**Particle Acceleration and Detection** 

# Ingo Hofmann

# Space Charge Physics for Particle Accelerators



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# Space Charge Physics for Particle Accelerators



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### Preface

The motivation for this book on space charge in particle accelerators has emerged from the continuing interest in the understanding and controlling of space charge effects in operating high-intensity particle accelerators and the numerous projects still under construction or in development, many of which are the world's largest instruments at the frontier of scientific and technical development.

This book focuses to a large extent on the author's angle on theoretical concepts of resonances and instabilities, in particular their coherent expressions, and attempts to connect them with simulation results and – in a limited number of cases – with experiments. Although these topics are well known in the accelerator community in the broad context of impedances or wake fields, their application to direct space charge is not yet equally well established. Utilizing terms like *coherent space charge resonances* or *coherent parametric instabilities* may be sometimes challenging; it is hoped that they will be useful to create a more systematic and differentiating picture of space charge effects, which is the primary scope of this book. It is thus complementary to existing textbooks on accelerators and beam dynamics with their much broader scope, which are needed for understanding themes that could not be adequately addressed in the format of this book.

The application of the material presented here is seen in the field of *linear* hadron accelerators at non-relativistic energies, but also in space charge issues in *circular* accelerators, like injector synchrotrons, all at basically non-relativistic energies, where direct space charge issues are of concern.

A personal remark: In preparing this manuscript, I have found time and again how challenging it is to map theoretical concepts of space charge effects to the boundary conditions of real accelerators. This is particularly true for linear accelerators, where a high level of space charge is embedded in often quite complex and transient acceleration structures.

Nonetheless it is hoped that the material presented here may be useful to all those who find that running a computer simulation code is not enough and who believe that trying to understand the gap between analytical concepts, multiparticle simulations and experiments is the best way to advance. I am grateful to many colleagues at GSI Darmstadt, Goethe-Universität Frankfurt and Technische Universität Darmstadt, among them Oliver Boine-Frankenheim, Giuliano Franchetti, Lars Groening, Vladimir Kornilov and Jürgen Struckmeier, for the many valuable discussions. Among my international colleagues, I am particularly indebted to the late Martin Reiser, who shared his insight into space charge over many years, which I cannot adequately value. I am also grateful to Bob Jameson, who stimulated the early work on beam anisotropy; to the late Bob Gluckstern and to Robert Ryne and Tom Wangler for many inspiring discussions; and to Didier Uriot for his support, in particular by developing helpful diagnostic features (such as stability charts and tune footprints) in TRACEWIN. Last but not least, I am extremely grateful for having received highly constructive and appreciated comments on the full manuscript by Morteza Aslaninejad, Bob Jameson and Uwe Niedermayer and by Yong Liu, Ji Qiang and Rob Ryne on selected topics. Thanks go to Alex Chao also for his valuable conceptual comments.

Darmstadt, Germany June 2017 Ingo Hofmann

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## Chapter 1 Introduction

No one believes the simulation results except the one who performed the calculation, and everyone believes the experimental results except the one who performed the experiment.

[Quote from: Martin Greenwald, Massachusetts Institute of Technology, in [1]]

**Abstract** A short historical account of the early development on space charge in accelerators since the 1960s is followed by a list of books of reference on the wider field of accelerator physics. A proper distinction of *incoherent* and *coherent* in the context of resonant effects in space charge dominated beams is crucial. An equally important distinction is that between externally excited resonances – for example by error fields – and parametrically driven resonant instabilities, furthermore between isotropic and anisotropic beams. A conceptualization of these terms – by presenting a kind of guideline for the following chapters – is presented in the introduction and intended as a hopefully useful framework for interpretation of theory, simulation and experiments.

Modern particle accelerators are not thinkable without the tremendous progress in computer simulation for beam dynamics since the 1970s. Narrowing the gap between simulation models and observation of beams in real accelerators has remained a challenging task. The above quote from the context of hydrodynamic and plasma simulations, which have also prepared the ground for a great deal of accelerator beam dynamics simulation, has remained valid until today.

