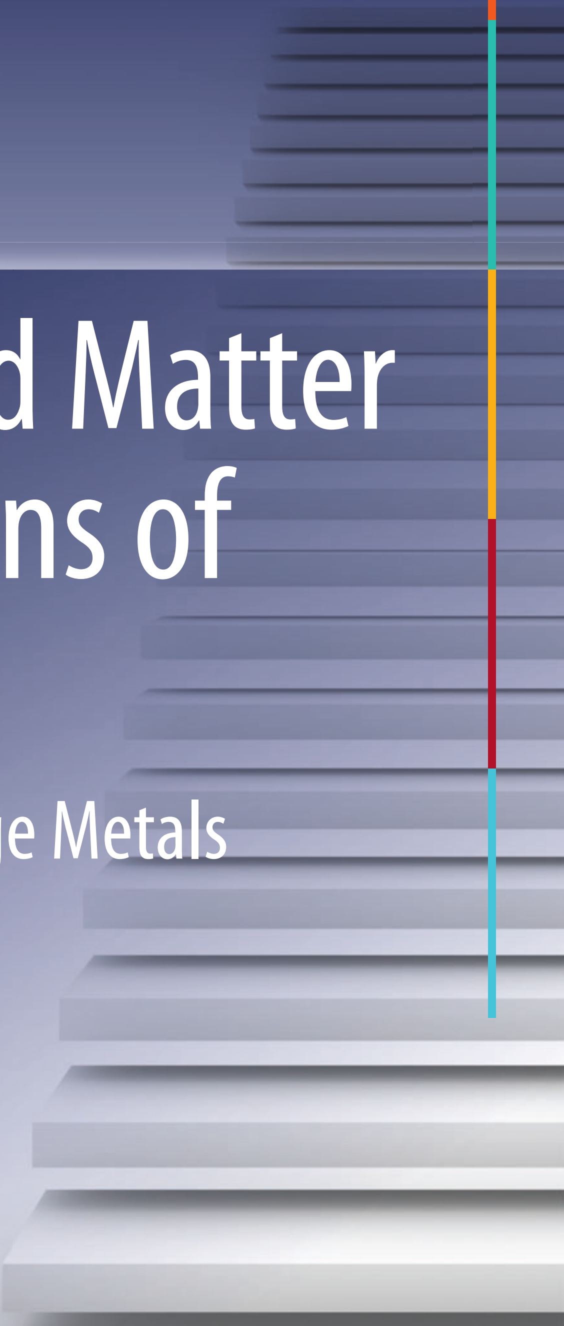
Springer Theses
Recognizing Outstanding Ph.D. Research

Andrea Amoretti

Condensed Matter Applications of AdS/CFT

Focusing on Strange Metals



Springer

Springer Theses

Recognizing Outstanding Ph.D. Research

Aims and Scope

The series “Springer Theses” brings together a selection of the very best Ph.D. theses from around the world and across the physical sciences. Nominated and endorsed by two recognized specialists, each published volume has been selected for its scientific excellence and the high impact of its contents for the pertinent field of research. For greater accessibility to non-specialists, the published versions include an extended introduction, as well as a foreword by the student’s supervisor explaining the special relevance of the work for the field. As a whole, the series will provide a valuable resource both for newcomers to the research fields described, and for other scientists seeking detailed background information on special questions. Finally, it provides an accredited documentation of the valuable contributions made by today’s younger generation of scientists.

Theses are accepted into the series by invited nomination only and must fulfill all of the following criteria

- They must be written in good English.
- The topic should fall within the confines of Chemistry, Physics, Earth Sciences, Engineering and related interdisciplinary fields such as Materials, Nanoscience, Chemical Engineering, Complex Systems and Biophysics.
- The work reported in the thesis must represent a significant scientific advance.
- If the thesis includes previously published material, permission to reproduce this must be gained from the respective copyright holder.
- They must have been examined and passed during the 12 months prior to nomination.
- Each thesis should include a foreword by the supervisor outlining the significance of its content.
- The theses should have a clearly defined structure including an introduction accessible to scientists not expert in that particular field.

