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Stefania Centrone *Editor*

Essays on Husserl's Logic and Philosophy of Mathematics

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To the memory of Richard Tieszen

Preface

In my *Logic and Philosophy of Mathematics in the Early Husserl* (Synthese Library, 2010), I set out "to restore the level of the real discussion between Husserl and his important early interlocutors, some of whom made definitive contributions to the development of formal logic as an autonomous discipline in the last two centuries." To this end, I considered Husserl's relationship to the algebraists of logic, in particular George Boole and Ernst Schröder, as well as many connections between his work on logic and philosophy of mathematics and the work of Georg Cantor, Bernard Bolzano, Gottlob Frege, and David Hilbert. However, a sense of dissatisfaction remained upon my completion of this project. Clarifying the relationships between Husserl and his most important interlocutors in the fields of logic and philosophy of mathematics was an impossible task for a single scholar; moreover, Husserl's reception and influence remained almost unexplored. During the years, also thanks to the annual meetings of the Husserl Circle, which were becoming more and more international due to the indefatigable activity of its secretary, Burt Hopkins, I had the opportunity to meet and attend lectures of many scholars and colleagues who seemed to look at Husserl's work from a perspective similar to my own, that is, who seemed to believe that a logico-historical and a phenomenological reading glass do not necessarily exclude each other when it is a matter of making sense of Husserl's writings. Meanwhile, the appearance of *Phenomenology and Mathematics* (Phenomenologica, 2010) by Mirja Hartimo, which "gathers the contributions of the main scholars of the field," most of whom also are contributors to the present volume, seemed to me an important further step in this direction, and, moreover, it gave me the courage to address the question of the relationships of Husserl's key ideas in logic and philosophy of mathematics to other figures. This volume presents contributions from the analytical and phenomenological perspective. It does not aim to give a comprehensive final judgment on Husserl's work but rather to open new and perhaps novel interpretative perspectives, from which to look at that work.

Very special thanks go to all authors in this volume. The encouragement at a decisive moment and the friendly advice I received from Professor Otávio Bueno, the editor in chief of *Synthese*, and from Christi Jongepier-Lue, the assistant publishing editor of Springer Science and Business Media, were truly invaluable. The criticisms and suggestions made by an anonymous referee for Synthese Library who read the penultimate draft of this book were very helpful. To her/him also is my warmest thanks.

Oldenburg, Germany Stefania Centrone

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Stefania Centrone is currently *privat Dozent* at the University of Hamburg and research assistant at the Carl von Ossietzky University of Oldenburg. In 2012, she was awarded a DFG-Eigene Stelle for the project "Bolzanos und Husserls Weiterentwicklung von Leibnizens Ideen zur Mathesis Universalis" at the CvO University of Oldenburg. In 2016 she was deputy professor of theoretical philosophy at the Georg-August-Universität Göttingen. She is author, among others, of the volumes *Logic and Philosophy of Mathematics in the Early Husserl* (Springer 2010) and *Studien zu Bolzano* (Academia Verlag 2015).

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Carlo Ierna is currently working as lecturer in history of philosophy at the University of Groningen. After working at the Husserl Archives Leuven since 2004, in 2012 Dr. Ierna was awarded a prestigious Dutch NWO VENI grant for his research project on the ideal of "Philosophy as Science" in the School of Brentano. In 2014, he was visiting fellow in philosophy at Harvard where he discovered an unknown letter from Husserl to Brentano. Dr. Ierna is completing his book on early Husserl, which will also contain an edition of Brentano's 1884/1885 lectures on elementary logic.

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Richard Tieszen was professor of philosophy at San José State University, located in California's Silicon Valley. He is the author of *After Gödel: Platonism and Rationalism in Mathematics and Logic* (Oxford University Press 2011); *Phenomenology, Logic, and the Philosophy of Mathematics*; and *Mathematical Intuition: Phenomenology and Mathematical Knowledge* (Cambridge University Press 2005), co-editor of a book on the philosophy of mathematics and another on comparative philosophy, and guest editor of several special issues of *Philosophia Mathematica*. He has published numerous papers and reviews on Husserl, Gödel, and other figures and issues in the philosophy of mathematics, logic, and phenomenology. He has been a visiting professor at Utrecht University in the Netherlands and at Stanford University. Richard Tieszen passed away shortly after completing his contribution to this volume. The volume is dedicated to his memory.

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Judson Webb is professor of philosophy at Boston University. He is the author of *Mentalism, Mechanism, and Metamathematics* (Reidel 1980). He has written a number of articles on Kant, Hilbert, and Gödel. His most recent publications are "Hintikka on Aristotelian Constructions," "Kantian Intuitions," and "Peircean Theorems," in *The Philosophy of Jaakko Hintikka* (Open Court 2006).

Introduction

This volume sets out to locate precisely Husserl's work in the field of logic and the philosophy of mathematics – a goal surely worth to be pursued, especially in the light of the developments in these fields during the past century. The aim is to provide an in-depth reconstruction and analysis of the discussion between Husserl and his most important interlocutors and to clarify pivotal ideas of Husserl's by considering their reception and elaboration by some of his disciples, such as Oskar Becker and Jacob Klein, as well as their influence on some of the most significant logicians of the past century, such as Rudolf Carnap and Kurt Gödel. Most of the papers focus on Husserl and another scholar – e.g., Leibniz, Bolzano, Kant, Brouwer, and Frege – and trace out and contextualize lines of influence, points of contact, and points of disagreement.

In the following, I will outline the main issues and historical movements and will conclude with a brief overview of each chapter in this volume.

