations that describe MHD are a combination of the Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. These differential equations must be solved simultaneously, either analytically or numerically.

A nanofluid is a fluid containing nanometer-sized particles, called nanoparticles. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid. The nanoparticles used in nanofluids are typically made of metals, oxides, carbides, or carbon nanotubes. Common base fluids include water, ethylene glycol, and oil. Nanotechnology is being used or considered for use in many applications targeted to provide cleaner, more efficient energy supplies and uses. While many of these applications may not affect energy transmission directly, each has the potential to reduce the need for the electricity, petroleum distillate fuel, or natural gas that would otherwise be moved through energy transmission system. More efficient energy generation and use may decrease the amount of construction, maintenance, repair, and decommissioning activities. Examples of how nanotechnology may be integrated into each of these technological areas are: engine cooling, engine transmission oil, in diesel electric generator as jacket water coolant, boiler exhaust flue gas recovery, heating and cooling of buildings, cooling of electronics, cooling of welding, nanofluids in transformer cooling oil, nuclear systems cooling, solar water heating, nanofluids in drilling, refrigeration (domestic refrigerator, chillers), defense, space, high-power lasers, microwave tubes, biomedical applications, drilling, lubrications, thermal storage, drag reductions. Nanofluids in solar collectors are another application where nanofluids are employed for their tunable optical properties.

In analysis such as computational fluid dynamics (CFD), nanofluids can be assumed to be single-phase fluids. However, almost all of new academic papers use two-phase assumption. Classical theory of single-phase fluids can be applied, where physical properties of nanofluid are taken as a function of properties of both constituents and their concentrations. An alternative approach simulates nanofluids using a two-component model (Saidur et al. 2011).

The spreading of a nanofluid droplet is enhanced by the solid-like ordering structure of nanoparticles assembled near the contact line by diffusion, which gives rise to a structural disjoining pressure in the vicinity of the contact line. However, such enhancement is not observed for small droplets with diameter of nanometer scale, because the wetting time scale is much smaller than the diffusion time scale.

The problem of solute transport in porous media has become a major research area in hydrology over the last five decades. Two fundamental worldwide issues are directly concerned (1) the selection of repository sites for radioactive waste in deep geological formations, and (2) groundwater pollution in fractured/unfractured reservoirs. The objective of this section is to investigate the roles of pore-water velocity, water content, and soil textural properties on solute dispersion in saturated/unsaturated porous media. There are many authors who have examined the process of dispersion in fully saturated porous media. Their finding suggested a linear relationship between the longitudinal dispersion coefficient and the pore-water velocity. In this section, seepage of water through earth dams, underneath hydraulic structures and problems related the supply of drinking water, a mathematical models are employed to predict the fate of hazardous chemicals environmental problems. Waterjet is a generic term used to describe equipment that uses a high-pressure stream of water for cutting or cleaning purposes. Abrasive jet is a subcategory of waterjet in which abrasive is introduced to accelerate the process. In other words, abrasive jets and pure waterjet are kinds of waterjet which is a kind of machinery. We can find the various applications in geophysics, earthquakes, astrophysics, sensors, engineering, magnetic drug targeting, etc.

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Part I
Fluid Dynamics

Unsteady MHD Nanobioconvective Stagnation Slip Flow in a Porous Medium Due to Exponentially Stretching Sheet Containing Microorganisms

R. Kumar and S. Sood

Abstract This communication deals with the numerical investigation of bioconvection induced by an unsteady MHD stagnation point flow of a nanoliquid containing suspension of microorganisms over a stretching sheet. The sheet is stretched in an exponential fashion and set at the right side of porous medium saturated with nanofluid, and permeability of porous medium is considered to have a specified form. The setup deals with the velocity slip and thermal slip at the sheet surface. Here, water is considered as the carrier liquid. Similarity transformations are used to convert the governing coupled nonlinear partial differential equations into ordinary differential equations and solved numerically by employing implicit finite difference scheme known as Keller box method. The effects of nanofluid parameters and bioconvection parameters on non-dimensional velocity, temperature, nanoparticle concentration, and motile microorganism concentration are presented through graphs. The effects of related parameters on local skin friction, Nusselt number, Sherwood number, and density number of microorganisms are exhibited through tables. The substantial influence of bioconvection parameters is noticed on the profiles of velocity, temperature, nanoparticle volume fraction, and density of microorganisms.