More information about this series at http://www.springer.com/series/8790

Andrea Amoretti

Condensed Matter Applications of AdS/CFT

Focusing on Strange Metals

Doctoral Thesis accepted by
University of Genoa, Italy

Springer

Author
Dr. Andrea Amoretti
Mathematical Physics of Fundamental Interactions
Université Libre de Bruxelles
Brussels
Belgium

Supervisors
Prof. Nicola Maggiore
Dipartimento di Fisica
Università di Genova
Genoa
Italy

Prof. Nicodemo Magnoli
Dipartimento di Fisica
Università di Genova
Genoa
Italy

ISSN 2190-5053 ISSN 2190-5061 (electronic)
Springer Theses
ISBN 978-3-319-61874-6 ISBN 978-3-319-61875-3 (eBook)
DOI 10.1007/978-3-319-61875-3

Library of Congress Control Number: 2017944314

© Springer International Publishing AG 2017
This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.
The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.
The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

To the memory of Lucio Calzia,
a good friend, a great warrior

Supervisors' Foreword

Gauge/gravity duality is one of the most powerful, ways to systematically tackle problems regarding strongly correlated field theories. It was discovered in 1997 by Juan Maldacena in the context of string theory, but physicists very quickly realised that this (still conjectured) duality might have a wider range of possible applications. To date it has been applied to analyse problems related to quark/gluon plasma as well as condensed matter systems.

The work of Andrea Amoretti deals with the application of gauge/gravity duality to the physics of high-temperature superconductive materials. These peculiar systems were discovered a long time ago, in the early 1980s. At present, despite strong efforts made by the physics community to understand the mechanisms which govern the physics of these materials, a complete theoretical explanation of their behaviour is still lacking. In particular, the most inexplicable features appear in the non-superconductive phase, where the temperature dependence of the thermo-electric transport coefficients differ significantly from the one predicted by Fermi liquid theory. It is commonly believed that a comprehension of this exotic behaviour is fundamental in order to understand the pairing mechanism which controls the superconductive transition. In this book, Andrea Amoretti tackles the problem from a phenomenological point of view by analysing the thermo-electric transport properties of a wide class of gauge/gravity models. Specifically, since in order to compare theoretical predictions with experimental results one needs to analyse DC transport properties, mechanisms of momentum dissipation need to be included in the holographic analysis in order to make the DC transport coefficients of the model finite. Andrea Amoretti has analysed carefully these mechanisms of momentum dissipation in the context of gauge/gravity duality and has clarified, for the first time, the advantages and limitations of these models if applied to the phenomenology of the phenomenon of high-temperature superconductivity.

The work of Andrea Amoretti has initiated unusual collaborations between experimentalists and string theorists. Moreover, the present book constitutes one of the few examples in the literature in which this topic is carefully reviewed both from experimental and theoretical points of view, including not only holographic results but also standard condensed matter achievements developed over the past decades. This work might be extremely useful both scientifically and pedagogically.

Genoa, Italy
April 2017

Prof. Nicola Maggiore
Prof. Nicodemo Magnoli

Abstract

In this book we analyse the properties of certain kinds of condensed matter systems in which the electrons are strongly correlated. Among these systems, the most well-known representatives are probably high-temperature superconductors, whose properties are carefully analysed in Part I. The most exotic phase of these materials is the non-superconducting one, where the behaviour of the thermo-electric transport coefficients abruptly deviate from those predicted by Fermi liquid theory—the standard paradigm according to which we understand the majority of the existing condensed matter systems. It is commonly thought that this exotic behaviour can be explained by the strong correlations between electrons, even though, after forty years since the discovery of the first example of this class of material, a complete theoretical explanation is still lacking. The main topic of this treatise is to analyse the properties of these materials using AdS/CFT (or holographic) correspondence—mathematical tools developed in the context of string theory—representing one of the most powerful techniques known to help understand strongly correlated systems. Part II carefully introduces the basics of AdS/CFT which is required to analyse strongly correlated condensed matter systems. Part III analyses the DC themo-electric transport properties of certain kinds of holographic systems which exhibit mechanisms of momentum dissipation. The holographic results are compared with the phenomenology of high-temperature superconductors in order to understand if these materials fit in the class of strongly correlated systems described by these holographic models.

Acknowledgements

When writing a scientific manuscript, be it a journal article, a book, or a review, the most challenging part is to consider who to acknowledge. It is extremely difficult to select, among the multitude of people I have interacted with, those who deserve acknowledging the most. I think that the most fair thing to do is to start by acknowledging all the human beings that have interacted with me over the past four years, whether that be in the form of a small chat on the street, a complicated philosophical conversation in a pub accompanied by a couple of beers, or a simple scientific discussion in front of a blackboard. All of these people have contributed and must be mentioned here.

