

# PROBABILISTIC PHYSICS OF FAILURE APPROACH TO RELIABILITY

**Modeling,  
Accelerated  
Testing,  
Prognosis and  
Reliability  
Assessment**

**Mohammad Modarres  
Mehdi Amiri  
Christopher Jackson**

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# Probabilistic Physics of Failure Approach to Reliability

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**Scope:** A true performance of a product, or system, or service must be judged over the entire life cycle activities connected with design, manufacture, use and disposal in relation to the economics of maximization of dependability, and minimizing its impact on the environment. The concept of performability allows us to take a holistic assessment of performance and provides an aggregate attribute that reflects an entire engineering effort of a product, system, or service designer in achieving dependability and sustainability. Performance should not just be indicative of achieving quality, reliability, maintainability and safety for a product, system, or service, but achieving sustainability as well. The conventional perspective of dependability ignores the environmental impact considerations that accompany the development of products, systems, and services. However, any industrial activity in creating a product, system, or service is always associated with certain environmental impacts that follow at each phase of development. These considerations have become all the more necessary in the 21st century as the world resources continue to become scarce and the cost of materials and energy keep rising. It is not difficult to visualize that by employing the strategy of dematerialization, minimum energy and minimum waste, while maximizing the yield and developing economically viable and safe processes (clean production and clean technologies), we will create minimal adverse effect on the environment during production and disposal at the end of the life. This is basically the goal of performability engineering.

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Modeling, Accelerated Testing, Prognosis  
and Reliability Assessment

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# Preface

This book is result of the compilation of class notes from several years of teaching a graduate course on physics-of-failure and accelerated testing to the graduate students pursuing Master of Science, Master of Engineering and PhD degrees in Reliability Engineering at the University of Maryland. The book provides probabilistic and highly technical approaches to the physics-of-failure and mechanistic based reliability prediction and assessment. It relies on various methods and techniques published in the open literature regarding the development and practice of physics-of-failure analysis, accelerated life testing and accelerated degradation testing. The authors first discuss the overall concepts, objectives and framework for accelerated life assessment through the use of formal probabilistic physics-of-failure models. They review important failure mechanisms to demonstrate the process of examining and developing appropriate physics and mechanistic models that describe the degradation and failure phenomena in accompanying accelerated testing and accelerated degradation testing methods, including step-stress testing. The book presents data analysis methods to evaluate the probabilistic physics-of-failure models based on the observed data obtained from accelerated reliability tests. Further, it discusses the steps and methods of probabilistic life assessment and integrity of structures, components and systems based on the probabilistic physics-of-failure models. Since the book is intended for graduate-level students and for highly trained reliability engineers, it provides supplementary solved examples to clarify complex technical topics within each chapter. Some of these examples are benefitted directly or with some modifications from other sources, including Bannantine, et al. (1997), Collins (1993), Stephens, et al. (2003), Meeker & Escobar (1998), Nelson (2004), and Dowling (1998), which are referenced extensively in the book. Although qualitative accelerated tests such as the Highly Accelerated Life Test (HALT) and Environmental Stress Screening (ESS) have been briefly reviewed, the book is mainly about the quantitative methods in probabilistic physics-based and accelerated testing life assessment of structures, components and systems. A companion website under the auspices of the Center for Risk and Reliability at the University of Maryland ([www.crr.umd.edu](http://www.crr.umd.edu)) provides downloadable support files for additional information and computational tools in form of MATLAB, R and OpenBUGS scripts to perform some of the more involved computational analyses discussed in the book. These files will be updated and conformed to the most recent versions of these tools. The companion website also includes a section on testing equipment and resources needed for accelerated testing. This book benefitted from contributions of many students who enrolled in the accelerated testing courses over many years at the University of Maryland. Particularly, inputs and solved example from Wendell Fox, Jonathan DeJesus, Reuel Smith, Reza Azarkhail, Andrew Bradshaw, and Taotao Zhou have been significant and are much appreciated.

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# Chapter 1: Overview of Probabilistic Physics-of-Failure Approach to Reliability

## 1.1. INTRODUCTION

To address risk and reliability challenges in design, manufacturing and operation, reliability engineering has gone through a number of transformations over the past few decades. Reliability methods have progressively become more realistic by incorporating both data and information from real causes, and the modeling of failure phenomena. The evolution of reliability modeling from constant hazard rates to more representative life distributions (such as the Weibull and lognormal) was the first step towards better addressing wear-out and aging failure mechanisms in structures, systems and components. This trend was followed by the use of physics and mechanistic principles, as well as thermodynamic laws. Accelerated testing has borrowed concepts from materials degradation and fracture mechanics, through which the aggregate effects of operational and environmental conditions were formally accounted for in the life models.

The formal consideration of physics and mechanistic methods in reliability engineering is referred to as a “physics of failure” (PoF) approach. The PoF approach is a science-based means to reliability engineering and prediction as well as prognosis and health management, in contrast to the traditional statistical approach that relies on historical data. It uses physics-based modeling and simulations to assess design and reliability. The approach can be used to evaluate and predict system performance while reducing subjectivities in reliability assessments by modeling failure mechanisms such as fatigue, fracture, wear, and corrosion. The PoF approach is a comprehensive representation of wear out and aging, and is capable of bringing relevant physical factors into the life assessment and reliability models of the structures, components and systems.