Frege is reported to have said that there is gold in his *Nachlass*. Husserl seems to think that much the same is true of Gottfried Wilhelm Leibniz and really gets many of his own key ideas from him. Chapter 1 of the volume aims at showing how Husserl's idea of formal mathematics as theory of abstract structures has an important source of inspiration in Leibniz. From him, as well as from the reception of Leibniz by the Bohemian mathematician and philosopher Bernard Bolzano, Husserl gets the idea that abstract mathematics is, at root, a priori ontology, "the totality of the laws of possible being," as Ettore Casari once put it.[1] Indeed, already in his youthful *Contributions to a Better-Grounded Presentation of Mathematics* (1810), Bolzano defines mathematics as the "science which deals with the general laws (forms) to which things must conform in their existence"[2] and understands by "things" not only those that actually exist "but also those which simply exist in

[1] Casari 2004.

[2] Bolzano 1810, §8.

our *imagination* [. . .],"[3] thereby meaning "things that are possible," things whose concept is not contradictory: "How must things be made in order that they should be possible?"[4]

Against the framework of these conceptual questions about the *mathesis*, Chap. 1 points out how Husserl's account of *symbolic thinking*, which has such important implications in Husserl's conception of algorithms as well as in his later account of knowledge in the *Logical Investigations*, actually traces back to Leibniz and not to Brentano, in spite of what Husserl himself declares in his first published work, *The Philosophy of Arithmetic* (1891).[5] Even so, among the main topics dealt with in the volume is the relation between Husserl and his teacher Franz Brentano (Chap. 7) as well as a very close reading of Husserl's claims, distinctions, and arguments with respect to Kant (Chap. 2).

The issue of abstract mathematics as *mathesis universalis* is resumed in Chap. 16 (*Husserl and Gödel*) along with that of Leibniz's influence on Husserl. Here we apprehend how the "realization of the Leibnizian idea of a universal ontology as the systematic unity of all conceivable a priori sciences" is thought of to be realizable on the basis of the transcendental methodological method, i.e., according to the late Husserl (in a draft of the *Encyclopedia Britannica* entry), "on a new foundation which overcomes 'dogmatism'." The phenomenological method is shown here "as a way to develop and defend a new kind of rationalism that avoids the excesses of older forms of rationalism but also avoids any kind of mysticism." Furthermore, Chap. 16 points out that the realization of Leibniz's dream of a *universal characteristic*, if the latter is interpreted as a formal system, is not possible in the light of Gödel's incompleteness theorems. The chapter also highlights how Gödel himself does not interpret his own incompleteness theorems as negative results that exclude that every clearly posed mathematical yes-or-no question is solvable by reason and claims that Gödel is here relying on Husserl to show, this time at a variance with Leibniz, that it is possible to solve gradually any intelligible question by reflecting on the concepts and on the way we use them.

Let us now come back to early Husserl and discuss another key point. One of the distinctive traits of early Husserl's work is the simultaneous presence in his logical and mathematical reflections of two different directions of research, (1) the project of a substantial mathematization of logic and (2) a conception of logic as the study of objective relations occurring among certain abstract logical entities, such as concepts and propositions.

As far as point (1) is concerned, we find Husserl's interest in specifically logico-formal issues: he succeeds in grasping with great clarity and insight the implications of the formal-abstract trend in mathematics and, in particular, of its tendency toward algebrization, which he is able to transfer to and elaborate at the logico-theoretical level. Viewed from this perspective, the volume proceeds

[3]Loc. cit.

[4]Casari 2004, 161.

[5]Husserl 1891, 215 fn. (Engl. transl., 2003, 205).

by outlining some important similarities and differences between early Husserl's work and that of the algebraists of logic, in particular George Boole (Chap. 5) and Ernst Schröder (Chap. 6), as well as between early Husserl's work and that of the German mathematician, linguist, and physicist Hermann Günther Grassmann (Chap. 4), thus setting out to understand more closely some contributions that may have anticipated Husserl's ideas or influenced them. One of the main issues concerns the abstraction and generality of algebra, which culminate in various attempts by Husserl to characterize universal algebra.[6] In the studies for the second volume of the *Philosophy of Arithmetic*, which actually never saw the light of the day, Husserl characterizes universal algebra as a system of operations on a certain set. The calculus is made up of certain general symbols of operations, which are defined by certain general definitions and obey to certain very general laws.[7] Husserl's account of universal algebra and some of his previous investigations on the concept of system of numeration in a given base[8] very closely resemble the work of the algebraists of logic. Indeed, already in the work of the English algebraists in Cambridge (C. Babbage, G. Peacock, J. W. Herschel) in the period 1830–1840, a distinction was made between (i) abstract algorithm of computation, by means of which conclusions are drawn in a deductive-algorithmic way, and (ii) the possible systems of entities that can satisfy such formal conditions. With the same abstract algorithm of computation, one can provide a unitary treatment for systems of heterogeneous entities that manifest a similar structural behavior. Later on, with the contributions of scholars such as W. R. Hamilton, H. Grassmann, and A. Cayley, there was a progressive distancing from the idea of algebra as "symbolic algebra of magnitudes," which culminated in the explicit disengagement of algebraic research from the quantitative dimension, in particular with Boole and his creation of the algebra of logic. Algebra no longer only treated numbers or magnitudes but also propositions, concepts, and, in general, *qualitative data*. The laws under which they fall are independent from any specific interpretation of the symbolism, and the structural properties of the operations that are reflected in such laws are unleashed from numerical elements and assume the character of abstract algorithmic procedures for "calculations" performed with symbolic expressions. The explicit separation between laws of calculus – purely *formal* laws – and their interpretations is more or less the distinctive trait of modern abstract algebra and mathematics. Chapters 5 and 6 present Husserl's investigations about abstract algorithms of computation along with his reflections on Boole's and Schröder's calculi, while Chap. 4 enlightens Grassmann's *formalization of computation* and traces back to Grassmann another key idea in Husserl's reflection, that of recursive process. Surprisingly enough, Husserl, as far as we know, turns out to be the

[6]See *Das Imaginäre in der Mathematik* (December/January 1901/1902), in HGW XII, 430–51 (Engl. transl., 2003, 409–432), and the critical edition Schuhmann & Schuhmann 2001.