This is particularly true for the large-scale accelerators at high intensities, which are in operation or in planning/construction phases in a number of places in Europe, America and Asia – most of them linear accelerators. They enable many developments at the forefront of basic or applied sciences, from neutron scattering to energy, industry and environment, including future nuclear waste management by accelerator driven transmutation (see Fig. 1.1).

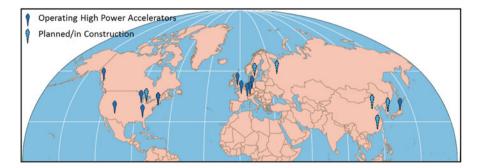


Fig. 1.1 Worldwide locations of major high intensity and high power accelerators in operation, planning or construction

For contrast, the first room-size particle accelerators from before World War II were practically hand-calculated – and yet they led to the discovery of new particles. Today's accelerators are not only designed by computers; operation at high beam power requires the use of computers to model and optimize beam behaviour. Predicting and controlling beam loss at high intensity, with space charge as major source of loss, is crucial for minimizing radioactivity in such facilities. This is a must for optimizing their performance and carrying forward the intensity frontier.

Hence, an in-depth understanding of the physics behind space charge – using analytical theory, simulation as well as experiments – is essential and the primary motivation behind this book.

#### 1.1 Historical Remarks

A brief historical account may be in place here. In the early time of accelerators – the 1950s and 1960s – it was understood that the demand for higher intensity would soon be increasing, and understanding and controlling of beam space charge would become more and more important. Steady improvements in intensity were made, but this was primarily a technical challenge at the accelerator "front end", for example how to improve the performance of proton or ion sources and of low beta acceleration. Gradually, first analytical-numerical concepts on space charge were developed in the 1960s: Significant contributions were the exploration of gradient errors with space charge by Smith in [2]; the rms envelope equations by his student Sacherer in [3]; and the first analysis of 2D oscillation modes by Gluckstern in [4] – just to mention some of them.

The field of space charge and its limiting effects on beam intensity received an important boost with the proposal in the mid-1970s to use heavy ion accelerators as drivers for inertial fusion energy production, which required pushing intensity

several orders of magnitude beyond state-of-the-art.<sup>1</sup> The first self-consistent Vlasov analysis of "space-charge induced transverse instabilities" in 2D beams in periodic focusing in [6] emerged from this project.

Since the late 1990s linear accelerator based high intensity spallation neutron sources triggered enhanced interest in space charge problems, also for rings, and efforts were increased towards a better understanding of the issues as well as laying a safe ground for the control of space charge effects. At the Shelter Island Workshop (Shelter Island, New York, 1998) Baartman in [7] justified that space charge deserves its own language: "Forces arising from the beam itself are not the same as external forces ... any theory which treats the two types of forces in the same way is incorrect and will make incorrect predictions."

#### 1.2 Important Other Sources of Reading

The scope of this book is not to enter into details of the numerous fundamental concepts of accelerators and the large diversity of important facets of particle motion or collective beam interaction beyond direct space charge, which is the focus. Many other sources of reading exist for the field at large.

One of the early and remarkable books on space charge, among the very few in this direction that existed over three decades ago, was written by Lawson [8]. Computer simulation played no role at all, but Lawson helped understanding many questions in his own stimulating language. His book and ideas inspired Martin Reiser at Maryland University to build – from the 1990s onwards – several very compact electron devices. They helped addressing experimentally many questions about space charge up to the present day. Many of these findings, along with theoretical models and computer simulation examples, went into the book by Reiser [9], which become one of the indispensable sources of beam physics with space charge in a broader context.

In the field of rf linear accelerators, the book by Wangler [10] grants insight into beam dynamics and the role of space charge as well as design issues of high current linear accelerators.

Readers interested in nonlinear beam dynamics in storage rings will find a profound treatment of this subject in the book by Forest [11].

A broader selection of topics, also including the vast fields of collective effects, with selected topics on space charge, is found in the books by Chao et al. [12], Wiedemann [13], Lee [14] and others.

Students in particular may find it useful to use the published lectures from the CERN Accelerator School in [15] and the U.S. Particle Accelerator School in [16].