The development of PoF models is still typically based on limited information. The uncertainties associated with this limitation have led to a Probabilistic Physics of Failure (PPoF) approach that formally addresses and incorporates uncertainties about the PoF models and their outputs.

Physics and mechanistic-based failure models can be categorized into three core frameworks: stress-strength, damage-endurance, and performance-requirements. In all these PoF modeling frameworks, metrics representing failure-inducing agents such as applied loads and environmental attack properties should be identified. Mechanical, thermal, electrical, chemical, and radiation-induced forces can cause stresses on an item. The passage of time drives the accumulation of damage. Both load and time may either be analyzed deterministically (e.g., identifying and studying the sources of stresses) or probabilistically (e.g., treating stress variation as a random variable). Substantial uncertainties associated with failure-inducing agents can emanate from environmental and operating (use) conditions and from the emergence of failure mechanisms that were not considered or well understood at the time of design.

Because of cost and time limitations, great emphasis has been placed on capturing reliability information from field data with minimal effort. As a result, design and assessment methodologies that address failures mechanistically have emerged as popular and powerful cost saving techniques. Accelerated life testing (ALT), an approach to mechanistic modeling of wear-out, damage process and failure, is a direct outcome of this movement. Unlike the reliability models developed on the basis of field data that suffer from wide variation in operating conditions and practices, reliability models based on PoF, developed using accelerated life or degradation tests, take into account operational conditions (applied stresses) that permit flexibility in applied stresses, leading to more relevant models.

## 2 PROBABILISTIC PHYSICS OF FAILURE APPROACH TO RELIABILITY

Before performing the accelerated test, a stress agent, which could be an aggregate effect of a single or multiple physical and operational conditions, should be identified. The next step involves accelerating this stress agent and applying it to samples of the structure, system or component in a test environment. Models of failure, damage and degradation are developed by using accelerated test data for a more flexible and representative description of the damage, failure phenomena, performance and life as compared to the traditional probabilistic approach to failure modeling.

Failure interdependency can also be a critical factor in reliability modeling of mechanical systems and components. In the study of system behavior, there are situations in which progressive failure of one component may activate or accelerate other failure mechanisms or the failure of other components. There are usually many links between different components by means of their properties and common environmental conditions. The PoF approach properly incorporates these interdependencies in complex structures, systems and components.

### 1.2. OVERVIEW OF PHYSICS-OF-FAILURE MODELING

Physics of failure modeling initially evolved out of examination of fatigue and fracture of materials. Reliability work related to fatigue and fracture of materials showed significant progress through the 1950s and early 1960s. In 1957, George R. Irwin proved that the fracture of materials was due to plastic deformation at the crack tip and generalized Griffith's Theory (Irwin 1957) that described the relationship between applied nominal stress and crack length at fracture. Between 1955 and 1963, Waloddi Weibull produced several publications related to modeling of fatigue and creep mechanisms that also discussed evaluating associated data (Weibull 1959). In 1961, Weibull published a book on materials and fatigue testing while working as a consultant for the U.S. Air Force Materials Laboratory (Weibull 1961). Building on Irwin's work on stress intensity factor, Paris et al. (Paris, Gomez and Anderson 1961) introduced methods for predicting the rate of fatigue crack growth.

Given this background in mechanistic-based life models (particularly to assess fatigue and fracture failures), Rome Air Development Center (RADC—the predecessor to the U.S. Air Force Rome Laboratory) introduced a PoF program in 1961 to address the growing complexity of military equipment and the consequent increase in number of failures observed. In 1962, researchers from Bell Labs published a paper on “High Stress Aging to Failure of Semiconductor Devices” that justified using the kinetic theory's interpretation of the Arrhenius equation: a simple yet accurate formula for the temperature dependence of the reaction rate constant as a basis for assessment of temperature-induced aging of semiconductor devices (Dodson and Howard 1961). Later, the RADC and Armor Research Foundation of the Illinois Institute of Technology (now IIT Research Institute) organized the first PoF symposium in electronics in Chicago in September 1962. This symposium laid the groundwork for future research and development activities related to PoF by RADC and several other organizations. Numerous original papers and ideas introducing and explaining the PoF concepts and methods were presented in these symposia, which continue today in IEEE International Reliability Physics symposia and Reliability and Maintainability Symposia (RAMS).

The PoF approach to reliability utilized scientific knowledge of damage and degradation processes and the load profile applied to an item, its structure, material properties and environmental conditions to identify potential failure mechanisms that individually or in combination lead to the item's failure. The PoF models would then be used to assess reliability, expended life and remaining life. Using PoF diminishes the need for enormous amounts of use-level life data and uses smaller sets of accelerated test data and other relevant data to present a more representative model. The PoF approach employs the available well-developed knowledge about the mechanisms of failure. The PoF models show how and why items fail, reducing the need for large (and expensive) quantities of life data.