If I really have to select specifically, there are probably three people that deserve to be listed more than the others. This is because they have contributed the most to the writing of this book. The first two are *Nico* and *Nicola*, my friends, collaborators, and supervisors, who have patiently tolerated my complaints about searching for postdoctoral jobs over the last year. The third person, probably the most important, is my close friend and collaborator *Daniele Musso*. I shared with him most of my doubts, passions, and moments of happiness and sadness over the last four years. It would not have been the same without him.

A final thought goes to *my family*, to whom I am grateful for the support and encouragement that has always been given to me when following my dreams and passions.

Contents

Part I Condensed Matter Background

Part II Introduction to Holography

Introduction

In the past century there has been great deal of progress made in understanding the world around us by means of quantum field theory, an extremely powerful tool which allows us to understand many different physical phenomena, from elementary particle physics to condensed matter physics. Our ability to extract results from quantum field theory mostly relies on perturbation theory. In this framework, the physical observables are usually evaluated as an expansion in powers of the coupling constant, i.e., a dimensionless parameter which measures the departure from free-field theory.

However, in the last decades it has been realised that nature cannot always be investigated by means of perturbation theory. One of the most well-known examples of this fact is quantum chromodynamics (QCD). The asymptotically free nature of QCD makes perturbation theory reliable at high energies. On the other hand, at low energies, QCD becomes strongly coupled to phenomena such as confinement and chiral symmetry, which are non-perturbative in nature. In the condensed matter framework, the prototypical example of a non-perturbative phenomenon is that of high-T_c superconductors, where strong coupling causes the physical behaviour of these materials to abruptly deviate from the standard paradigm according to which we understand normal metals in nature—the Fermi liquid theory. Recently, one of the predominant ideas for explaining the strong coupling nature of these materials has been that proposed by Sachdev [6], who suggested that strong interactions between electrons at finite temperatures are governed by the existence of critical points at zero temperature. Systems in the vicinity of a critical point are actually described by scale invariant quantum field theory because of the infinite correlation lengths which arise. Then, in this sense, the problem reduces to that of finding a suitable strongly interacting scale invariant quantum field theory which describes the physics of such strange metals. Unfortunately, at present we know very few techniques to analyse strongly interacting quantum field theory. To this end, most recently a mathematical tool has been developed in the context of string theory, the so called Anti-de Sitter/conformal field theory (*AdS/CFT) correspondence* (or gauge/gravity duality), which has acquired a prominent role in helping to understand the general properties of strongly coupled systems. In brief, the tool is based on a conjectured duality, discovered by Juan Maldacena [4] in 1997, between certain strongly coupled regimes of ordinary quantum field theories in d spacetime dimensions and classical (i.e., weakly coupled) theories of gravity in at least $d+1$ dimensions. As a result, the correspondence maps difficult quantum problems in field theory to easier, classical ones on gravity. In its simplest form, the correspondence relates a strongly coupled CFT to classical gravity on AdS backgrounds.

To date, the gauge/gravity duality provides the closest connection between string theory and the observable world. Simultaneously, it constitutes an extremely promising environment for enlarging our theoretical understanding of strongly interacting quantum systems and string theory itself. Despite the fact that this tool was born out of string theory, in the last decade it acquired a prominent role to help understand strongly interacting quantum systems by means of providing an effective description. Such an effective description does not take into account the string theory origin of the duality. The main goal of this approach is to construct effective toy models with features which are believed to be universal to many other strongly interacting systems either with or without a stringy origin. Within this framework, the duality has been used extensively to describe phenomena analogous to those which occur in QCD and in high-temperature superconducting materials.

The holographic correspondence provides a relatively easy set-up for computing properties of systems in and out of equilibrium, as well as phases with non-zero fermionic densities and transport coefficients. The latter are all difficult to achieve with other standard non-perturbative approaches. The main limitation of AdS/CFT is that, at present, realistic field theories like QCD cannot be directly explored. However, despite its limitation to toy models, the correspondence has provided valuable insights at both the quantitative and qualitative level on properties of strongly coupled systems realized in nature.