[7]*Das Imaginäre in der Mathematik*, cit., 433 (Engl. transl., 2003, 412). Hereto cp. Centrone 2010, 159–61.

[8]Husserl 1891, 243–4 (Engl. transl., 2003, 230–1).

first scholar who, having insisted on the algorithmic meaning of arithmetical operations, explicitly specifies a number of general procedures by means of which new arithmetical (computable) operations are generated from given ones and at the same time attempts to investigate the question concerning the characterization of the class of *computable* arithmetical functions *as a whole*.[9]

As regards point (2) above, a conception of logic is present in early Husserl's work as the study of objective relations occurring among certain abstract semantic entities. A source of inspiration for this claim is the theory of notions (*Vorstellungen an sich*) and propositions (*Sätze an sich*) in Bolzano's *Wissenschaftslehre*, and one of its more remote ancestors is the Stoic doctrine of *sayables* (*lektá*). Chapter 3, *Husserl and Bolzano*, dwells on Bolzano's logical universe, which might turn out to be unfamiliar to the English-speaking scholar, since a complete translation of Bolzano's masterwork, *The Theory of Science*, has been published only in 2014. It is a fact of intellectual history that Bolzano's theory of notions and propositions and Frege's theory of the *senses* of our non-propositional and propositional mental acts or states and linguistic utterances appear to be very close. Both Frege and Bolzano believed in an objective realm of abstract logical entities, distinct from mental states and processes and from the objects people think and talk about. Like Bolzano's notions and propositions, Frege's *senses can* be the contents of our mental acts or states and/or linguistic utterances, can subsist independently of us, and are "capable of being the common property of many."[10] Both Frege and Bolzano take propositions to have a complex internal structure, and it is of the essence of their non-propositional parts to be parts of propositions. Both Bolzano's and Frege's propositions are bearers of *unrelativized* truth or falsity. Both endorse the principle of *bivalence*, though Frege only for propositions expressible in his conceptual notation (*Begriffsschrift*) and not for those expressed in natural languages. Finally, both philosophers take the concept of propositional truth to be *epistemically unconstrained*. Dagfinn Føllesdal conjectured in 1958 that Frege was an important factor in Husserl's conversion from the psychologism of the *Philosophy of Arithmetic* to the anti-psychologism of the *Prolegomena*.[11] This claim has been contested by J. N. Mohanty (1982) and is strongly contested in Chap. 10 of this volume by G. Rosado Haddock. Yet it is a fact that Husserl acknowledges his indebtedness to both Frege and Bolzano. In his *Prolegomena to Pure Logic*, he finds important to stress that his investigations were not "in any sense mere commentaries upon, or critically improved expositions of, Bolzano's thought patterns" but that they "[had] been crucially stimulated by Bolzano ...."[12]

[9]See Husserl 1891, Ch. XIII. This point has been first made by E. Casari (Casari 1991, 46). For a mathematical reconstruction of Husserl's intuitions, see Centrone 2006 (and 2010, 54–61), where I have defended the thesis that the generation procedures that Husserl studies in the 13th chapter of the *Philosophy of Arithmetic* give indeed rise to a class of numerical functions that is extensionally equivalent to the one known in contemporary logic as the class of partial recursive functions.

[10]Frege, "Über Sinn und Bedeutung" (Frege 1892), fn. 5.

[11]See Føllesdal 1958 and Føllesdal 1982, which is a reply by Føllesdal to one of his critics.

[12]Husserl 1900, 227 (English translation, 1970, 224).

Bolzano had, in Husserl's eyes, the great merit of having characterized pure logic as a discipline that is concerned "with the most general conditions of *truth* itself"[13] and deals with the relations among the *contents* of our thoughts. He praised Bolzano's *Wissenschaftslehre* as "a work which ... far surpasses everything that world-literature has to offer in the way of a systematic contribution to logic."[14] In the same *Prolegomena*, he mentions Frege in a footnote and writes: "I need hardly say that I no longer approve of my own fundamental criticism of Frege's antipsychologistic position set forth in my *Philosophy of Arithmetic*. I may here take the opportunity, in relation to all of the discussion of these Prolegomena, to refer to the Preface of Frege's later work *Die Grundgesetze der Arithmetik*, vol. I (Jena 1893)."[15] Indeed, Frege's *Preface* contains much of what Husserl presents on a wider screen in the *Prolegomena*. We find a further hint in the *Husserl-Chronik* (Schuhmann 1977). It is a remark from H. Spiegelberg's *Scrap-Book* (a manuscript quoted with the permission of the author). There we read[16]:

> Andrew Osborn visited H. 1935 in Black Forest to ask him about Frege's influence on the abandonment of the psychological approach of the "Philosophie der Arithmetik." H. concurred, but also mentioned his chance discovery of Bolzano's work in a second-hand book store.

The already mentioned Chaps. 3 and 9 (*Husserl and Frege*) explore in detail the relationship between Husserl and Bolzano, respectively Husserl and Frege as to logical objectivism and the validity of logical laws, while Chap. 10 rejects the claim of Frege's influence. The final judgment is left to the reader.

It is indeed clear that each chapter of the volume brings out specific themes of Husserl's work. At the same time, it is the peculiarity of a collective volume that it could not adopt a harmonic preconception neither with respect to the author's thought nor with respect to certain interpretative questions. The good thing is that the aspects of internal contradiction that are often present in each thought that aspires to call himself philosophical are not given up in favor of a reading that inclines to obscure them on the account of a comprehensive interpretative stance that *privileges* one point of view. Actually, as Chaps. 3 and 9 on the one side and Chap. 10 on the other side nicely disagree, so Chap. 12 (*Husserl and Brouwer*) and Chap. 16 (*Husserl and Gödel*) disagree as to Gödel's project of founding classic mathematics on transcendental phenomenology. As well, Chap. 7 (*Husserl and Brentano*) disagrees with what Ierna calls "the mainstream account of Edmund Husserl's early works," advocated, among others, by Ettore Casari (above all in Casari 1991) and myself (in Centrone 2010). Such account sees Husserl's *Philosophy of Arithmetic* mainly as a work fitting into the general framework of the so-called *research on the foundations* of mathematics, into which two separate influences flow: a mathematical one coming from Karl Weierstrass, whose project of

[13] Bolzano 1837, I, 65.