1 Introduction

Bioconvection caused by the suppression of self-driven microorganisms in a base fluid is an area in fluid mechanics which have attracted the attention of several scientists and engineers due to its various applications in industry, technology, and environmental sciences. It is a mesoscale phenomenon, in which the motion of

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motile microorganisms induces a macroscopic motion in the fluid. These microorganisms have active response to external stimuli such as light, gravity, chemical reactions and oxygen, and the responses are termed as 'taxes'. The pioneering work of Pedley et al. (1988) on the bioconvection growth patterns due to suspension of gyrotactic microorganisms framed the basis for its detailed analysis. Later on, the paper of Bees and Hill (1997, 1999) and Ghorai and Hill (2000) on the wavelength of bioconvection patterns appeared in the literature. Very recently, in linear stability analysis, Hwang and Pedley (2014) showed that uniform shear can significantly alter the gravitaxis microorganisms. Simultaneously, the work on heat transfer analysis of nanofluid (coined by Choi 1995) also developed to a great extent. The suspension of metallic oxides including copper, aluminum, titanium as nanoparticles in base fluid is known to enhance the thermal conductivity of the base fluid tremendously. Moreover, nanoparticles like silver have hygiene features, and can also be utilized in wound treatment (Lee et al. 2007) and can be used as an anti-bacterial agent (Lee et al. 2003). The concept of bioconvection in nanofluid was introduced by Kuznetsov (2010), in which he claimed that the self-driven microorganisms enhance mixing and prevent nanoparticles from agglomeration in nanofluids. The suspension of nanoparticles and motile microorganisms in base fluid will not only be helpful in enhancing the thermal performances but would also be useful in producing next-generation biofuels with green and sustainable features (Chiu et al. 2011). Mutuku and Makinde (2014) presented a numerical solution on the hydromagnetic bioconvection flow of nanofluid along a vertical permeable plate due to gyrotactic microorganisms. Further, the introduction of porous medium into a flow field plays a vital role in dampening the convective effects caused by bioconvection. New developments in bioconvection in porous medium can be found in Vadasz (2008). In this sequence, Aziz et al. (2012) investigated the effects of free convection on the boundary layer flow of a nanofluid over a horizontal flat plate embedded in porous medium which is filled by nanoparticles and gyrotactic microorganisms. Shaw et al. (2014) extended this problem by analyzing the magnetohydrodynamics and soret effects and considering the flow across an inclined plate to justify the influence of gravity. Their study further demonstrates the substantial effect of magnetic field on the microorganisms behavior.

In the above cited investigations, emphasis has been given on the steady-state problem, which relies on no-slip conditions. But in certain situations, velocity may slip on a stretching boundary when the fluid contains particulates in the form of suspensions, polymer solutions, and emulsions. Due to this, the Navier slip model has been employed by various researchers to depict the jump in velocity and temperature on the surface of the wall. Navier slip effects on the MHD boundary layer flow of a nanofluid containing gyrotactic microorganisms over a vertical plate have been discussed by Khan et al. (2014). Very recently, Uddin et al. (2016) investigated the Stefan blowing and multiple slip effects on the bioconvection nanofluid flow. This study revealed that microorganisms' propulsion can be enhanced, higher flow velocity can be achieved, and nanoparticle distributions can be improved by the introduction of Stefan blowing through the stretching/shrinking surface. On the other hand, the flow can turn out to be time dependent due to the impulsive stretching of surface or when there is step change in the temperature of the surface. Raees et al. (2015) analyzed unsteady mixed nanobioconvection flow in a horizontal channel and found monotonic decrease in velocity components and continuous decrease in temperature with increasing time. The influence of magnetic field and bioconvection on the stagnation point flow of nanofluid over a stretching surface has been presented by Ahmed et al. (2015).