In the context of condensed matter, this duality has been applied to the study of unconventional superconductivity. Assuming the existence of a quantum critical point in the phase diagram of these materials, according to Sachdev's proposal, the theory governing these systems in the vicinity of this point has to be scale invariant, allowing the gauge/gravity duality to be applied in its simplest form. Using holographic techniques, one can study the perturbations within the quantum critical region in a controlled way using a holographic dictionary. As an example, studying the system at finite temperature is equivalent to placing a black hole at the center of AdS spacetime. Analogously, one can analyze the system at finite charge density or include the effects of an external magnetic field by introducing a Maxwell field in the gravitational model. At present this is a very active field of research with a number of good reviews and books already available in the research literature (see, e.g., [1–3, 5, 7–8])

The aim of this book is to consider how the AdS/CFT correspondence might help in gaining some insight of strongly coupled condensed matter systems. In particular, in the following Chapters 6, 7, 8 and 9, the holographic correspondence is used to try to understand the peculiar properties of high-T_c superconductors.

Since this is a line of research where techniques from very different fields of physics come together, in order for the discussion to be self-consistent and comprehensible to scientists from different branches of physics, a large introductory text is necessary. Consequently, the book is organised into three parts. Parts I and II are introductory while Part III contains the original result of the treatise.

In Part I the basic features of high-T_c superconductors are described. Initially the book analyses the experimental properties of these peculiar materials, focusing in particular on in-plane thermo-electric transport properties. At each step, the differences between the behaviour of the strange metals and the Fermi liquid prediction is commented upon. In the last chapter of Part I we focus on some theoretical attempts, introduced in the past, to explain the exotic behaviour of these materials, concentrating on the idea of the existence of a quantum critical point in their phase diagram.

In Part II the AdS/CFT correspondence is introduced. This Part is constructed in order to introduce the holographic duality as a series of computational rules, the so called *holographic dictionary*, the knowledge and comprehension of which is a necessary step in order to understand the discussion in Part III of the book.

Part III contains the original results of the manuscript. In particular, the thermo-electric transport properties of a strongly coupled bi-dimensional plasma in the presence of an extrinsic mechanism of momentum dissipation is deeply analysed using holography. The holographic results are compared with the phenomenology of strange metals. In the last chapter of Part III a conclusion to the book, and the physical implications of these holographic toy models, are discussed in details.

References

1. Martin Ammon and Johanna Erdmenger, *Gauge/Gravity Duality* (Cambridge University Press, Cambridge, UK, 2015)
2. Sean A. Hartnoll, Lectures on Holographic Methods for Condensed Matter Physics. Class. Quant. Grav. **26**, 224002 (2009)
3. Sean A. Hartnoll, Andrew Lucas, Subir Sachdev, *Holographic Quantum Matter*, Dec 21, (2016), pp. 178. arXiv:1612.07324 [hep-th]
4. Juan Martin Maldacena, The large N limit of superconformal field theories and supergravity. Int. J. Theor. Phys. **38**, 1113–1133 (1999). [Adv. Theor. Math. Phys. 2,231(1998)]
5. Makoto Natsuume, AdS/CFT Duality User Guide. Lect. Notes Phys. **903**, 1–294 (2015)
6. S. Sachdev, *Quantum Phase Transitions* (Cambridge University Press, 2001)
7. Jan Zaanen, Ya-Wen Sun, Yan Liu, Koenraad Schalm, *Holographic Duality in Condensed Matter Physics* (Cambridge University Press, 2015)
8. A. Zaffaroni, Introduction to the AdS–CFT correspondence. Class. Quant. Grav. **17**, 3571–3597 (2000)
9. A. Amoretti and A. Braggio and N Maggiore, N Magnoli. Thermo-electric transport in gauge/gravity models. In: Adv. Phys. X 2.2 (2017), pp. 409–427
10. A. Amoretti, N. Magnoli, *On Conformal Perturbation Theory*, May 9, (2017), pp. 20. arXiv: 1705.03502 [hep-th]