The most critical step in a PoF approach is to understand failure mechanisms (such as corrosion or fatigue) in order to appropriately model degradation and time that a failure occurs. Accelerated life testing based on PoF models is an approach that can reduce long and costly life testing. In this approach, one seeks to relate the fundamental physical and chemical properties of materials to reliability metrics (such as degradation, life or cycles-to-failure). To eliminate (or reduce) the occurrence of failures, one must eliminate (or reduce) their root causes. To do that, one must also understand the physics of the material and failure mechanisms involved (Vaccaro 1962). Sometimes it is impossible to build several identical units or prototypes for reliability testing. Cases in point include large-scale systems (like buildings and space vehicles), one-of-a-kind or highly expensive systems, and units that must work properly at the first time. In these cases, performance and field data are not available, and a PoF approach to degradation and life assessment is the most appropriate. As such, the PoF approach is particularly useful in the design stage when there are limited prototypes or test facilities. Finally, the PoF approach has great utility when dealing with highly reliable units, when there is very little failure data to analyze.

PoF techniques can be used to interpret and extrapolate field data for failure prediction for in-service components. This field data might include parameters that are related to traditional physical measures but can only be used as a loose model for failure prediction. A good example of this is vibration of a bearing. The vibration is suggestive of a flaw, but since the flaw itself cannot be tracked, the vibration can be used to estimate failure. This obviously presents potential for high uncertainty because changes in the model cannot be easily detected; for example, there is no assurance that if a new flaw develops it would expedite the failure, so that it can be detected and incorporated into the model. However, this method at least presents some means of tracking the degradation of the component. This is useful for maintenance practitioners, as it provides a means of failure estimation when traditional methods cannot be used due to the lack of measurable PoF model parameters.

There is no single methodology for performing PoF-based reliability analysis. Chapter 2 discusses in more detail the steps involved in developing a PoF model of an item. If an item involves multiple subassemblies (parts and components), each subject to different failure mechanisms, then the combined effect of applicable failure mechanisms should be modeled. Figure 1.1 depicts the structural and dynamic hierarchy of PoF analysis elements for a multi-component system. The lowest level in this hierarchy is inter- and intra-environmental factors. The intra-environmental factors refer to conditions resultant from unit operation itself. This includes, for example, heat dissipation or vibration caused by an imbalanced rotating shaft. The inter-environmental factors are those imposed externally from its design boundary. Examples include relative humidity and prevalence of dust particles. There may be a causal chain among inter- and intra-environmental factors such that one may lead to another or vice versa in a synergistic manner. For example, a low temperature may cause condensation, leading to accelerated corrosion.

All environmental factors potentially lead to various forms of stress. For example, high temperature (as either an inter- or intra-environment factor) leads to thermal expansion, and (if the unit is confined) can cause mechanical stresses. Such stress agents are key actors in activating or accelerating degradation through corresponding failure mechanisms. While one failure mechanism may also accelerate another (such as corrosion accelerating fatigue), failure mechanisms can also produce new stresses. For example, wear in a journal bearing can cause vibration-induced fatigue. The top part of the hierarchy in Figure 1.1, known as the structural hierarchy, depicts the formal organization and topology of the system showing the functional and support relationships among parts, components and the whole systems. On the other hand, the lower part of the figure, the systems dynamics hierarchy, shows the underlying processes (failure mechanisms) and conditions that lead to the occurrence or acceleration of such mechanisms.

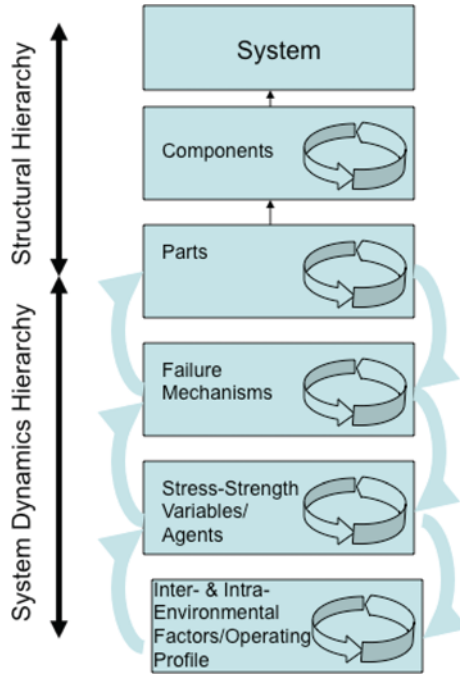


Figure 1.1: System hierarchy used in PoF analysis

### 1.3. IMPORTANT FORMS OF POF MODELS

As noted above, there are three possible PoF modeling frameworks subject to the nature of underlying failure and degradation mechanism. Each is described briefly below.

**Stress-Strength Model.** In this model, the item (e.g., a structure, system or component) fails if the applied stresses caused by design, operation and the external environment exceed its strength (see Figure 1.2). This failure model may depend on environmental conditions, applied operating loads and the occurrence of critical events, rather than the passage of time or cycles. Stress and strength are treated as a random variable encompassing variability in all conditions. Two examples of this model include a steel bar under a mean tensile stress lower than its yielding point but which will be randomly subjected to load that exceeds the yielding point over time.