Though many research paper can be found in literature on the nanofluid bioconvection concepts but many characteristics of these latest fluids are yet to be explored at fundamental level. Hence, an attempt has been made to reveal the effects of velocity and thermal slip on the unsteady mass stagnation flow of nanofluid over a vertical exponentially stretching sheet embedded in a porous medium which is filled with nanoparticles and microorganisms.

2 Mathematical Formulation of the Problem

We considered an unsteady boundary layer stagnation point flow of a water-based nanofluid having a suspension of gyrotactic microorganisms over a vertical sheet embedded in a porous medium. The porous medium is filled with nanoparticles and microorganisms. A Cartesian coordinate system is considered, in which x-axis is measured along the surface and y-axis normal to it. The flow is induced by the exponential stretching of the surface and slip effects under the following assumptions: (i) suspension of nanoparticles is stable and dilute; (ii) nanoparticles do not alter the direction and velocity of swimming microorganisms; (iii) shape of microorganisms is uniform and no extra cellular polymers are produced on the surface of microorganisms; (iv) pore size of porous medium is significantly larger so that microorganisms can penetrate freely through the pores; and (v) porous matrix does not absorb microorganisms. Under above said assumptions, the boundary layer equations utilizing Boussinesq approximation and extended Darcy model for porous medium are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\begin{aligned}\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= \frac{\partial U_e}{\partial x} + \frac{1}{\rho_f}[(1 - C_\infty)\beta g(T - T_\infty) \\ &\quad -(\rho_p - \rho_f)g(C - C_\infty) - (n - n_\infty)g\gamma(\rho_m - \rho_f)] \\ &\quad + U_e\frac{\partial U_e}{\partial x} + \vartheta\frac{\partial^2 u}{\partial y^2} + \frac{\vartheta}{k_0}(U_e - u) + \frac{\sigma B_0^2}{\rho}(U_e - u),\end{aligned} \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left\{D_B\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}\right\}, \tag{3}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

$$\frac{\partial n}{\partial t}+u\frac{\partial n}{\partial x}+v\frac{\partial n}{\partial y}=D_B\frac{\partial^2 n}{\partial y^2}-\frac{b_1 W_c}{(C_w-C_\infty)}\left[\frac{\partial}{\partial y}\left(n\frac{\partial C}{\partial y}\right)\right], \tag{5}$$

where u and v are the respective components of velocity along x and y directions. Here, $v, \mu, k_0, \sigma, \rho_f, g, T, \alpha, \tau = \frac{(\rho C)_p}{(\rho C)_f}, D_B, D_T, D_m, C, n, b_1$ and W_c are kinematic viscosity, dynamic viscosity, permeability, electrical conductivity, density of the base fluid, gravity, volume expansion coefficient, average volume of microorganisms, temperature, thermal diffusivity, volumetric heat capacitance ratio of nanoparticle to base fluid, coefficients of Brownian diffusion, thermophoretic diffusion, microorganisms diffusion, nanoparticle volume fraction, concentration of motile microorganisms, chemotaxis constant, and maximum swimming speed of microorganisms, respectively.

The relevant boundary conditions are as follows:

$$\begin{aligned}
&u=v=0,\quad T=T_\infty,\quad C=C_\infty,\quad n=n_\infty \quad \text{at} \quad t\leq 0,\\
&v=0,\quad u=U_w(x,t)+\vartheta N_1\frac{\partial u}{\partial y},\quad T=T_w+D_1\frac{\partial T}{\partial y},\\
&C=C_w(x,t),\quad n=n_w(x,t) \quad \text{at} \quad y=0,(t>0)
\end{aligned} \tag{6}$$

$$u=U_e(x,t),\quad T=T_\infty,\quad C=C_\infty,\quad n=n_\infty \quad \text{at} \quad y\to\infty,\ (t>0), \tag{7}$$