11. A. Amoretti, D. Arean, R. Argurio, D. Musso, Z. Pando, A. Leopoldo. *A holographic perspective on phonons and pseudo-phonons*. In: JHEP 1705 (2017) 051. arXiv:1611.09344 [hep-th]
12. A. Amoretti, M. Baggioli, N. Magnoli, D. Musso. *Chasing the cuprates with dilatonic dyons*. In: JHEP 06 (2016), p. 113. arXiv:1603.03029 [hep-th]
13. A. Amoretti, D. Musso. *Magneto-transport from momentum dissipating holography*. In: JHEP 09 (2015), p. 094. arXiv:1502.02631 [hep-th]
14. A. Amoretti, A. Braggio, N. Magnoli, D. Musso. *Bounds on charge and heat diffusivities in momentum dissipating holography*. In: JHEP 07 (2015), p. 102. arXiv:1411.6631 [hep-th]
15. A. Amoretti, A. Braggio, G. Caruso, N. Maggiore, N. Magnoli. *Introduction of a boundary in topological field theories*. In: Phys. Rev. D90.12 (2014), p. 125006. arXiv:1410.2728 [hep-th]
16. A. Amoretti, A. Braggio, N. Maggiore, N. Magnoli, D. Musso. *Analytic dc thermoelectric conductivities in holography with massive gravitons*. In: Phys. Rev. D91.2 (2015), p. 025002. arXiv:1407.0306 [hep-th]
17. A. Amoretti, A. Braggio, N. Maggiore, N. Magnoli, D. Musso. *Thermo-electric transport in gauge/gravity models with momentum dissipation*. In: JHEP 09 (2014), p. 160. arXiv:1406.4134 [hep-th]
18. A. Amoretti, A. Braggio, G. Caruso, N. Maggiore, N. Magnoli. *Holography in flat spacetime: 4D theories and electromagnetic duality on the border*. In: JHEP 04 (2014), p. 142. arXiv:1401.7101 [hep-th]
19. A. Amoretti, A. Braggio, N. Maggiore, N. Magnoli, D. Musso. *Coexistence of two vector order parameters: a holographic model for ferromagnetic superconductivity*. In: JHEP 01 (2014), p. 054. arXiv:1309.5093 [hep-th]
20. A. Amoretti, A. Braggio, G. Caruso, N. Maggiore, N. Magnoli. *3+1D Massless Weyl spinors from bosonic scalar-tensor duality*. In: Adv. High Energy Phys. (2014), p. 635286. arXiv:1308.6674 [hep-th]
21. A. Amoretti, A. Blasi, G. Caruso, N. Maggiore, N. Magnoli. *Duality and Dimensional Reduction of 5D BF Theory*. In: Eur. Phys. J. C73.6 (2013), p. 2461. arXiv:1301.3688 [hep-th]
22. A. Amoretti, A. Blasi, N. Maggiore, N. Magnoli. *Three-dimensional dynamics of four-dimensional topological BF theory with boundary*. In: New J.Phys. 14 (2012), p. 113014.

Part I
Condensed Matter Background

Chapter 1
Preamble: Transport Coefficients Definition

In this manuscript we will analyse the thermo-electric transport properties of two-dimensional condensed matter systems. Then, in order to fix the notations in this brief preamble we will define the transport coefficients for the case at hand.

We will consider a two-dimensional system living in the plane $x - y$ and, in some cases, we will analyse also the effects due to an external magnetic field B applied in the direction perpendicular to the $x - y$ plane, z (see Fig. 1.1).

We are interested in the response of the electrical current $\vec{J}$ and the heat current $\vec{Q}$ to an applied electric field $\vec{E}$ and a temperature gradient $\vec{\nabla} T$. By definition, the transport coefficients relate the previous quantities in the following way:

$$\begin{pmatrix} \vec{J} \\ \vec{Q} \end{pmatrix} = \begin{pmatrix} \hat{\sigma} & \hat{\alpha} \\ T\hat{\alpha} & \hat{\bar{\kappa}} \end{pmatrix} \begin{pmatrix} \vec{E} \\ -\vec{\nabla} T \end{pmatrix} . \tag{1.1}$$

In the presence of an external magnetic field B in the z-direction (see Fig. 1.1) the transport coefficients $\hat{\sigma}$, $\hat{\alpha}$ and $\hat{\bar{\kappa}}$ are matrices, which, due to Onsager reciprocity, assume the following form:

$$\hat{\sigma} = \sigma_{xx} \hat{1} + \sigma_{xy} \hat{\varepsilon} , \tag{1.2}$$