<sup>&</sup>lt;sup>1</sup>A recent review of this project is found in [5], where also other energy related accelerator applications are reviewed.

#### **1.3 What Are Space Charge Dominated Beams?**

The notion of *space charge dominated* beams is not sharply defined and differs between linear and circular accelerators. In this book only "direct" space charge is considered assuming only electrostatic interaction of the charged particles within a bunch, or between line charges in a coasting beam model. Bunch-to-bunch, image, impedance or wake field effects would go beyond the scope of this book.

In synchrotrons an intensity limiting criterion used in the early days was guided by the idea not to have space charge tune shifts exceed the  $\frac{1}{4}$  tune separation between fourth order resonances. It was learnt later that such a definition is not well-justified and often too conservative. In view of the diversity of space charge effects more specific criteria needed to be defined, including the role of coherent space charge effects.

In linear accelerators an early hand-waving argument, without real justification, was to let space charge – in the average – cancel about half of the external focusing force. This amounts to  $\approx 30\%$  reduction of the zero-current tune by space charge. In modern high current accelerators, though, much higher peak values are achieved on the basis of criteria, which consider the diverse relevant space charge effects.

#### 1.4 Incoherent and Coherent Effects

*Incoherent* and *coherent* space charge effects are a central issue in the following chapters. These terms are not used in an unambiguous way in beam dynamics literature, however. Here they are understood as characterising the difference between single particle and collective response behaviour.

In circular accelerators *coherent mode* is frequently understood as dipole mode of oscillation causing a displacement of the beam as a whole, and *incoherent* as betatron oscillations of single particles. We ignore dipole mode instabilities here – they are not governed by direct space charge only – and focus on second order and even higher order modes, which may be resonantly driven by the lattice in combination with space charge and possibly beam anisotropy. The distinctive feature of *coherent* is a clear, observable frequency associated with the specific kind of mode. This leads to a coherent frequency shift entering into its condition of resonance. *Incoherent* motion is much more difficult to measure – often impossible – as it is part of the equilibrium beam; this is understood as a modulation following the periodic pattern of the focusing lattice.

In principle, this is not very different in high intensity linear accelerators, where the strength of space charge forces – relative focusing forces – is even more pronounced. Coherent motion of the beam core may result from lattice transitions or focussing discontinuities, but also from the resonant action of the lattice including space charge. As in circular accelerators, coherent space charge modes introduce new frequencies, which are not present in the matched beam and can be observed – at least in simulations.

#### 1.5 Terminology of Resonance, Coherence and Instability

The terms of resonance, coherence, instability and parametric play a key role in this book. Generally speaking, they are not always used in an unambiguous way in the available literature. The following nomenclature is intended to be a consistent guide through the various chapters of this book hoping that it may also be useful beyond it.

Instabilities are understood as feedback processes growing exponentially from the noise level. Resonances require periodic action on single particles or eigenmodes – usually due to an *external driving force*, which can be also the space charge self-field of the beam.

We distinguish between a number of cases. First, note that a *coherent resonance condition* expresses the appearance of a coherent space charge shift in the resonance condition, in addition to the space charge shifted rms tunes of single particles; second, space charge structure resonances are driven by the periodic *matched beam space charge force*, which by itself is *not* a coherent feature.

- Incoherent, also called single particle resonances: they can be
  - (1) error resonances due to error magnet multipoles;
  - (2) structure resonances due to a magnet multipole with lattice structure periodicity; but also due to a space charge pseudo-multipole with lattice structure periodicity, in which case they are called *space charge structure resonances*; both cases are described as *incoherent* or as alternative nomenclature *single particle* resonance based on the space charge shifted tune of single particles (but *not* a coherent frequency shift as in the subsequent item).
- Coherent resonances:
  - coherent eigenmodes of oscillation not just oscillating single particles

     can be driven by error magnet multipoles<sup>2</sup>; described by a *coherent resonance condition*, which includes a coherent frequency shift (otherwise an incoherent resonance).
  - (2) alternatively, coherent eigenmodes in beams with more than one dimension can be driven by anisotropy, e.g. different emittances and/or average focusing constants; described by *coherent difference resonance* conditions.

