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Anthony Teolis

# Computational Signal Processing with Wavelets

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# Dedication

To my two favorite playmates, Spencer and Trevor, who relinquished, without their consent or pardon, many hours, Saturdays, and weeknights spent with Dad.

# Contents

# Preface

## Overview

For over a decade now, wavelets have been and continue to be an evolving subject of intense interest. Their allure in signal processing is due to many factors, not the least of which is that they offer an intuitively satisfying view of signals as being composed of *little* pieces of *waves*. Making this concept mathematically precise has resulted in a deep and sophisticated wavelet theory that has seemingly limitless applications.

This book and its supplementary hands-on electronic component are meant to appeal to both students and professionals. Mathematics and engineering students at the undergraduate and graduate levels will benefit greatly from the introductory treatment of the subject. Professionals and advanced students will find the overcomplete approach to signal representation and processing of great value. In all cases the electronic component of the proposed work greatly enhances its appeal by providing interactive numerical illustrations.

A main goal is to provide a bridge between the theory and practice of wavelet-based signal processing. Intended to give the reader a balanced look at the subject, this book emphasizes both theoretical and practical issues of wavelet processing. A great deal of exposition is given in the beginning chapters and is meant to give the reader a firm understanding of the basics of the discrete and continuous wavelet transforms and their relationship. Later chapters promote the idea that overcomplete systems of wavelets are a rich and largely unexplored area that have demonstrable benefits to offer in many applications.

In addition to the text, there is also supporting MATLAB based software that is graphically oriented and provides a computational platform for exploration and illustration of many of the ideas and algorithms presented here. The software includes comprehensive graphical interfaces for high level interaction as well as hundreds of low-level object-oriented methods for general signal processing.

## Organization and Features

The book while written for senior or beginning graduate students in mathematics or engineering, is also accessible to professionals and practitioners in the signal processing community. Technical prerequisites include an undergraduate level knowledge of linear algebra, linear systems theory, Fourier transform theory, and a working knowledge of MATLAB basic functionality. Additional familiarity with operator theory and real analysis is helpful but not required.

Beginning chapters are expository in nature and describe basic notation, concepts, orthonormal wavelets, and frames. Later chapters depart slightly from the mainstream of wavelet theory and instead emphasize *overcomplete* representations of signals as opposed to the more widely used *orthonormal* representations associated with the discrete wavelet transform. Finally, the presentation becomes more numerically oriented in the last chapters where the benefits of overcomplete wavelet representations are explored in various applications. These numerical explorations are fully reproducible and extensible using the available software. The impatient and/or curious reader is encouraged to start there.

This work is geared towards practical application and numerical implementation of wavelet-based algorithms supported by a solid mathematical foundation. Some of its main features are listed as follows.

- An expository treatment of the following topics are included:
  - continuous and discrete Fourier transforms,
  - orthonormal and biorthogonal bases,
  - frames, wavelet frames, and reconstruction,
  - discrete wavelet transform and orthonormal wavelets,
  - classical sampling theorem, and
  - regular and irregular sampling and reconstruction.
- A frame-based theory of the discretization and reconstruction of analog signals is developed in terms of the sampling of a continuous transform.
- The continuous wavelet and Gabor transforms are introduced in a unified group-theoretic setting.
- Concepts and techniques are numerically demonstrated through
  - software reproducible examples,
  - interactive graphical user interfaces, and
  - over 120 traditional static figures.
- Problem exercises are given at the end of each major chapter to reinforce concepts and ideas.
- A new and efficient *overcomplete* wavelet transform is introduced and applied to the tasks of

- noise suppression,
- compression,
- digital communication, and
- identification.

Chapter 1 describes the motivations and objectives of the entire book and provides an overall perspective to the material. Chapter 2 introduces the notation and basic mathematical concepts used throughout the text. Chapter 3 discusses mathematical frames and their use as signal representations as well as algorithms for reconstruction of signals from their frame representations. Chapter 4 presents the continuous wavelet and Gabor transforms and also provides a unified view of them in terms of frame representations. Chapter 5 reviews the discrete wavelet transform, multiresolution analysis, and the construction of compactly supported orthonormal wavelet bases. The fast wavelet transform is also described there. Chapter 6 introduces the overcomplete wavelet transform, its inverse, and their filter bank implementations. Chapter 7 presents several applications of wavelet-based signal processing including noise suppression, signal compression, and identification. Finally, Chapter 8 describes the supporting object-oriented MATLAB code which has been used to numerically illustrate the material.