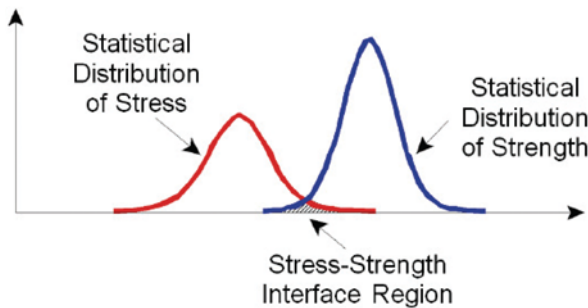


Figure 1.2: Stress-strength modeling

The second is a transistor with a mean voltage applied across the emitter-collector remaining below a failed level but which may randomly exceed the limit. In the case of the steel bar, the likelihood of failure is estimated from the probability that the stress random variable exceeds the strength random variable, which is obtained from a convolution of the two respective distributions.

**Damage-Endurance Model.** This model differs from the stress-strength model in that the *stress* (load) causes degradation in the form of irreversible cumulative damage through, for example, corrosion, wear, embrittlement, creep, or fatigue. The stress (load) aggregate drives the cumulative damage metric. Cumulative damage may not degrade performance; however, the item fails when the cumulative damage exceeds its endurance limit. For example, a crack grows on a structure until it reaches a critical length beyond which the growth will be catastrophically rapid. Accumulated damage does not disappear when the *stresses* are removed, although sometimes treatments such as annealing can repair cumulative damage. Variables representing damage and endurance may be treated as random and represented by probability density functions to capture distributions of initial damage, model parameter uncertainties, and model errors. Therefore, at any time or cycle (see Figure 1.3) the likelihood of failure may be represented by the exceedance of the damage distribution from the endurance probability density functions. If endurance is not a random variable and remains constant, then the distribution of the time to failure may be obtained when cumulative damage values randomly exceed the constant value of the endurance (see Figure 1.3). The distribution of the time-to failure shown in Figure 1.3 is based on the assumption of a constant endurance limit around the median of the distribution of the endurance. Clearly, at a given time or cycle,  $N$ , the probability that the damage distribution exceeds endurance level (or distribution of endurance), would be equal to the probability that the random variable, time to failure (as represented by the time to failure distribution) is lower than  $N$ .

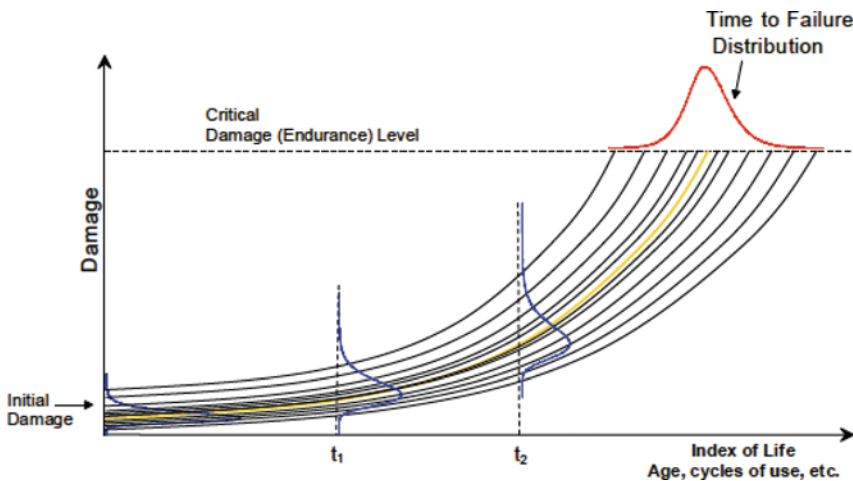


Figure 1.3: Damage-endurance model

**Performance-Requirements Model.** In this modeling approach, a system performance characteristic (such as system output capability, efficiency or availability) is satisfactory if it remains within acceptable tolerance limits. Examples include rotating machinery efficiency and printer print quality (such as one that is based on a level of efficiency or output at the pump head). Systems start with a positive margin of performance that cumulatively and irreversibly degrades due to the underlying failure mechanisms. These mechanisms cause degradation and damage until performance falls below the minimum requirement level (i.e. fails). As the stress applied to the unit increases the rate of performance degradation, the time to failure (the point at which the system reaches minimum or acceptable performance limit) is reduced. The concept is depicted in Figure 1.4.