 $<sup>^{2}</sup>$ For example a gradient error driving an envelope mode in a circular accelerator; this can also occur with the structure gradient, in which case the condition would correspond to the first "Mathieu" stopband – see Sect. 7.1.1.

• Coherent parametric instabilities<sup>3</sup>:

coherent eigenmodes of oscillation are resonantly growing due to the parametric action of a system parameter – here the periodic modulation of the focusing force; they are called here *parametric instabilities* if they are associated with a half-integer (1:2) frequency relationship<sup>4</sup>; described, accordingly, by *coherent half-integer resonance* conditions.

In beam dynamics literature such coherent parametric instabilities are also called *structure space charge instabilities*; the more accurate term *parametric* is preferable as it also helps to adequately describe the phenomenon of sum coherent parametric instabilities in Chap. 7. Sum parametric resonances/instabilities are well-known in parametric resonance theory in theoretical mechanics; in beam dynamics, and as a *coherent* phenomenon, they have only been recently considered.

In circular accelerator literature the emphasis is primarily on magnet error driven resonances. In linear accelerators, however, coherent instabilities, to some extent also incoherent structure resonances by space charge, are a major source of beam degradation. The lower degree of periodicity in linear accelerators is easily outweighed by the higher level of space charge and its intrinsic strong nonlinearity.

Note that the direct space charge driven instabilities discussed here are practically always subject to resonance conditions. Non-resonant instabilities can be driven by dissipative mechanisms, like the resistive wall instability, which are not considered here.

#### 1.6 Analytical and Simulation Approaches

Progress in the control of space charge in design, optimization and operation of high intensity and high power accelerators is owed to both, analytical studies as well as advanced particle-in-cell computer simulations. An in-depth understanding of the diverse effects of space charge is the basis for the interpretation of simulation and experiments.

Beam dynamics at high intensity is an interplay of single-particle nonlinear dynamics with various coherent and parametric resonance effects. The goal of comparisons between analytical results, simulation and experiments must be to clarify the importance of this interplay under realistic conditions.

Several important topics cannot be adequately covered here, although they are interwoven with space charge: among them the role of errors in linear accelerators, which are a driving source for emittance growth and halo; out of the broad field of magnet error driven resonance effects in circular accelerators only selected

<sup>&</sup>lt;sup>3</sup>To be distinguished from *single particle* parametric instabilities, see Sects. 7.1 and 6.1.

<sup>&</sup>lt;sup>4</sup>Where the eigenmode oscillates at half the lattice periodicity. Note that these modes have been addressed under the name "space-charge induced transverse instabilities" in [6], where also other (not necessarily practically significant) frequency relationships were considered, like 1:1 etc.

examples are presented; and the wide area of impedance driven collective effects in circular accelerators, where space charge also plays a role, is beyond the scope of this book.

#### 1.7 Overview

The material is presented in the following way:

Chapters 2 and 3 review basic concepts on phase space dynamics leading to Vlasov's equation as important analytical tool needed further on; the concept of matched equilibrium beams is outlined in Chap. 4.

Chapter 5 exposes the nature of different modes of interaction when dealing with space charge on a general level; this includes the role of resonance vs. instability and of incoherent vs. coherent oscillations. Followed by a review of the Vlasov theory of anisotropic coherent eigenmodes and a discussion of the role of negative energy waves and of Landau damping, this chapter lays the ground for the later Chaps. 7, 8 and 9. Chapter 6 applies part of this to analyse space charge in mismatched beams.

A central theme of space charge interaction in this book is that of coherent parametric instabilities dealt with in Chap. 7; it is followed in Chap. 8 by a discussion of selected coherent and incoherent resonance effects in circular machines driven by magnet errors; and by Chap. 9 on anisotropic beams in linear or circular accelerators, where the role of coherent effects on emittance exchange between planes due to different emittances and/or focusing strengths is discussed in some detail.

Chapter 10 summarizes the relevance of the discussed space charge effects in the design of circular and linear accelerators, and an Epilogue offers a brief outlook.

Literature is found sorted by chapter. No claim of completeness is made, and I therefore apologize for having omitted relevant and important contributions to the field.

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