## Computational Aspects

Numerical examples presented in this book have all been computed using a suite of object-oriented tools developed in MATLAB 5 called the Wavelet Signal Processing Workstation (WSPW). The material includes a demonstration copy of the WSPW and is available through the Internet. This object-oriented wavelet signal processing software (MATLAB) is available at the Web site

`www.birkhauser.com/book/ISBN/0-8176-3909-8`

The software requires MATLAB version 5.0 or later to run and has been tested on operating systems including Windows 95 and the various flavors of UNIX and LINUX.

## Acknowledgments

Many people have aided the production of this project and I am greatly indebted to all. There are several individuals and organizations whose support demands special mention and they are listed in the following.

First, and foremost, is my wife, Carole. In addition to being my wife and best friend, she has served as my main supporter, in both the technical and emotional senses. She has tolerated my frequent absences from the affairs of

family and friends all for the benefit of this project. Balancing the demands of our two children, her own career, and myself during this time has been no small sacrifice. To her I cannot be too grateful.

I am grateful to the Naval Research Laboratory (NRL) in Washington, DC, where I have worked as a contractor for the past four years in the Tactical Electronic Warfare Division. Many of the aspects of my work there are reflected in this book. In particular, I am indebted to Dr. Charles Heider, Mr. Roger Oxley, and Dr. Joseph Lawrence for providing me with the intellectual freedom, resources, and general support needed to pursue and complete this project. Specifically, NRL branches 5740 and 5720 have provided a stimulating and exciting working environment that has contributed positively in many ways to the production of this work.

I thank my company, AIMS, Inc., and its founders John and Mary Baras, as well as all my fellow employees for their continued support during this project. Deserving of special note is Rob Mentle who, in addition to being my coworker and friend, has served as proofreader, reviewer, system administrator, and stock advisor.

For introducing me to frames and irregular sampling from a mathematically sophisticated viewpoint, John Benedetto deserves special thanks. Much of the material presented here on noise suppression and compression has grown from joint work on those subjects.

This work has also benefited greatly from the comments of numerous anonymous and nonanonymous reviewers. To them all I am very much indebted. Of special note is Hans Feichtinger of the University of Vienna. His deep understanding, unique perspective, and pervasive and penetrating commentary on early versions of the manuscript have resulted in this much improved version of the current work.

Any errors which may remain are the sole responsibility of the author. Any comments, suggestions, bug reports, errors, or thoughts of any kind, would be most welcome by the author. Please send your email to

`tonyt@palindrome.nrl.navy.mil`

Special thanks is owed to all the people at Birkhäuser who helped make this project become a reality. These include Wayne Yuhasz, who first suggested this project to me and has provided invaluable guidance throughout its preparation, and Lauren Lavery who has handled many of the details of its production.

In addition, the contributing support of the following contracts is gratefully acknowledged: (NRL) N00014-C-93-2093, (NRL/DARPA) N00014-C-95-2043, (ONR/DARPA) N00014-C-97-0364, and (CECOM) DAAB07-97-C-A753.

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# Introduction

## 1.1 Motivation and Objectives

Although the theory of wavelet analysis is a relatively new and still evolving discipline, there is a deep and sophisticated body of work currently available. Much of this work, however, requires a fairly in-depth knowledge of several areas of advanced mathematics and hence limits its accessibility. It is a main objective of this work to strike a balance between accessibility and mathematical rigor that sacrifices as little as possible of both. To help achieve this goal, the dissemination of the material is provided by a hybrid combination of traditional (text) and nontraditional (Internet and electronic) media.

Despite the project's multifaceted nature, the traditional text component is designed as the primary vehicle for delivery of the material. This has been done with the intent that the text be useful as a standalone reference. Supporting the text is the electronic component of the material that provides a dynamic and interactive aspect. It consists of both software and (Web-accessible) hypertext documents. In this way interactive illustration of signal processing concepts and techniques are provided in an effective, compelling, and practically useful way.