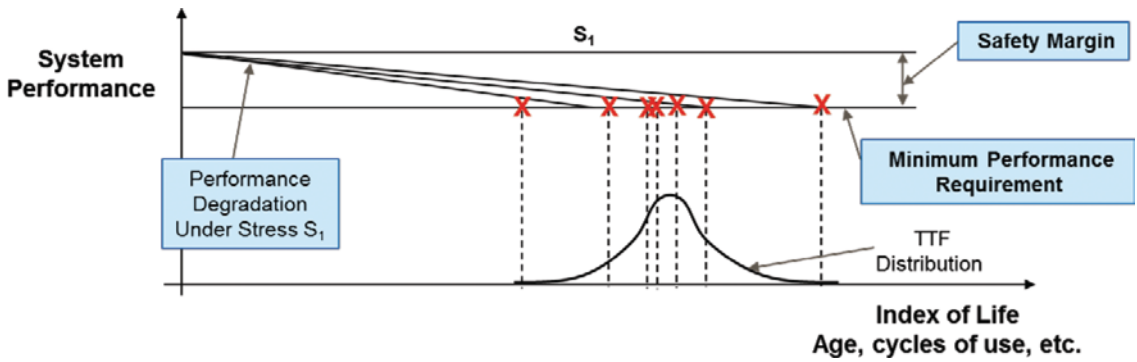


Figure 1.4: Performance-requirement model

#### 1.4. PPOF APPROACH TO LIFE ASSESSMENT

Due to the inevitable stochastic variations of the many factors involved in degradation and failure processes described by PoF models, probabilistic physics-of-failure (PPoF) models can be used to formally account for the uncertainties and model errors. Factor variations include uncertainties in environmental and operational stresses, mission profile and variability in materials properties, and stress agents. The earliest effort in PPoF modeling was by Haggag, et al. (Haggag, McMahon, Hess, Cheng, Lee, & Lyding, 2000) who presented a PPoF approach to reliability assurance of high-performance chips by considering common defect activation energy distribution. Hall and Strutt (Hall & Strutt, 2003) have presented PPoF models for component reliabilities by considering parameter and model uncertainties. Azarkhail and Modarres (Azarkhail & Modarres, 2007) have presented a Bayesian framework for uncertainty management in physics-based reliability models. Matik and Sruk (Matik & Sruk, 2008) highlighted the need for PoF to be probabilistic in order to include inevitable variations of variables involved in processes contributing to the occurrence of failures in the analysis. Finally, Chatterjee and Modarres (Chatterjee & Modarres, 2012) have presented PPoF modeling of integrated steam generators in small modular reactors. Although substantial research has been done on PPoF modeling for reliability assessment, more research in this area is necessary.

The element of a PPoF model that assesses time-to-failure of a component (such as a ball bearing under fatigue-wear degradation mechanism) is illustrated in Figure 1.5. The lowest element in this figure shows the inter- and intra-environmental factors that produce the stresses that cause degradation and failure. The next three elements are drawn from the PoF models that describe the degradation. The final element (top level) is the probabilistic life assessment that formally accounts for parameter uncertainties in the PoF models and model errors. In Figure 1.5, a probabilistic approach (such as Bayesian inference) is shown to characterize the corresponding PoF model uncertainties. The arrows in Figure 1.5 show the direction of influences, such as how external ambient temperature may affect viscosity. Usually the direction of influences is upward (i.e., sequential causal relationships), but it is possible to have some influences going downward, causing a circular synergy among variables. For example, certain operating conditions, such as high internal temperature generated by poor lubrication during operation, lowers lubricant viscosity, which in turn increases the friction that further exacerbates the high internal temperature.

There are two basic types of uncertainties that can be described by a PPoF model of failure mechanisms: aleatory and epistemic uncertainty. Aleatory uncertainty is the inherent randomness of the phenomena that the model attempts to predict. This type of uncertainty is intrinsic and cannot be reduced. Examples of aleatory uncertainty include random environmental variations, random vibration in stress amplitude and certain material properties such size and density of flaws. Epistemic uncertainty is about

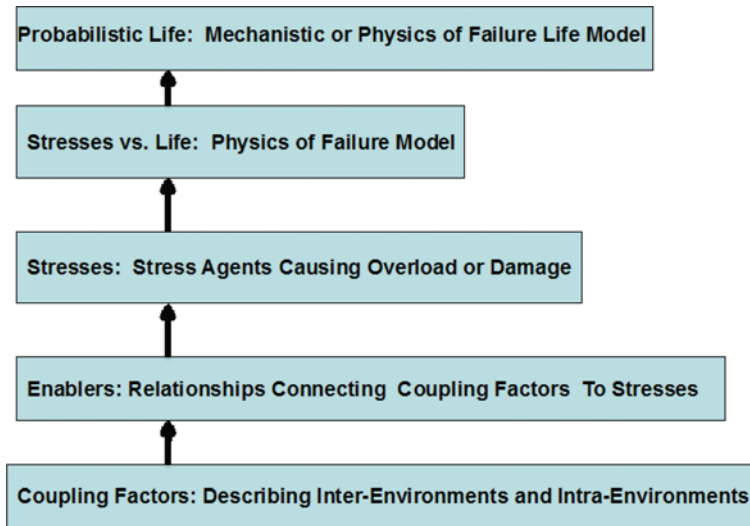


Figure 1.5: System hierarchy in probabilistic-mechanistic reliability life model

our lack of knowledge and consists of incomplete description of the modeled phenomena, measurement errors, and a lack of sufficiently accurate measurements to fully capture the phenomena. Incorporating additional PoF model data and information reduces this type of uncertainty: as such this uncertainty is reducible, whereas aleatory uncertainty is not. Since there can be uncertainties associated with failure-inducing agents (i.e., stresses), model parameters and the model itself, the prediction of failures is inherently a probabilistic problem requiring PPoF models for applicable primary failure mechanisms (Modarres, Kaminskiy, & Krivtsov, 2017). Each failure, damage or degradation mechanism should have its own PPoF model. All applicable PPoF models applied to an item need to be combined to find the overall degradation process. Methods for combining multiple PPoF models include the use of the weakest link approach, which assumes that one of such degradation mechanisms causes damage that will exceed the endurance limit before the other applicable mechanisms.