[14] Husserl 1900, 226 (English translation, 1970, 223).

[15] Loc. cit., 169 fn. (318 fn. 6).

[16] Schuhmann 1977, 463. Cp. Føllesdal 1982, 55.

founding analysis on a restricted number of simple and primitive concepts Husserl would have inherited, and a philosophical one coming from Franz Brentano, whose method for identifying the primitive concepts by describing the psychological laws that regulate their formation Husserl would have adopted.

A short overview of each chapter in the volume follows.

Chapter 1. "Husserl and Leibniz: Notes on the Mathesis Universalis" by Stefania Centrone and Jairo da Silva considers Husserl's intellectual debt to Leibniz with respect to two key ideas, the idea of symbolic thinking and the idea of universal mathematics (*mathesis universalis*). In his first major work, the *Philosophy of Arithmetic* (1891), Husserl maintains that the genesis of arithmetic is to be found in the fact that we are almost always forced to limit ourselves to symbolic number presentations. Arithmetic, as a whole, is nothing but *a collection of artificial means* to alleviate the essential incapacity to have a proper presentation of all numbers. As mentioned above, Husserl claims to have taken over the distinction between *proper* (*eigentliche*) and *symbolic* (*symbolische*) presentations from Brentano, but, at root, it is an adaptation of Leibniz's distinction between *cognitio intuitiva* and *cognitio caeca vel symbolica*. Thus, the authors explore the uses Husserl puts the Leibnizian concept of symbolic cognition and how he develops it to explain and justify the sense and functioning of algorithms. Next, they look at another key idea of Leibniz's appropriated by Husserl, namely, the idea of *mathesis universalis.* They start out from Leibniz's conception of mathematical disciplines as branches of the *mathesis* and of the latter as a general science of forms applicable not only to magnitudes but to any object whatever *that exists in our imagination*, i.e., to all objects that are, in principle, *possible*. Thereafter they outline the development of this idea in Husserl, going through early Bolzano's conception of mathematics as the "totality of the laws of possible being."

Chapter 2. "Husserl and His Alter Ego Kant" by Judson Webb begins with the presentation of background material in the works of Kant and Lambert that are relevant to Husserl's lifelong concern with Kant. This intensifies when, in the midst of his own transcendental turn, he recognizes that by virtue of his Copernican turn, Kant became the first to detect the secret longing of modern philosophy for a phenomenological clarification at the sense of being. Kant's transcendental idealism and deductions presuppose a pure ego that he does not adequately analyze but which Husserl finds must survive the phenomenological reduction as pure subjectivity. Husserl's verdict is that Kant does not achieve such a genuine reduction, without which he is unable to eliminate things in themselves from his epistemology or account for intersubjectivity. Husserl improves upon Kant's inadequate accommodation of meaning to intuition in mathematics with his theory of categorical intuition, as well as a method of clarification of concepts that occur in seemingly unintuitive impossibility proofs.

Chapter 3. "Husserl and Bolzano" by Ettore Casari is a contribution to our understanding of Husserl in relation both to earlier and more mature Bolzano as well as to Lotze. The chapter focuses firstly on the idea that Bolzano takes up from the Leibnizian-Wolffian tradition that there is a certain objective connection among truths, independent of the cognitive activity of the subject: certain truths are the "grounds (*Gründe*)" of others and the latter are "consequences (*Folgen*)" of the former. This idea is taken up more than once by Husserl, for instance, in his *Prolegomena to Pure Logic*, when he talks of *Begründung*, and it also has an effect on his conception of proofs and theories. The chapter presents, then, Bolzano's logical universe, in particular, the three main conceptual areas of Bolzano's logical system, namely, *lektology*, that is, the theory of the possible contents of our mental act or states and linguistic utterances; *consecutivity*, that is, his theory of the ground-consequence relation; and his *theory of knowledge*, the theory of the relations between mental acts or states and their abstract logical contents, such as concepts and propositions. Casari then explains how Bolzano's conceptual system was made accessible to Husserl by his study of Lotze and shows how much Bolzano is present in Husserl's *Logikvorlesung 1996*.

Chapter 4. "Husserl and Grassmann" by Jan von Plato distinguishes two late-nineteenth-century approaches to the foundations of arithmetic: one as in Frege, who tries to answer the ontological question of what numbers are, and another as in Grassmann, with numbers as undefined basic concepts but instead definitions of the arithmetic operations that allow their properties to be proved by induction. Husserl's *Philosophie der Arithmetik* stands at the crossroads of these two traditions. Reading it in this light, the lack of a development of formal arithmetic is striking. Husserl tries instead to decide on the basis of general criteria what is reasonable and defensible and what is not. He sees that Grassmann's approach leads to an extremely useful "art of computing and arithmetic" but also criticizes it as a "pure mechanics of computation" that would not be philosophically satisfying.

Chapter 5. "Husserl and Boole" by Stefania Centrone and Pierluigi Minari confronts Husserl's representation of the problem-solving processes with the analysis of "symbolic reasoning" proposed by George Boole in the *Laws of Thought* (1852). This chapter focuses, in particular, on Husserl's lecture "Über die neueren Forschungen zur deduktiven Logik" of 1895, in which an entire section is devoted to Boole. As corroborated by the previous and by the next chapter, Husserl seems to offer a similar criticism of Grassmann's approach as well as of Boole's and Schröder's. On the one hand, according to Husserl, Boole and Schröder are masters in the development of the *logical calculus*, and on the other hand, they are "bad philosophers of mathematics"; they don't grasp the real sense and meaning of the calculus they are setting up. The chapter also focuses on the different reception by Husserl and Boole of the Leibnizian idea of "parallelism" between algorithm and field of experience to be interpreted by the algorithm. While Boole explicitly admits the possibility that not all the steps of the problem-solving processes are *interpretable*, Husserl's position appears to be the opposite: the parallelism between the symbolic-algorithmic level of the problem-solving process and the conceptual one should be preserved along the entire process.