An underlying goal of the material presented in this work is to provide a bridge between the theory and practice of signal processing with particular emphasis on generally overcomplete wavelet techniques. Despite the fact that the *practice* of signal processing is a wide and varied one, the term *practical signal processing* is used in the context of this material with a very specific meaning: the numerical implementation of techniques for signal manipulation on a finite precision digital machine.

## 1.2 Core Material and Development

A main thread of this work is the idea that overcomplete systems of wavelets are a rich and largely unexplored area that have great benefits to offer in



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many applications. Starting from the continuous wavelet transformation, a mathematically sound theory of the discretization of analog signals is developed. The development yields a rich family of signal representations and leads naturally to computer implementations. The discussion is at that of a senior or beginning graduate student level and is accessible to professionals in the signal processing community.

Numerical illustration of concepts and techniques are facilitated through software reproducible examples, interactive graphical user interfaces, as well as traditional static figures in the text. This work is geared towards practical application and numerical implementation of wavelet-based algorithms. As such it includes working interactive software demonstrations available from the Internet-accessible Web site:

`www.birkhauser.com/book/ISBN/0-8176-3909-8`

## 1.3 Hybrid Media Components

This text is but one piece of a larger body of material presented in a hybrid media form consisting of print, electronic, and software components. Specifically the material consists of

- an expository theoretical treatment of the discrete and continuous wavelet transformations with an emphasis on discretization through sampling of the continuous wavelet transform;
- an applications-oriented presentation illustrated via numerical examples on synthetic and real data; and
- an electronic component in the form of an Internet-accessible Web page that includes down-loadable MATLAB-based code with the following capabilities:
  - (i) reproducing examples presented in the text,
  - (ii) conducting numerical experiments as suggested in the text,
  - (iii) applying algorithms described in the text on data provided by the user, and
  - (iv) designing new algorithms from component modules on user-defined signal processing tasks.

Despite the fact that this text is just part of the entire hybrid media work, it is meant to be a self-contained document. The electronic components, however, are an invaluable supplement to the text and shall be updated and modified continually.

## 1.4 Signal Processing Perspective

### *1.4.1 Analog Signals*

A main philosophy adhered to in this book is the idea that the fundamental underlying objects of interest are analog (as opposed to discrete or digital) signals.

In most real-world applications the fundamental signals of interest are analog in nature. For example, the variation in air pressure caused by a sound source or the intensity of electromagnetic energy reflected by an object illuminated by an active source may be considered as analog signals. An analog signal is one whose domain has no measurable gaps, for example, $f(t)$ where $t$ may take arbitrary values over the whole real line. In contrast a discrete signal is one whose domain is restricted to a countable set of points, for example, $f(t_n)$ where $n$ is restricted to be an integer.

Whether signals come from the audio, visible and/or infrared, or microwave areas of the electromagnetic spectrum, *signal processing* generally involves a prescribed manipulation in order to achieve some useful goal such as communication, compression, or information extraction. Depending on the area of the electromagnetic spectrum of interest, such manipulations may be identified with sound, image, and radar signal processing.

### *1.4.2 Digital Processing of Analog Signals*

Although the signals of interest in many applications are inherently analog in nature, digital platforms have fast become the primary vehicle for implementation of signal processing algorithms and techniques. This situation is due not only to the proliferation and ever-increasing computational power to cost ratio of digital platforms, but also to the established and demonstrable benefits of digital processing. Perhaps the best example of this comes from the audio (voice and music) reproduction industries ([Gib93]) in which the superiority of digital coding for both communication and archival (via compact disc) has been firmly established. Reproductions based on digital techniques attain a level of fidelity unsurpassed by former analog techniques. Other benefits associated with digital based signal processing include the abilities to

- manipulate signals via digital processors,
- store/archive signals on digital media,
- propagate signals via digital networks (e.g., the Internet), and
- achieve a high degree of noise robustness.

Digital techniques and processing offer a host of desirable qualities. On the other hand, signals of interest are fundamentally analog in nature.

Clearly, there exists a gap that must be bridged in order to process analog signals via digital platforms. In order to manipulate a signal via a digital platform it is not only necessary to discretize the domain and range of the signal, but also to restrict the extent of those discretizations to some finite interval. The process of doing so yields a discrete representation of the underlying analog signal.

In practice, the analog-to-digital gap is routinely bridged by the direct digital sampling of analog signals. Theoretically, the direct sampling and reconstruction (of bandlimited signals) is completely understood via the classical sampling theorem (viz. Theorem 2.6 on page 23); and what's more, the theory is successfully and widely implemented in practical systems.