PPoF models are formulated considering all the variables that can initiate and propagate degradation in the item under study. As part of this process, one should identify important degradation causing variables such as (for a rotating tube example) normal loads, displacement amplitudes, and material properties contacting surfaces. In this example, tube degradations may be measured in terms of volume of material lost and then correlated with the stress variables. Experimental degradation data from accelerated testing would be needed to determine the PoF-based correlation between degradation and causal stress variables. In some cases, well-established correlations from the literature are used. In developing PoF models, other important variables (such as geometry) may also be considered. The next step is to characterize all forms of uncertainties associated with the PoF models and data, and estimate model parameters. This step converts the PoF models into PPoF models. A suitable regression approach should be developed to formally characterize all uncertainties. Bayesian regression is a powerful technique for estimating probability distributions of model parameters. In the tube example above, one requires experimental degradation data under prevailing environments experiencing operational conditions corresponding to each degradation mechanism.

Other factors that can lead to uncertainties in the likelihood of failures (such as manufacturing methods and material properties) also need to be considered. Most of these uncertainties should be accounted for when evaluating the stress agents acting on the tubes. Flow-induced vibratory stresses and thermal stresses that propagate tube degradations are two such examples for the tube example above. Each failure mechanism has specific stress agents that cause degradation. Agents like fatigue stress are alternating stresses, whereas stress agents of stress-corrosion-cracking (SSC) involve constant tensile

stresses. In order to determine the stress agents, a detailed finite element analysis of the tube geometry and material properties is required, with prevailing operating conditions applied. The input parameters for the finite element analysis, (e.g., geometry, material properties) need to be entered probabilistically—not deterministically—with corresponding stresses estimated as probability distributions.

A Monte Carlo simulation approach complements PPOF models by propagating all the associated uncertainties (such as those associated with the model, its parameters, and initial material flaws) to estimate probability distribution of the unit failure or amount of damage as a function of time under the prevailing stresses. Monte Carlo simulation is the leading method for simulating systems with many coupled input variables. Appropriate failure criteria need to be defined for each failure mechanism considering the operability. For example, a failure criterion for a normal operating condition-induced fatigue mechanism can be defined as the through-wall cracks reaching the wall thickness of tubes.

**1.5. ACCELERATED TESTING IN PPOF MODEL DEVELOPMENT**

To develop the PoF models and estimate their parameters and model uncertainties, it is imperative to rely on failure and degradation (damage) versus time data. These data can be obtained from life and degradation testing or from valid field data. Many of today's structures, systems and components are capable of operating under benign environmental stresses for thousands of hours without failure. This makes normal life (non-accelerated) testing of such equipment difficult and costly. Field data in many cases are scarce, and even when they are available it is hard to judge their uniformity and accuracy. Alternatively, accelerated testing provides far more quickly a better understanding of equipment life and degradation processes, and generates data for development of PoF and PPOF models. As such, generating degradation data in the shortest possible time can be achieved by relying on formal accelerated testing methods.

In essence, accelerated testing gathers more reliability and life information in a shorter span of time by utilizing a more severe test environment than what would otherwise be experienced under normal use conditions. Accelerated testing increases the stressors that are known to dominate the causes of failure of a system in order to test it in a compressed timeframe. Importantly, accelerated testing ensures that failure modes and mechanisms that would not be encountered under normal use are not inadvertently introduced in the test. The trajectory of cumulative degradation shown in Figure 1.3 and Figure 1.4 would by be shifted to the left under an accelerated testing regime, as failures occur faster than under normal (or use) operating conditions.

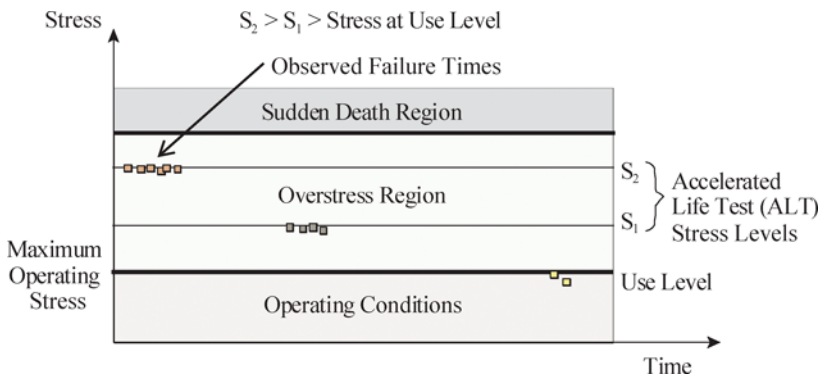


Figure 1.6: Conceptual acceleration of stress agents at two overstress conditions and corresponding data points generated from ALT

Accelerated tests are used to develop PPoF models, which in turn can be used to estimate and predict the equipment life or degradation and damage under normal operating conditions. This step in PPoF analysis underlines the importance of formally characterizing all the uncertainties in the PPoF models so as to reflect such uncertainties in the predicted life from such models.