Chapter 6. "Husserl and Schröder" by Stefania Centrone and Pierluigi Minari pursues the goal of clarifying to what extent the work of the German mathematician Ernst Schröder (1841–1902) on the algebra of logic is taken into consideration and criticized in the work of early Husserl, focusing on Husserl's 1891 *Review* of the first volume of Schröder's monumental *Vorlesungen über die Algebra der Logik* (*Exakte*

Logik) and on Husserl's text *Der Folgerungskalkül und die Inhaltslogik* written in the same year of Husserl's *Review*. Husserl levels the same kind of criticism against Schöder that, as the two previous chapters show, he had leveled against Grassmann and Boole: while praising Schröder's calculus, he strongly criticizes Schröder's attempt at a philosophical clarification and justification of it. The chapter also considers Frege's famous letter to Husserl dated 24.5.1891 and briefly compares Frege's and Husserl's objections to Schröder.

Chapter 7. "The Brentanist Philosophy of Mathematics in Edmund Husserl's Early Works" by Carlo Ierna begins by challenging the mainstream account of Edmund Husserl's early works as a combination of two separate influences: a mathematical one in Berlin from i.a. Karl Weierstrass and a philosophical one in Vienna and Halle from Franz Brentano and Carl Stumpf. Instead, this chapter builds a case for a preexisting framework of a Brentanist philosophy of mathematics. Thus, rather than an original application of Brentano's psychology to the foundations of mathematics, Husserl's early writings represent a further elaboration on topics that had already been discussed previously in the School of Brentano. Starting from Brentano's own discussion of mathematical topics in his lectures and considering the various works on the philosophy of mathematics by his students, including Stumpf and Ehrenfels, Ierna then analyzes Husserl's position in the *Philosophy of Arithmetic* against this background. In the last section of the chapter, Ierna then traces the continuities and discontinuities of Husserl's position during the period leading up to the *Logical Investigations*. The result is that through the reconstruction of the Brentanist framework, Husserl's early works gain stronger connections both to their background in the School of Brentano as well as to the development of his early phenomenology.

Chapter 8. "Husserl and Cantor" by Claire Ortiz Hill seeks to shed light on the complex period in the development of Husserl's thought from 1886 to 1900, during which he maintained close personal and collegial ties with the creator of set theory, Georg Cantor. She looks how their ideas about psychologism, Platonism, set abstraction, metaphysics, arithmetization, manifolds, actual consciousness, and pure logic overlapped and crisscrossed during those years. She considers that although Husserl's ideas changed dramatically and definitively during that time, and while it might be tempting to think that those changes were conditioned, if not actually induced by Cantor's bold experiments in mathematics, metaphysics, and epistemology, Husserl himself described the time as one of intellectual crisis, of lonely hard work during which he saw all around him only ambiguously defined problems and profoundly unclear theories. He said that, sick of the confusion and afraid of sinking into an ocean of endless criticism, for the sake of philosophical self-preservation, he had felt compelled to draw to strike out on his own. It was, he said, Lotze's more sophisticated conceptions about Platonic ideas that were responsible for his radical rejection of psychologism, espousal of Platonism, and newfound comprehension of Bolzano's more sophisticated work on pure logic that led him to adopt metaphysical and epistemological views that he had been taught to consider odious and despicable.

Chapter 9. “Husserl and Frege on Sense” by Christian Beyer begins by describing some aspects of the first (1891) phase of a debate between Husserl and Frege, in their correspondence, regarding what Frege calls “sense.” Following this, Frege’s main arguments for the distinction between sense and reference and some of his most important theses regarding sense are presented, in order to set the stage for both the presentation of the second (1906) and last phase of their debate and for some further comparisons between Frege and Husserl in the course of a presentation of some important features of Husserl’s conception of sense. It is argued that while the similarities between their views speak in favor of the so-called Fregean interpretation of Husserl’s notion of noematic sense, there are also important differences. With regard to the latter, it is argued that Husserl’s view yields a more general criterion of propositional difference and also provides a more detailed conception of the use of indexicals and (other) non-descriptive singular terms and of (what determines) their reference. In this context, Husserl’s conceptions of constitution and genetic constitution analysis, respectively, are invoked and interpreted in terms of the epistemic notion of processing mental files or individual “concepts” (as Husserl calls them in *Erfahrung und Urteil*).

Chapter 10. “Husserl and Riemann” by Guillermo E. Rosado Haddock traces the origins of Husserl’s conception of mathematics as a *Mannigfaltigkeitslehre* to Riemann. Certainly, it has been difficult for scholars to trace the possible influences on Husserl’s mature views on logic, mathematics, and their relation in his *opus magnum*, *Logische Untersuchungen*, some erroneously believing that Frege’s review of 1894 of Husserl’s youth work, *Philosophie de Arithmetik*, represented a decisive influence, as presumably was Frege’s distinction between sense and referent. Both assertions have been proven to be false. Frege simply played for Husserl a similar role as that of Hume for Kant. It has also been thought that Cantor exerted a decisive influence on his younger friend Husserl, and it is not excluded that he had some influence, though not so much on Husserl’s conception of *Mannigfaltigkeit*, a term used by Cantor more or less as a synonym for what we now call “set.” The fact of the matter is that the student and assistant of the great Karl Weierstrass and friend of Cantor used the word *Mannigfaltigkeit* in the sense that Bernhard Riemann had used it, and his conception of mathematics as a *Mannigfaltigkeitslehre* can very well be conceived as a generalization of Riemann’s usage in his duly famous “Über die Hypothesen, welche der Geometrie zu Grunde liegen.” Moreover, it is shown that Riemann influenced Husserl in another respect, namely, in Husserl’s conception of physical space as a manifold whose structure is to be determined empirically, not by a priori considerations. By the way, Husserl is a counterexample to the simplistic distinction made by historians of mathematics between mathematical schools: his formation was in the Berlin school but was more decisively influenced by a prominent member of the Göttingen school.