where N_1 and D_1 are the velocity and thermal slip respectively. The velocity, temperature and concentration at the surface of the sheet are considered in the following syntax: $U_w(x,t)=\frac{b\exp(x/l)}{1-ct}$, $T_w=T_\infty+\frac{T_0\exp(x/2l)}{1-ct}$, $C_w=C_\infty+\frac{C_0\exp(x/2l)}{1-ct}$, $n_w=n_\infty+\frac{n_0\exp(x/2l)}{1-ct}$, respectively. $U_w(x,t)=\frac{a\exp(x/l)}{1-ct}$ is external flow velocity. The following similarity transformations have been introduced to reduce the set of Eqs. (1)–(7) into self-similar form:

$$\begin{aligned}
&\eta=\sqrt{\frac{a\exp(x/l)}{2\vartheta L(1-ct)}}y,\quad \psi=\sqrt{\frac{2\vartheta La\exp\left(\frac{x}{l}\right)}{(1-ct)}}f(\eta),\quad \theta(\eta)=\frac{(T-T_\infty)}{(T_w-T_\infty)},\\
&\phi(\eta)=\frac{(C-C_\infty)}{(C_w-C_\infty)},\quad \xi(\eta)=\frac{(n-n_\infty)}{(n_w-n_\infty)},\quad u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}.
\end{aligned} \tag{8}$$

In above similarity transformations, we assumed that $(1-ct)>0$, and hence, the similarity transformation is valid only for $t<\frac{1}{c}$.

The following set of modified equations is obtained after the application of above transformations:

$$\begin{aligned} & f''' + ff'' - 2f'^2 + 2 + M(1 - f') + K(1 - f') \\ & \quad - A(\eta f'' + 2f' - 2) + \lambda(\theta - Nr\phi - Rb\xi) = 0 \end{aligned} \tag{9}$$

$$\theta'' - APr(\eta\theta' + 2\theta) + \Pr(\theta' f - f'\theta) + Nb\phi'\theta' + Nt\theta'^2 = 0, \tag{10}$$

$$\phi'' - ASc(\eta\phi' + 2\phi) + Sc(\phi' f - f'\phi) + \frac{Nt}{Nb}\theta'' = 0, \tag{11}$$

$$\xi'' - ALb(\eta\xi' + 2\xi) + Lb(\xi' f - f'\xi) - Pe[\phi''(\xi + \omega) + \xi'\phi'] = 0. \tag{12}$$

The boundary conditions will take the following altered form:

$$\begin{aligned} & f(0) = 0, \quad f'(0) = \frac{b}{a} + Nf''(0), \\ & \theta(0) = 1 + D\theta'^{(0)}, \quad \phi(0) = 1, \quad \xi(0) = 1. \end{aligned} \tag{13}$$

$$f'(\infty) = 1, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0, \quad \xi(\infty) = 0, \tag{14}$$

where prime signifies differentiation with respect to h, and in the meanwhile, following dimensionless parameters are ensued: $A = \frac{Lc}{U_e}, M = \frac{2\sigma B_0^2 L}{U_e \rho_f}$, $K = \frac{2\vartheta L}{kU_e}$, $\lambda = \frac{2L(1-C_\infty)\beta g(T_w - T_\infty)}{U_e^2}, Nr = \frac{(\rho_p - \rho_f)(\phi_w - \phi_\infty)}{\rho_f \beta(1-C_\infty)(T_w - T_\infty)}, Rb = \frac{(\rho_m - \rho_f)(n_w - n_\infty)}{\rho_f \beta(1-C_\infty)(T_w - T_\infty)}, Pr = \frac{\vartheta}{\alpha}, Nb = \frac{\tau D_B(\phi_w - \phi_\infty)}{\alpha}$, $Nt = \frac{\tau D_B(T_w - T_\infty)}{T_\infty \alpha}$, $Sc = \frac{\vartheta}{D_B}$, $Lb = \frac{\vartheta}{D_m}$, $Pe = \frac{bW_c}{D_m}$, $\omega = \frac{n_\infty}{(n_w - n_\infty)}$, $\varepsilon = \frac{b}{a}$, $N = N_1\sqrt{\frac{\vartheta U_e}{2L}}$, $D = D_1\sqrt{\frac{U_e}{2\vartheta L}}$ which represent unsteady parameter, magnetic parameter, permeability parameter, mixed convection parameter, buoyancy ratio parameter, bioconvection Rayleigh number, Prandtl Number, Brownian motion parameters, thermophoresis parameter, Schmidt number, bioconvection Lewis number, bioconvection Peclet number, microorganisms concentration difference parameter, velocity ratio parameter, velocity slip parameter, and thermal slip parameter, respectively. In this study, the quantities of engineering interest are c_f, Nu, Sh, and Nn, which are termed as coefficient of skin friction, Nusselt number, Sherwood number, and density number of motile microorganisms, respectively. These physical quantities are defined mathematically as follows:

$$\begin{aligned} & c_f = \frac{\mu}{\rho_f U_e^2}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad Nu = -\frac{Lk}{k(T_w - T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0}, \\ & Sh = -\frac{L}{(C_w - C_\infty)}\left(\frac{\partial C}{\partial y}\right)_{y=0}, \quad Nn = -\frac{L}{(n_w - n_\infty)}\left(\frac{\partial n}{\partial y}\right)_{y=0} \end{aligned} \tag{15}$$

After substituting Eq. (8) in (15), we reckoned that

$$\begin{aligned} c_{fx} &= \sqrt{2Re_x}c_f, \quad Nu_x = \frac{Nu}{Re_x} = -\sqrt{\frac{x}{2L}}\theta'(0), \\ Sh_x &= \frac{Sh}{Re_x} == -\sqrt{\frac{x}{2L}}\phi'(0), \quad Nn_x = \frac{Nn}{Re_x} = -\sqrt{\frac{x}{2L}}\xi'(0) \end{aligned} \tag{16}$$

Here, k is thermal conductivity of nanofluid. Above stated quantities are the local skin friction which specifies the wall shear stress, local Nusselt number which connotes the heat transfer rate, local Sherwood number which indicates the mass transfer rate, and local density number of motile microorganisms which represents the microorganisms transfer rate respectively.

3 Results and Discussion

In order to gain physical insight into the flow problem, rigorous numerical computations have been performed for different values of pertinent parameters that describe the flow features. Implicit finite difference method (IFDM) also known as Keller box method (KBM) has been employed for the numerical solutions of strongly coupled boundary layer equations due to the limitations of closed-form solutions. The maximum values for η_∞ have been chosen as $\eta_\infty = 2$ to $\eta_\infty = 7$, which lies well outside the boundary layers and provides the mesh-independent numerical results. By choosing a grid size $\Delta\eta$ = 0.005, the whole computational procedure is administered with the aid of computational language MATLAB. The numerical computations are accomplished for various values of parameters, which have been displayed through figures and tables. During the whole computational process, the values to related parameters are assigned as $D = 0.1$, $A = M = K = \lambda = Sc = Pe = Lb = 1$, $N = \varepsilon = 0.5$, $Pr = 6.2$, $Nr = Nb = Nt = Rb = \omega = 0.1$, unless otherwise stated.

The variations of local skin friction coefficient $f''(0)$, heat transfer coefficient, $-\theta'(0)$, mass transfer coefficient $-\phi'(0)$, and local density number of motile microorganisms $-\xi'(0)$ with respect to nanofluid, and bioconvection parameters have been shown in Tables 1 and 2. It is noticed that coefficient of local skin friction $f''(0)$ and wall heat transfer rate $-\theta'(0)$ are decreased with increasing values of $N = D = K = M = \lambda = Nr = Rb = Nt = Sc = Pe$ and Lb, whereas an enhancing influence of unsteadiness parameter A is observed on $f''(0)$ and $-\theta'(0)$. On the other hand, augmentation in wall mass transfer rate $-\phi'(0)$ and local density number of motile microorganisms $-\xi'(0)$ is observed with an increment in the values of parameters like $A = N = D = K = M = \lambda = Nr = Rb = Sc = Pe$