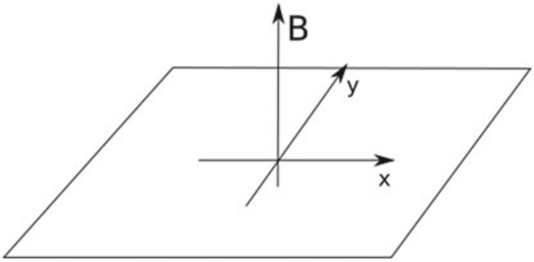
where $\hat{1}$ is the identity, and $\hat{\varepsilon}$ is the antisymmetric tensor $\varepsilon_{ij} = -\varepsilon_{ji}$. σ_{xx} and σ_{xy} describe the longitudinal and Hall conductivity, respectively. The resistivity $\hat{\rho}$ is defined as the inverse of the conductivity matrix, namely $\hat{\rho} = \hat{\sigma}^{-1}$. Similarly, the thermo-electric conductivity $\hat{\alpha}$ has a form analogous to (1.2), and determines the Seebeck coefficient S via the relation:

$$S = \frac{\alpha_{xx}}{\sigma_{xx}} , \tag{1.3}$$

Finally $\hat{\bar{\kappa}}$, which governs thermal transport in the absence of electric fields, assumes a similar structure to that described before for $\hat{\sigma}$ and $\hat{\alpha}$. In contrast to $\hat{\bar{\kappa}}$, the thermal

A. Amoretti, *Condensed Matter Applications of AdS/CFT*, Springer Theses,
DOI 10.1007/978-3-319-61875-3_1

Fig. 1.1 Schematic illustration of a typical experimental setup



conductivity, $\hat{\kappa}$, is defined as the heat current response to $-\vec{\nabla}T$ in the absence of an electric current, namely, in the presence of electrically isolated boundaries. It is given by

$$\hat{\kappa} = \hat{\bar{\kappa}} - T\hat{\alpha} \cdot \hat{\sigma}^{-1} \cdot \hat{\alpha} \; . \tag{1.4}$$

Eventually, the Nernst coefficient is defined as the electric field induced by a thermal gradient in the absence of an electric current. It is defined by the linear response relation $\vec{E} = -\hat{\theta}\vec{\nabla}T$, with

$$\hat{\theta} = -\hat{\sigma}^{-1} \cdot \hat{\alpha} \; . \tag{1.5}$$

With these definitions at hand, we are now ready to start the analysis of the exotic and exciting properties of the cuprates superconductors, starting by understand how they differ from the Fermi Liquid theory.

Chapter 2
Standard Metals and the Fermi Liquid

One of the milestones and great results of the 20th century is *Landau's Fermi liquid theory*, which underlines our present understanding of the majority of the known states of matter, like normal metals, semi-conductors, superconductors and superfluids. To better understand the differences between the predictions of this great theory and the behaviour of the cuprate superconductors, strange states of matter discovered since the early 80s, it is necessary to recall its basic properties in this Introduction, referring the reader interested in the technical aspects to Appendix A or to standard condensed matter textbook (e.g. [1, 2]).

Let us start by recalling the basic properties of a system of free fermions in a box, where the Pauli exclusion principle controls everything. In the ground-state of this system all the single-particle states inside a sphere in momentum space with radius k_F[1] are filled, while the state outside the sphere are empty. The external surface of this sphere is called the *Fermi surface*. The system has two types of low energy excitations. One can in fact fill a state slightly outside the Fermi surface, creating a particle, or remove a fermion from a filled state slightly inside the Fermi surface, creating what is typically called a hole. These excitations are gapless by definition, and have linear dispersion (for $k - k_F \ll k_F$):

$$\varepsilon(k) = \frac{k^2}{2m} - \mu = \frac{k_F}{2m}(k - k_F) \equiv v_F(k - k_F) \ , \tag{2.1}$$

where m is the mass of the fermions, $\mu \equiv \frac{k_F^2}{2m}$ is the chemical potential, the quantity $v_F \equiv \frac{k_F}{m}$ is called the Fermi velocity and particles and holes are distinguished by the sign of $k - k_F$. Rephrasing the previous statements in a more formal language,

---

[1] k_F is fixed by the density of Fermions, as we will see below.

A. Amoretti, *Condensed Matter Applications of AdS/CFT*, Springer Theses,
DOI 10.1007/978-3-319-61875-3_2