Chapter 11. “Husserl and Hilbert” by Mirja Hartimo presents Husserl’s approach to mathematics as complementary to but also critical of Hilbert’s approach. The paper examines first Husserl’s and Hilbert’s approaches around the turn of the century in terms of “the division of labor between mathematicians and philosophers” advocated by Husserl in 1900. The division of labor reflects Husserl’s, as well as

Hilbert's, non-revisionist approach toward mathematics. According to it, mathematics comes first, whereas the philosophers' task is to examine its conceptual essence. In his later writings, Husserl was more critical toward Hilbert's approach. The paper argues that for both Husserl and Hilbert mathematics was a study of structures. While Husserl's view of formal mathematics in the *Formal and Transcendental Logic* is mainly in line with that of Hilbert's, he was critical of Hilbert's attempt at founding it formalistically on sensuous signs. In contrast to Hilbert, Husserl wanted to show it is reducible to judgments about individuals.

Chapter 12. "Construction and Constitution in Mathematics" by Mark van Atten argues that Brouwer's notion of the construction of purely mathematical objects and Husserl's notion of their constitution by the transcendental subject coincide. Various objections to Brouwer's intuitionism that have been raised in recent phenomenological literature (by Hill, Rosado Haddock, and Tieszen) are addressed. On this basis, an objection to Gödel's project of founding classical mathematics on transcendental phenomenology is presented. The problem for that project lies not so much in Husserl's insistence on the spontaneous character of the constitution of mathematical objects, or in his refusal to allow an appeal to higher minds, as in the combination of these two attitudes.

Chapter 13. "Husserl and Weyl" by Jairo da Silva focuses on the influence Husserl's teachings might have had on the philosophical and scientific ideas of the great mathematician, physicist, and philosopher of science, Hermann Weyl. Husserl is explicitly mentioned in two important works of Weyl, *Das Kontinuum* and *Raum, Zeit, Materie*, both published at roughly the same time (1918 and 1919, respectively). The author's goal, however, is not simply to measure the extent of Husserl's influence on Weyl, although this is an ever-present concern, but to clarify the views of one by contrasting them with those of the other. To accomplish the task, da Silva carries out a comparative study of the views of both thinkers on issues such as mathematical existence, mathematical intuition, the validity of classical logic, the concept of logical definiteness, the nature of symbolic mathematics, the role of mathematics in empirical science, scientific theories vis-à-vis perception, space representation, the philosophy of geometry, and intentional constitution in general. Da Silva concludes that despite some points of divergence, Husserl is an influence to be reckoned with, although sometimes in an elusive way. As da Silva shows, this influence is not restricted to the field of philosophy, extending to Weyl's scientific ideas as well.

Chapter 14. "Paradox, Harmony, and Crisis in Phenomenology" by Judson Webb tries to analyze to what degree the discovery of logical paradoxes factors into Husserl's view of the crisis of modern science, as well as the extent to which they may have weakened his confidence that his transcendental phenomenology could account for the wonderful affinity between mathematical thought and things that the natural sciences evince – in short, that pure phenomenological consciousness should actually constitute nature without simply assuming a doctrine of pre-established harmony. Husserl develops to this end a supramathematics of all possible deductive theory forms for whose arithmetical theories he offers completeness proofs. Close attention to these proofs seems to show that he was vaguely aware of the difficulties that lead to incompleteness theorem, but he did not pursue his examination of the logical paradoxes to a point that might have highlighted these difficulties. Finally, just as the work of Weyl in general relativity invoking phenomenological themes encouraged Husserl to believe that his phenomenology could bring clarity a priori to physical science, the rise of quantum mechanics convinced Weyl that intuitionism and phenomenology could not account for it after all. For his students Becker and Mahnke, this created “a crisis of phenomenological method” itself, but Husserl concluded that it did so only for Becker’s mantic phenomenology, not his own.

Chapter 15. “Husserl and Carnap on Regions and Formal Categories” by Ansten Klev is concerned with what may be called Husserl’s doctrine of categories. This chapter offers a detailed explanation of Husserl’s technical definition of a region in the *Ideas* (1913). The explanation displays close similarities between Husserl’s doctrine of essences and the doctrine of concepts in traditional logic. Moreover, both the notion of region and the notion of formal category are elucidated by means of concepts from Husserl’s philosophy more generally. The second part of the chapter considers how the notions of region and formal category can be made sense of within Rudolf Carnap’s so-called constitution system in his *The Logical Structure of the World* (1928). This constitution system is divided into different sections corresponding to different kinds of objects and, therefore, it is noted, also to regions in Husserl’s sense. Formal categories in this context will be the elements of the logical framework, namely, simple type theory, used in the construction of the constitution system.

Chapter 16. “Husserl and Gödel” by Richard Tieszen is concerned with Kurt Gödel’s study of the philosophy of Husserl, going back to 1959. Gödel turned to Husserl to find a philosophical foundation for logic and mathematics, along with a methodology that would deepen and expand his efforts to develop a rigorous philosophy that would include an account of rational intuition, meaning clarification, an analysis of mind that would not be mechanistic, a kind of Platonism, and a monadology that would improve upon Leibniz’s “preliminary” version. The logician Hao Wang tells us that Gödel was interested primarily in the work that Husserl did after 1906, which is when Husserl took his turn into transcendental eidetic phenomenology and in which transcendental egos were soon to be described by Husserl as “monads.” In the chapter, Richard Tieszen discusses what is known about Gödel’s study of Husserl, based on Gödel’s 1961 lecture manuscript on Husserl, some items in the Gödel *Nachlass*, Gödel’s comments about Husserl recorded by Wang, and Wang’s own notes of his discussions with Gödel about Husserl. It is clear that, starting in 1959, Gödel was influenced by Husserl in a number of ways. The famous philosophical supplement to the 1964 version of “What Is Cantor’s Continuum Problem?,” for example, was influenced by his reading of Husserl, although hardly any of the literature on these passages has displayed awareness of this fact, due to lack of knowledge about Husserl’s work among most “analytic” philosophers. Several other notes and letters that Gödel wrote after 1959, apart from the 1961 paper, also show Husserl’s influence. There is some irony in the fact that one of the greatest logicians of all time would have turned to Husserl’s transcendental phenomenology of logic when such a view would have been reviled as meaningless or irrelevant in certain traditions in philosophy with which Gödel was very familiar.