Because of these facts there is strong justification for focusing attention purely on the digital domain processing of discrete signals under the three-step processing model of

1. sample all analog signals as prescribed by the sampling theorem (A/D),
2. manipulate data in the digital domain, and,
3. (possibly) transform back to the analog domain (D/A).

Taking this view necessarily has the consequence of ignoring the analog origins of the signals. A contention held here is that there are both practical benefits and theoretical insights to be gained by considering the process as a whole from its analog domain of origin. From a theoretical point of view, in many respects, it is easier to deal directly with the original analog space than its discrete counterpart. In fact, there is a wealth of existing theory and understanding associated with analog spaces, for example, spaces of bandlimited or finite energy functions.

In this more general context, methods other than direct sampling may be considered for discretizing an analog signal. One drawback of direct digital sampling is that a fixed frequency extent (bandwidth) is supposed on the signal over its (infinite) duration. Accordingly, the sampling theorem requires that the signal be (uniformly) sampled at a rate inversely proportional to this fixed frequency extent.

Thus, both periods of high frequency content and low frequency content are sampled at a rate that is governed by the highest frequency content of the signal. This leads to the intuitively unappealing situation that many signals may be "oversampled" over most of their duration. As an example, consider a signal that has a burst of high frequency energy highly concentrated in time. The direct sampling of this signal is deficient in the sense that the sampling rate is required to be constant, resulting in critical sampling over the burst and oversampling elsewhere. Stated succinctly, direct sampling is insensitive to fluctuations over time in the frequency content of a signal.

Signals that exhibit structured time–frequency behaviors abound in natural and manmade systems. For such signals the fluctuation of their frequency content over time varies coherently. As an example consider the sound of a gong or steam engine whistle. In both cases their characteristic sound may be understood in terms of a coherent fluctuation of frequency content over time. Similar statements can be made for speech, radar, and many signals generated by mechanical means. Such signals are said to be *time–frequency coherent.*

For the discretization (and processing) of time–frequency coherent signals, the wavelet transform offers itself as a natural tool. For one-dimensional signals the (continuous) wavelet transform yields a two dimensional function of time and frequency. At any fixed value of time, the magnitude of the wavelet transform indicates the frequencies present in the signal. In this way, a wavelet transform has the ability to expose the time–frequency content of a signal. An alternative to the direct sampling of a time–frequency coherent signal is to first (in the analog domain) *continuously* wavelet transform the signal and then sample the wavelet transform. Sampling strategies may then be prescribed that are sensitive to the signal's time–frequency content.

### 1.4.3 Time–Frequency Limitedness

A basic assumption of direct sampling is that the analog signal to be sampled is bandlimited. On the one hand, it is intuitive that practical signals can have neither infinite duration nor infinite bandwidth; yet, on the other hand, fundamental mathematical considerations preclude the existence of simultaneously timelimited and bandlimited signals. This is the so-called paradox of simultaneously timelimited and bandlimited signals. One cause of this paradox comes from the very concept of limitedness itself, that is, the idea that a signal is *exactly* zero outside some finite interval. From a practical viewpoint, it is not possible to measure a signal to enough accuracy to determine if it is exactly zero and, hence, assuming so is nothing more than a mathematical convenience. An assumption of limitedness has ramifications that may lead to various paradoxes and must therefore be used with caution. Even so, it is undeniable that real-world signals are of finite duration. A possible resolution to this dilemma may be attained by taking a more practical, less stringent, definition of duration that allows the signal to be nonzero outside a finite interval yet requires that the most significant portion of the signal be resident in a finite interval.

Such a view is presented by David Slepian in [Sle76] who introduces the idea of $\epsilon$-distinguishability between two signals. In particular, two signals of time $f(t)$ and $g(t)$ are *distinguishable* if their difference has sufficient energy; that is,

$$\int_{-\infty}^{\infty} |f(t) - g(t)|^2 > \epsilon.$$

This concept leads, in turn, to obvious definitions of $\epsilon$ timelimitedness and bandlimitedness (cf. Section 7.3.4). In this realm, the prolate spheroidal wave functions ([SP61],[SL61]) play a key role. The interested reader is referred to [Sle76].