Figure 1.6 illustrates the stress regions of a conceptual accelerated test that generate several failure data points at two stress levels in the “overstress” region. These data are then used to develop the PoF model that best describes them, including the associated uncertainties to extrapolate the resulting PPoF models (associated with each quantile of the life) to the “use” stress level to estimate the corresponding life distribution (see Figure 1.7).

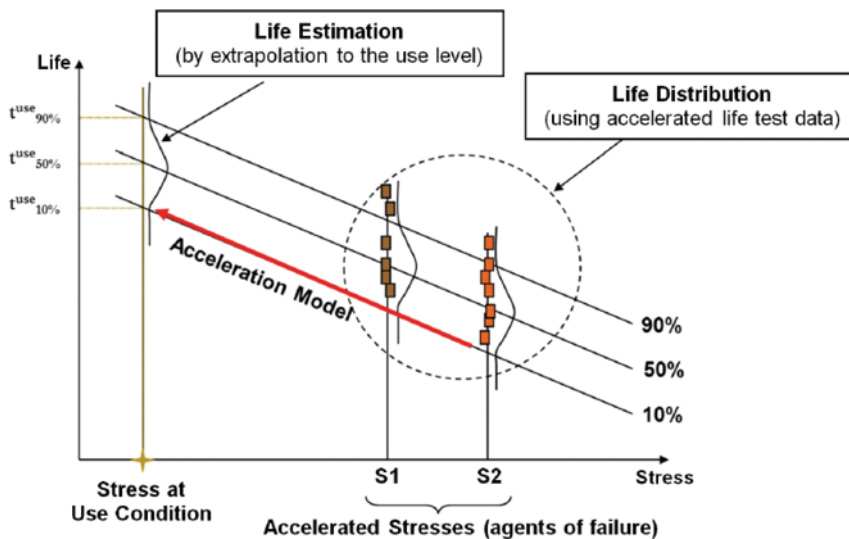


Figure 1.7: Conceptual PPoF model development and extrapolation in ALT

There are three fundamental approaches to accelerated testing: (1) field-testing of the unit under higher use frequency or higher operating stresses (loads); (2) laboratory testing of the unit, prototype, material samples or coupons under higher stress or higher use frequency; and (3) computer-based simulated acceleration using previously validated PPoF models. Acceleration of the stress variable is achieved either singly or in combination. Examples include:

- More frequent power cycling
- Higher vibration levels
- Higher humidity
- More severe temperature cycling
- Higher temperatures
- Higher load amplitudes

In addition to developing PoF and PPoF models, there are other motivations for accelerated testing to assure reliability. These motivations stem from, for example:

- The need to identify design failures. Results from accelerated tests can help eliminate or reduce design failures through redesign (e.g. intrinsic redundancy).
- The need for immediate verification of lifetime statistics (rather than waiting for field data). This is especially important for cases involving the prediction of performance of highly reliable

products where access to normal use failure data is not feasible or practical.

- The need for shorter life tests due to the fast technological evolution of products.
- The need to assess and demonstrate component reliability in the design stage.
- The need to certify components and detect failure modes so that they can be corrected.
- The need to compare different manufacturers and vendors in a condensed timeframe. Accelerated testing can aid in choosing designs, components, suppliers, rated operating conditions, and test procedures.
- The need to determine an appropriate service policy in a condensed timeframe. Accelerated testing can aid in determining the most appropriate approach to inspecting, servicing and replacing units.

There are two basic categories of accelerated tests: quantitative tests and qualitative tests. The former commonly refers to Accelerated Life Tests (ALT) and Accelerated Degradation Tests (ADT). The latter is characterized by tests that aim to enhance the reliability of the item during design and operation.

Quantitative tests, are conducted on items (structures, systems, components) and manufacturing processes. They can take a few weeks to a few months to complete. ALT is fundamentally based on the assumption that the unit under test will exhibit the same behavior under a shorter time frame (at a high stress) as it would in a longer time frame at use stress conditions. Hence, there are several important planning considerations when conducting ALT tests so that this assumption remains valid.

Qualitative accelerated tests are designed to find failures linked to design or manufacturing without providing any life or damage characteristics associated with the items. By accelerating failures of structures, components or systems, these tests can determine the robustness of the unit in its useful life. When a failure occurs during a qualitative accelerated test, one needs to determine the root cause of the failure and judge whether the failure mode and mechanism observed would occur under normal use conditions. The most common type of qualitative tests is known as Highly Accelerated Life Testing (HALT). Being a qualitative accelerated test, it is important to note that HALT is not a life test: its purpose is not to determine life characteristics. Rather, it is a test designed to promote the occurrence of failure modes (mechanical or electronic) that will occur during the life of the product under normal use conditions. HALT provides valuable information to determine design weaknesses as well as the product's upper and lower destruct limits.