Chapter 17. "Husserl and Jacob Klein" by Burt Hopkins explores the relationship between Husserl's and Klein's accounts of the foundations of modern symbolic mathematics. It argues that the problem of this foundation is twofold. On the one hand, it concerns the epistemological account of the origin of the concept of unity that characterizes the many-as-one. On the other hand, it concerns the gap between proper number concepts and symbolic ones that are actually used in calculations. Hopkins argues that Husserl's self-acknowledged failure to provide either of these foundational accounts in the *Philosophy of Arithmetic* is understandable when viewed from the perspective of Klein's philosophical-mathematical account of the historical origin of modern algebra in François Viète's *The Analytic Art*. Klein's account shows that, on the one hand, the problem of many-as-one was the very problem faced by ancient philosophies of mathematics and cannot be resolved by filtering it back thorough current logic technicalities. On the other hand, Klein shows that Husserl's attempt to ground symbolic mathematics on the concept of positive whole number was doomed to fail, because of an untenable (historical) identification of proper number concept and symbolic one, i.e., the one behind the origin of modern algebra. Moreover, Klein shows that this identification is something with which we live to this day.

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Chapter 1
Husserl and Leibniz: Notes on the *Mathesis Universalis*

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[L]a bonne caractéristique est.
une des plus grandes aides
de l'esprit humain (*[A] good symbolism*
is one the greatest aids to the human mind
Leibniz, *N.E. IV,* Ch.7, § 6)
[O]*hne die Möglichkeit symbolischer* [. . .] *Vorstellungen*
gäbe es kein höheres Geistesleben, geschweige denn eine
Wissenschaft
([W]*ithout the possibility of symbolic representations* [. . .]
there would not exist a higher spiritual life, and even less
science
Husserl, *Zur Logik der Zeichen, PdA* 349)

Abstract The notion of *mathesis universalis* appears in many of Edmund Husserl's works, where it corresponds essentially to "a universal a priori ontology". This paper has two purposes; one, largely exegetical, of clarifying how Husserl elaborates on Leibniz' concept of *mathesis universalis* and associated notions like *symbolic thinking* and *symbolic knowledge* filtering them through the lesson of the so called "bohemian Leibniz", Bernard Bolzano; another, more properly philosophical, of examining the role that the *universal mathesis* is allowed to play, and the space it occupies in Husserl's intuition-based epistemology.

Keywords Husserl • Leibniz • Bolzano • *Mathesis Universalis* • Symbolic • Sign

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1.1 Introduction

Under the heading "*mathesis universalis*" René Descartes and, later, G.W. Leibniz understood a most general science built on the model of mathematics. Though the term, along with that of "*mathesis universa*", was already used during the XVI century,[1] it is with Descartes und Leibniz that it became customarily to designate with it a universal mathematical science that unifies all formal a priori sciences.[2] In his comment to the *IV Rule to the Direction of the Mind*[3] Descartes talks of a general discipline "that should contain the primary rudiments of human reason and extend to the discovery of truths about any object whatsoever",[4] he maintains to see some traces of this true mathematics already in Pappus and Diophantus,[5] identifies its method with *algebra* or, at least, with algebra "divested of the multiplicity of numbers and incomprehensible figures which overwhelm it"[6] and, instead, endowed with that "abundance of clarity and simplicity [...] that the true *mathesis* ought to have"[7] and adds that this most general mathematical science should have as branches not only arithmetic and geometry but also astronomy, music, optics and mechanics. To the question as to the criterion according to which we decide whether a science belongs to this general mathematics or not, Descartes answers that the concern proper of this science "is with questions of order or mesure and it is

[1]Cp. Piccolomini's *Commentarium de certitudine mathematicarum* (1547), ch. 7; Dasypodius' mathematical writings (1564a, b, 1571, 1593); van Roomen's *Apologia pro Archimede* (1597). All these Authors, except for van Roomen, refer to Proclus' *Commentary on the First Book of Euclid's Elements*, especially to the first prologue of the *Commentary*, which deals with the mathematical sciences in general. In Proclo's commentary there are many hints at a common mathematical discipline that shall precede all other mathematical disciplines and has, therefore, a more general character. Crapulli 1969 offers a very interesting investigation about the development of the idea of the *mathesis* in the XVI century.

[2]Before Descartes and Leibniz there were mainly two criteria for gathering together mathematical disciplines, namely, that they all had, in some way, quantity as their object (think of the disciplines traditionally unified in the *Quadrivium*) and an higher degree of certitude in proofs with respect to non-mathematical disciplines.

[3]Descartes *Rule IV*, *AT.X*, 372–379, here 374f. Actually it is a recurrent theme by Descartes that "all sciences are concatenated", "that it is much more easier to apprehend them all together, then to separate one of them from the others" and focus solely on a unique one (*Rule I*, *AT.X*, 361). Hereto also cp. *AT.X*, 255: "[one single science] cannot be brought to perfection, without doing the same with the others". Responsibility for translations from German, French and Latin is ours, even when we refer to, benefit from, or simply echo published translations.

[4]*AT.X*, 376.

[5]Pappus of Alexandria (290–350 c. AD) and Diophantus of Alexandria (201/215(?)-285–299(?) AD) were Alexandrian Greek Mathematicians.

[6]*AT.X*, 378.

[7]loc. cit.

irrelevant whether the measure [or order] in question applies to numbers, shapes, stars, sounds, or any other object whatever".[8] He continues as follows[9]:

> [t]here must be a general science which explains all the points that can be raised concerning order and measure irrespective of the subject-matter, [. . .] this science should be termed *mathesis universalis* – a venerable term with a well-established meaning – for it covers everything that entitles these other sciences to be called branches of mathematics.

For our purposes, it is important to notice that talk about "mathesis universalis" and "symbolic", not only in Leibniz, but in Descartes as well, usually go together and are put to uses that are epistemological in nature. Descartes, in his "Géométrie",[10] developed a very efficient method for tackling geometric problems by *purely algebraic*, that is *symbolic* means. The geometric constructions required to solve a geometric problem are symbolically represented by an algebraic equation, which once solved by purely algebraic means reveals the sequence of steps required for the geometrical construction to be actually performed and the original geometric problem solved by geometric means. This is a powerful illustration of the efficacy of symbolic calculi as tools for *thinking* and *knowing* that Descartes could not fail to appreciate.

However, Descartes was not the first to realize the epistemic and heuristic efficacy of symbolic calculi. The Italian algebraists of the Renaissance (Cardano,[11] del Ferro,[12] Tartaglia,[13] Bombelli[14]) had already succeeded in developing algorithmic methods for dealing with algebraic equations of the third degree, some of them involving signs without meaning being manipulated as *bona fide* number-denoting symbols.[15] Whereas in Descartes' algebraic geometry algebraic symbols always denoted geometric entities,[16] in Bombelli's algebra signs like the square root of -1, which in those days did not denote anything, could nonetheless participate in

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[8]loc. cit.

[9]*AT.X*, 378–379.

[10]Descartes 1954.

[11]Gerolamo Cardano (1501–1576) was an Italian mathematician, physicians and astrologers. He occupies an important place in the history of Renaissance philosophy. He wrote more than 200 works in the most disparate fields, but especially, mathematics, medicine, philosophy and astrology. Among his works we remember his *Artis magnae sive de regulis algebraicis liber unus* (1545), commonly known as *Ars magna*, that draws the anger of Nicolò Tartaglia for having published the solution of third degree equations revealed to him by Tartaglia 6 years earlier, though under oath, not to reveal it.

[12]Scipione del Ferro (1465–1596) was an Italian mathematician who first discovered a method to solve the depressed cubic equations.

[13]Niccolo Fontana Tartaglia (1499/1500–1557) was an Italian mathematician famous for having been the first to translate Euclid and Archimedes into Italian as well as for his controversy with Cardano as to the solution of cubic equations.

[14]Raphael Bombelli (1526–1572) was an Italian mathematician. He is Author on a treatise on algebra (1572) and gave important contributions in the understanding of imaginary numbers.

[15]Cardano 2007.

[16]Descartes 1954.

the algebraic calculus on equal terms with denoting, meaningful symbols.[17] The surprising thing, which Leibniz certainly realized, was that by treating empty symbols as denoting symbols these algebraists succeeded in developing a very efficient calculus and put it to good *heuristic* uses.

Leibniz too invented (concomitantly with but independently of Newton) a symbolic method for dealing with geometric problems, the *infinitesimal calculus*, that shares in the spirit of both Descartes' and Bombelli's creations.[18] Leibniz' calculus operates with symbols for infinitesimals, i.e. non-vanishing quantities that, however, are smaller than any given quantity. Leibniz did not believe that infinitesimals actually existed, but likewise $\sqrt{-1}$ in Bombelli's calculus, to treat symbols for infinitesimal quantities on equal terms with symbols for real quantities (and operate with them) proved to be a very useful strategy. Leibniz's calculus is essentially a set of rules for operating with symbols for infinitesimal quantities, which Berkeley, referring to the correspondent quantities in Newton's calculus, called "ghosts of departed quantities".[19] Previously to Leibniz, Cavalieri[20] and Kepler,[21] among others, had already developed methods for dealing with geometrical problems (calculation of volumes of solids, for example) by supposing that bodies are *actually* formed by an infinity of infinitesimal parts.[22] The reliability of these methods, however, depended heavily on the mathematician's ability to sum infinite series. Leibniz calculus, instead, reduced everything to rule following.[23]

For Leibniz, however, mathematics was not an end in itself; in a letter to Countess Elizabeth, for instance, he says: "As for myself, I cherished mathematics only because I found in it the traces of an art of *invention in general*".[24] Leibniz considered his calculus superior to Descartes' analytic geometry, and so a proof of the superiority of his philosophical system over Descartes.[25] The success of the method pointed towards *generalization* and *extension* of its core idea. The *mathesis*

[17] See Nahin 1998.

[18] Leibniz 1684.

[19] Berkeley 1948–57.

[20] Bonaventura Cavalieri (1598–1647) was a pupil of Galileo. He developed Galileo's thoughts into a geometrical method and published in 1635 a work on the subject: *Geometria Indivisilibus Continuorum Nova quadam Ratione Promota*.

[21] Johannes Kepler (1571–1630) is one of the most representative figures in the Renaissance. He was strongly influenced by Copernicus and endorsed the Platonic conception that universe is ordered according to a pre-established mathematical plan. He his Author of many works, among them a *Mysterium Cosmographicum* (1596) and his *Astronomia Nova* (1609), in which we find him saying: "The chief aim of all investigations of the external world should be to discover the rational order and harmony which has been imposed on it by God and which He revealed to us in the language of mathematics" quoted after Pearcey & Thaxton 1994, 126.

[22] See Mancosu 1996.

[23] This comes out clearly in Leibniz's *practice* of the calculus he invented, see Leibniz 1684 note 18.

[24] Leibniz 1678, quoted after Ariew & Gaber 1989, 236.

[25] See Belaval 1960.