Another example of a qualitative accelerated test is known as Highly Accelerated Stress Screening (HASS). HASS tests are applied during the manufacturing phase and are used to screen marginal and defective units. HASS can expose infant mortality failures and other latent defects that would otherwise occur when the unit is being used. Whereas HALT is applied during the design phase to iron out potential design issues, HASS screens out defects (that can be fed back to designers) associated with the manufacturing process. HASS generally builds upon HALT but typically uses lower stresses.

### 1.6. ORGANIZATION OF THE BOOK

This book can be divided into three parts. Part one (chapters 1 and 2) introduces the PPOF approach more formally and provides a comprehensive coverage of key failure mechanisms, including their corresponding PoF models. Part two (chapters 3 to 6) focus on accelerated testing. Types of accelerated tests, accelerated degradation modeling, test planning, data gathering, PPOF model development, characterization and analysis methods will be covered. Part three (chapter 7) focuses on uncertainties in physics-based models in reliability and prognosis and health management.

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# Chapter 2: Summary of Mechanisms of Failure and Associated PoF Models

## 2.1. INTRODUCTION

Chapter 1 provided an overview and the motivation behind accelerated testing. As discussed, one of the key elements required in order to successfully conduct an accelerated life test is determining the most suitable accelerated life model (which is contingent on the most suitable PoF model), upon which the test can be based. The selection of the most appropriate accelerated life model is directly influenced by the mechanism of failure that is being considered for the applicable material. For this reason, the study of the PoF is pertinent in any accelerated life testing analysis. This chapter will discuss important failure mechanisms that illustrate the process of deriving the relationship between life and applied stresses from the physics of the failure of the mechanism. Some of the mechanical, thermal, electrochemical and electrical failure mechanisms include:

- Fatigue cracking
- Creep
- Pitting corrosion
- General corrosion
- Crevice corrosion
- Erosion
- Wear
- Radiation embrittlement
- Hydrogen embrittlement
- Electromigration
- Conductive filament formation
- Thermally-induced fatigue: cyclic creep-fatigue
- Fatigue induced by vibration
- Fracture induced by shock & drop
- Fretting-corrosion in connectors induced by vibration

Also, a few synergistic failure mechanisms are:

- Creep-fatigue
- Corrosion-fatigue
- Fretting-wear

Classification of material degradation by its root cause is the first step in any plan to protect a specific material in a specific situation. A classification of the known forms of material degradation as physical, chemical or biological phenomena is illustrated schematically in Figure 2.1 as a Venn diagram.

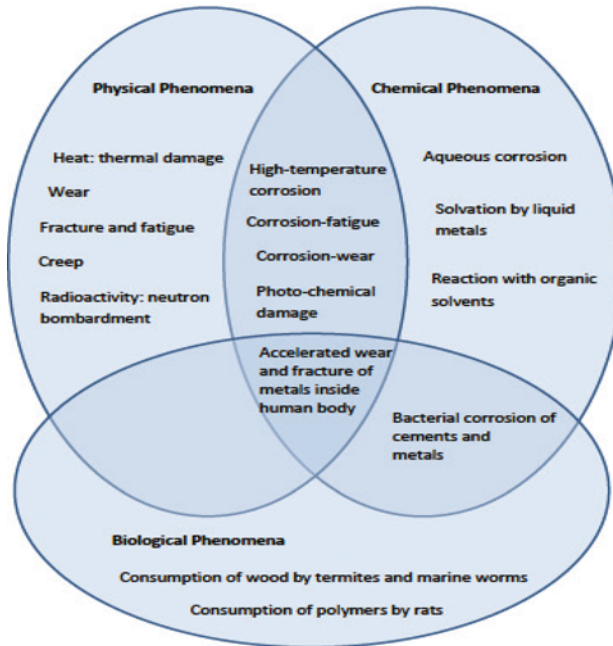


Figure 2.1: Classification of material degradation

For all materials (including electronics) there are three basic categories of material degradation: those that are (1) physical, (2) chemical and (3) biological in origin. A physical origin is one that involves the effect of force, heat or radiation. A chemical origin involves the destructive reactions between material and chemicals that contact it. A biological origin includes all interactions between life forms and engineering materials.

Physical and chemical material degradation (such as thermal damage or destructive chemical reactions) coexist with combined forms of material degradation (such as corrosive-wear). Environmental conditions strongly affect material degradation, meaning that any material degradation problem depends on locality. For example, a component operating in a corrosive environment is susceptible to corrosion, and a component that is inside of a living organism is more subject to biological damage. As will be shown in later chapters, there is a wide range of possible interactions or synergy between degradation processes, and considerable care is needed to predict which of these interactions are significant to any given situation.

An example of interaction between degradation processes is corrosive-wear, which is mechanical wear accelerated by chemical damage to the worn material. Biological phenomena, which can cause this corrosion, are either found in a pure form (such as organisms eating artificial materials) or in more complex relationships (such as bacterial corrosion where bacterial waste products are destructive to materials).

Damage that depends on just one phenomenon is easier to recognize than multi-phenomena damage such as those occurring in corrosive-wear. Damage measures are usually ineffective when dealing with multi-phenomena. In this book, single-phenomenon material degradation is described before more complex forms of degradation are discussed.

The PoF approach to ALT aims to use the science of physics to create analytical models to describe the failure of components and devices. In essence, the PoF methodology explains why and when a

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